

The Effects of Contracting and Labor Search on Risks in Financial Markets

by

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Abstract

My research employs theoretical modeling and quantitative analysis to study the interaction between market frictions and risks in the areas of Banking and Asset Pricing. Specifically, I examine how frictions in contracting and search affect systemic risk and asset prices.

Sparked by the recent crisis, linkages among financial firms are identified as a major source of systemic risk. In chapter one, “**Distress Dispersion and Systemic Risk in Networks**,” I present a model in which the cross-sectional dispersion of financial distress endogenously generates inefficiencies in network formation, creating excessive systemic risk. Financial firms face costly liquidation and strategically trade assets, thereby forming links. A link with a distressed firm can be socially costly as it increases system-wide liquidation risk. The model reveals that, when the dispersion of distress is high, the *network composition* is distorted in two ways: there are too many links with distressed firms and too few risk sharing links among non-distressed firms. The inefficiency arises from an externality due to contract incompleteness in the bilateral trades. Using insights from the model, I discuss policy implications for financial stability. I also show empirical evidence that the distress dispersion across financial firms provides a novel indicator for systemic risk.

Similar to the financial system, the interactions between frictions and risks also apply to the labor market. In chapter two, “**Asset Pricing with Dynamic Labor Contracts**,” I study asset prices in a two-agent production economy in which the worker has private information about her labor productivity. The shareholder offers an incentive compatible long-term labor contract, which partially insures the worker against labor income risk. I compare the model’s performance to settings with a competitive labor market, and with static labor contracts. My model successfully matches both asset returns data and business-cycle features, including a countercyclical and high equity premium, a low risk-free rate, procyclical labor input, and countercyclical labor share. The results highlight that the dynamic contracting feature in labor relations is quantitatively important in determining asset prices.

Risk allocation implied by labor market frictions also affects asset prices at the cross section. In the data, sorting firms according to their loadings on the aggregate vacancy-unemployment ratio, defined as the labor market tightness, generates a spread in future returns of 6% annually. To rationalize the finding, in chapter three, we propose “**A Labor Capital Asset Pricing Model**” (joint with Lars-Alexander Kuehn and Mikhail Simutin) and show that labor search frictions are an important determinant of the cross section of equity returns. In this partial equilibrium labor market model, heterogeneous firms make dynamic employment decisions facing labor search frictions. The insight is that loadings on labor market tightness proxy for priced time variation in the efficiency of the aggregate matching technology. Firms with low loadings are more exposed to adverse matching efficiency shocks and require higher expected stock returns.

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Chapter 1

Distress Dispersion and Systemic Risk in Networks

1 Introduction

The interconnectedness of financial institutions is a key feature of the modern financial system. Linkages are formed by a diverse range of transactions and contracts that connect firms to each other. A growing literature identifies these linkages as a major source of systemic risk (e.g. [Allen and Gale \(2000\)](#), [Caballero and Simsek \(2013\)](#), [Brunnermeier \(2009\)](#), and [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#)). The insights are evident in the financial crisis: initial losses caused the financial distress of a few firms, which then spread via the links that connect the distressed firms with otherwise healthy ones, resulting in systemic failures. Yet, these studies analyze contagion in given network structures and do not consider firms' strategic formation of links.

In this paper, I focus on endogenous linkage formation which allows firms to strategically build connections for profit and risk diversification purposes. A recent literature examines linkage formation among homogeneous firms and concludes that either over- or under-connections prevail in the financial system (e.g. [Castiglionesi and Navarro \(2011\)](#) and [Farboodi \(2014\)](#)).¹ In contrast, this paper studies the linkage formation among firms differing in financial distress levels. Such framework provides novel implications for efficiency and systemic risk by generating over- and under-connections simultaneously.

I show that the endogenously formed network features inefficiencies and leads to systemic risk as measured by the probability of joint failures. A link between two non-distressed firms creates gains from risk-sharing, whereas a link with a distressed firm can be socially costly as it increases systemic risk through balance sheet interdependence. I find that, when the dispersion of distress is high, the *network composition* is distorted in two ways: there are too many links with distressed firms and too few risk-sharing links among non-distressed firms. The inefficiency arises as firms write bilateral contracts that are not contingent on the entire network structure.

¹For example, [Castiglionesi and Navarro \(2011\)](#) show that decentralized network is under-connected when counterparty risk is high. [Farboodi \(2014\)](#) illustrates over-connection in an endogenous core-periphery network.

Hence, the non-distressed firms have incentives to link with distressed firms for profit, while failing to internalize negative spillovers. Such inefficient network generates contagion and loss in risk-sharing, creating excessive systemic risk. By embedding heterogeneity as a new dimension of links, my model provides unique predictions on the efficiency of network composition.

In my model, financial firms face costly liquidation risks and strategically trade assets, thereby forming a network. There are a finite number of firms financed by short-term debt and each invests in a long-term asset. A random fraction of the asset is liquid and can be used to repay debt. As in [Allen, Babus, and Carletti \(2012\)](#), if the amount of liquid asset falls short of the debt level, a costly liquidation is triggered.² To hedge the idiosyncratic liquidation risk, firms can strategically enter into bilateral forward contracts to trade liquid assets. A two-sided link in a network is formed when both parties decide to purchase a fraction of each other’s liquid asset claims. Firms *differ ex ante* in how liquid the asset is expected to be, which generates the key feature of the model: cross-sectional heterogeneity in financial distress levels. Differences in asset liquidity also implies a price of trade in each contract. Motivated by the incomplete contracts literature, I assume that prices in the bilateral trades are not contingent on the entire network structure. Specifically, I consider *local contingency*, that is, prices are contingent on which firms the two parties directly trade with. Given the network formed, the liquid asset holding of a firm depends not only on who its direct counterparties are, but rather on the entire network structure.³ As a benchmark for efficiency, I solve for the optimal network that minimizes total bank liquidations.

The pairwise stable network formed in equilibrium can be inefficient relative to the optimal benchmark: there can be excess links with distressed firms and insufficient risk-sharing links among non-distressed firms. When distress dispersion is high across firms, the optimal network requires that the non-distressed firms form risk-sharing links and that the most distressed firm be isolated. In comparison, the equilibrium network with four or more firms shows that the distressed firm is always connected with the most liquid firm. The suboptimal link between the liquid and the distressed firm (“*distress link*” hereafter) transmits risky assets in the network and leads to systemic risk, measured by the probability that all firms fail at the same time.

The inefficiency is caused by network externalities. Linking with a distressed firm potentially avoids liquidation, thus is *ex ante* profitable for the most liquid firm. However, when a firm is too distressed, linking with it can be socially costly because it contaminates the balance sheets of others in the network. Hence a liquid firm forming a distress link imposes a network externality, as distressed assets are then shared jointly by all connected firms. The distress link increases the risk of contagion, which in turn reduces risk-sharing participation among non-distressed firms. As such, two forces reinforce and lead to inefficiency: the transmission of distressed assets that should have been isolated and the insufficient risk-sharing among non-distressed firms.

²A firm with a low level of liquid asset has difficulty in repaying short-term debt and hence is *distressed*.

³Following [Cabrales, Gottardi, and Vega-Redondo \(2014\)](#), I model this balance sheet interdependence as an iterative swap process which represents asset securitization.

The necessary ingredients for the externalities are interconnectedness, distress heterogeneity, and local contingency. Interconnectedness transmits risky assets, thereby enabling spillovers. Firm heterogeneity generates distress dispersion and different incentives to form links. When there are only two firms or multiple identical firms, there is no externality. However, when there are trades between multiple firms differing in distress levels, the most liquid firm can profit from trading with the distressed firm and can shift risks away to its direct and indirect counterparties; hence, the most liquid firm has a greater incentive to link with the distressed firm than is socially desirable. But interconnectedness and heterogeneity are not enough. The externalities are not internalized because of local contingency. Firms that bear the externalities cannot jointly give incentives to the liquid firm via contingent payments. This occurs as long as one of the indirect counterparties of the most liquid firm cannot condition payments on the distress link. Thus, the liquid firm fails to internalize negative spillovers and forms the inefficient distress link.

While the prior literature largely focuses on the average soundness of the financial sector,⁴ my second primary result identifies a novel indicator for the level of network inefficiency: the distress dispersion across financial firms. In my model, inefficiency arises when the distress dispersion is sufficiently high and increases with the level of dispersion thereafter. This positive relation is due to changes in network composition. When distress dispersion is higher, a wider cross-sectional distribution implies more distressed firms in the left tail and more liquid ones in the right tail. It is precisely then that the most liquid firm has an incentive to form the socially costly distress link. Hence the disparity between individual and social incentives for forming a distress link is greater, which crowds out valuable risk-sharing links and increases inefficiency.

Using insights from the model, I discuss policy implications for financial stability. The links with distressed firms in the model can be interpreted as acquisitions of distressed firms. This interpretation is reasonable because distressed financial firms are commonly acquired by healthier institutions in the same industry.⁵ More than 1000 distressed financial firms were acquired during 2000-2013, including Countrywide Financial Corp. and Riggs Bank. The asset size of these acquisitions was \$2.2 trillion, about half the size of all current banking deposits. Despite the fact that acquisitions are a prevailing regulatory approach to improve financial stability,⁶ my findings imply that excess acquisitions may emerge precisely when more banks

⁴Atkeson, Eisfeldt, and Weill (2014) measure the median Distance to Insolvency of largest financial firms based on the Leland’s model of credit risk. Rampini and Viswanathan (2014) argue that the net worth of (representative) financial intermediaries is an important state variable affecting the cost of financing. Gilchrist and Zakrajsek (2012) show that the average credit spreads on outstanding corporate bonds has predictive power for economic activity.

⁵Acharya, Shin, and Yorulmazer (2010) argue that if a bank needs to restructure or be sold, the potential buyers are generally other banks. Almeida, Campello, and Hackbarth (2011) document that distressed firms are acquired by liquid firms in their industries for financial synergies. Such acquisitions are more likely when industry-level asset specificity is high and firm-level asset specificity is low, which applies to the financial sector.

⁶White and Yorulmazer (2014) provide a summary of resolution options for bank distress/failure. An acquisition “imposes the least cost since the franchise value is preserved, there is no disruption to the bank’s customers or the payment system itself, and there are no fiscal costs.” For this reason, acquisition is the primary choice by resolution authorities whenever there are willing acquirers.

are distressed, thus increasing systemic risk rather than reducing failures.

In the context of acquisitions of distressed firms, I show an acquisition tax that varies with the distress dispersion can prevent the inefficient acquisitions and reduce total liquidation costs. Based on this result, regulators can restore efficiency by supervising the acquisitions of distressed firms and using the purchase and assumption (P&A) method for distress resolution. In a model extension that allows for the analysis of *ex post* policies, I show that if the excess acquisitions are not banned *ex ante*, the too-connected-to-fail problem arises. In such a scenario, government bailout or subsidized acquisitions are *ex post* optimal remedies, thereby rationalizing the government interventions observed during the recent financial crisis.

Finally, I provide empirical evidence that the distress dispersion across financial institutions provides a novel indicator for systemic risk. Following [Laeven and Levine \(2009\)](#), I measure distress by estimating Z-scores of financial firms. The time series of distress dispersion displays large variations over time. Moreover, it has a countercyclical pattern and appears to lead recessions. Consistent with the model predictions, the empirical dispersion series significantly comoves with future economic activities and systemic risk, bank failures, acquisitions of distressed firms, and interbank risk sharing. I run forecasting regressions to evaluate whether the dispersion series conveys new information about aggregate indicators beyond what is contained in the average distress and existing systemic risk measures. The estimates confirm that the dispersion series has high predictive power for future indices of systemic risk.

1.1 Related Literature

This paper builds on network theory and its applications in economics and finance.⁷ Pioneered by [Allen and Gale \(2000\)](#), a growing literature argues that certain network structures among financial institutions can lead to risks of contagion.⁸ While powerful for analyzing how risks propagate under different connection properties, this stream of research treats the network structures as given. My paper studies network formation, hence contributes to the analysis of how links evolve in response to changes in policies or aggregate conditions.

The main contribution of this paper is to embed distress heterogeneity in linkage formation and to study the implications on efficiency and systemic risk. As such, my paper belongs to the recent literature on financial network formation, which examines how inefficient networks form due to various frictions.⁹ [Castiglionesi and Navarro \(2011\)](#) demonstrate network fragility when undercapitalized banks gamble with depositors' money. [Di Maggio and Tahbaz-Salehi \(2014\)](#) analyze the role of collateral on a similar moral hazard problem. [Zawadowski \(2013\)](#) studies a

⁷See surveys by [Jackson \(2003, 2008\)](#) and [Allen and Babus \(2009\)](#).

⁸See [Eisenberg and Noe \(2001\)](#), [Dasgupta \(2004\)](#), [Nier, Yang, Yorulmazer, and Alentorn \(2007\)](#), [Gai, Haldane, and Kapadia \(2011\)](#), [Greenwood, Landier, and Thesmar \(2012\)](#), [Caballero and Simsek \(2013\)](#), [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#), [Elliott, Golub, and Jackson \(2014\)](#), and [Glasserman and Young \(2015\)](#).

⁹See for example [Lagunoff and Schreft \(2001\)](#), [Castiglionesi and Navarro \(2011\)](#), [Gofman \(2011\)](#), [Babus \(2013\)](#), [Blume, Easley, Kleinberg, Kleinberg, and Tardos \(2013\)](#), [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2014\)](#), [Zawadowski \(2013\)](#), [Cabrales, Gottardi, and Vega-Redondo \(2014\)](#), and [Farboodi \(2014\)](#).

type of risk shifting stemming from banks' underinsurance of counterparty risk. [Gofman \(2011\)](#) and [Farboodi \(2014\)](#) highlight that bargaining friction and intermediation lead to welfare loss.

In the network formation literature, my paper is closest to [Farboodi \(2014\)](#) who illustrates that a core-periphery intermediation structure arises inefficiently due to a lending constraint and the opportunity to earn intermediation spreads. While my paper also generates excessive systemic risk due to certain types of inefficient links, I differ by studying linkage formation among firms differing in financial distress levels. Inefficiency arises from the incentive of liquid firms to link with distressed firms for profit under contract incompleteness. Moreover, I model links on the asset side of the balance sheet. The resulting asset cross-interdependence structure can be used to evaluate acquisition regulations. Finally, the novel finding that the distress dispersion is a critical state variable allows for a closer link to the data in forecasting systemic risk.

The key friction underlying the network inefficiency in my model is the failure to offer incentives conditional on the entire network structure. In this sense, my paper is related to the literature on incomplete contracts.¹⁰ From [Hart and Moore \(1988\)](#), agents cannot write contracts contingent on states that cannot be clearly specified, even if the states are perfectly foreseeable. The reason is that the states written in the contracts must be verifiable in court. In my setting, given that the links entered by other firms are not specifiable or verifiable, bilateral prices are contingent only on who the two firms directly trade with. This assumption is in line with [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2014\)](#) who show that inefficient networks can emerge in interbank lending markets with contingency debt covenants.

Finally, this paper adds to the studies on the trade-off between diversification and contagion. [Banal-Estanol, Ottaviani, and Winton \(2013\)](#) evaluate conglomeration with default costs in terms of this trade-off. I follow [Cabrales, Gottardi, and Vega-Redondo \(2014\)](#) and study the trade-off in a network setting. [Acharya \(2009\)](#), [Wagner \(2010\)](#), [Ibragimov, Jaffee, and Walden \(2011\)](#), and [Castiglionesi and Wagner \(2013\)](#) show that diversification may lead to greater systemic risk as banks tend to over-diversify by holding similar portfolios. These papers mostly assume costly joint failures among homogeneous agents. My paper complements these studies by showing that links among heterogeneous firms can result in both *over* and *under* diversification.

The rest of the paper proceeds as follows. Section 2 lays out the model environment and defines the equilibrium. Section 3 demonstrates the network inefficiencies and investigates the key friction. Section 4 examines the role of distress dispersion on inefficiency. Section 5 discusses the policy implications in the context of acquisitions of distressed firms. Section 6 presents empirical results, and Section 7 concludes. All proofs are in the Appendix.

2 Model

This section describes a model of network formation in which financial firms strategically trade assets via bilateral forward swap contracts.

¹⁰See for example [Hart and Moore \(1988, 1999\)](#), [Tirole \(1999\)](#), [Maskin and Tirole \(1999\)](#), and [Segal \(1999\)](#).

2.1 Environment

Consider a four-date economy with a finite number of levered financial firms, denoted by $i = 1, \dots, N$. All agents are risk neutral and there is no discounting.

At date 0, each firm borrows 1 unit of short-term debt from a continuum of creditors and invests in an asset with fixed return R . The asset is subject to liquidity risk. A random component $\tilde{a}_i$ becomes liquid at date 2 and can be used to repay debt, whereas the rest $R - \tilde{a}_i$ is illiquid and matures at date 3. Given this financing structure, a maturity mismatch arises. A firm can be interpreted as a financial institution, e.g., an investment firm investing in a certain class of securities, or a commercial bank issuing an unsecured loan.

At date 1, firms observe the vector ν , which is a public signal about how much liquid asset each firm expects to receive. Then they simultaneously decide to enter into bilateral forward swap contracts for risk-sharing purpose, thus forming links. Each forward swap contract promises a claim to a fraction of each other's liquid assets.

At date 2, firms observe the amount of their liquid assets, given by $\tilde{a}_i = \nu_i + \sigma\varepsilon_i$. The idiosyncratic shock ε_i is i.i.d. standard normal and is independent of ν_i .¹¹ Firms fulfill the forward swap contracts, and based on the overall linkage structure, firms obtain potentially diversified liquid asset holdings, which they use to repay debt.¹² If the liquid asset holdings fall short of debt, the firm liquidates its illiquid asset with a fixed cost c , for instance by selling at a discount to industry outsiders as in Shleifer and Vishny (1992).¹³

At date 3, if not liquidated, the illiquid component $R - \tilde{a}_i$ of the asset matures. Using this return, the payments associated with the forward swap contracts are paid in full.

Given the signal ν , firms differ at date 1 in the amount of expected liquid asset. This generates heterogeneity in financial distress. I follow Roy (1952) and define a distress statistic, z_i , as the number of standard deviations that firm i is expected to be away from liquidation ($z_i \equiv \frac{\nu_i - 1}{\sigma}$). A firm with high z_i has highly liquid asset and low financial distress. We say such a firm is *liquid*. In contrast, a firm is *distressed* if it has a low z_i . To highlight the role of heterogeneity, I assume that the vector z has mean $\bar{z}$ and is equally spaced with step size $\delta \geq 0$, i.e.

$$z_i = \bar{z} + \frac{N + 1 - 2i}{2}\delta, \quad i = 1, \dots, N. \quad (1.1)$$

$\bar{z}$ measures the average distance from liquidation. Let $\bar{z} > 0$ so that firms invest in positive NPV projects on average. δ is proportional to the cross-sectional standard deviation of z_i and proxies for the degree of distress dispersion.¹⁴

¹¹ $\tilde{a}_i$ being negative means further liquidity inflow needed in the asset investment.

¹²Introducing debt roll-over, renegotiation, or endogenous default boundary do not change the qualitative features. To separate from risk-shifting due to agency conflict between shareholders and depositors (Jensen and Meckling (1976)), limited liability is not particularly imposed for firm owners.

¹³The cost can result from deadweight loss in liquidation due to asset specificity, loss of franchise value, or disruption of credit and payment services associated with relationship banking (see White and Yorulmazer (2014)).

¹⁴I rank firms by z_i merely for expository purpose. Distress is modeled as exogenous, while in reality firms choose liquidity holding and risk-taking which endogenously determine distress levels. Acharya, Shin, and Yorulmazer

2.2 Network Formation

At date 1, firms strategically decide to enter into bilateral forward swap contracts. In this network formation game, a *strategy* of firm i includes a vector $l_i = (l_{i1}, \dots, l_{i,i-1}, l_{i,i+1}, \dots, l_{iN})$ and a vector $p_i = (p_{i1}, \dots, p_{i,i-1}, p_{i,i+1}, \dots, p_{iN})$. Firm i proposes to buy $l_{ij} \in \{0, \bar{l}\}$,¹⁵ where $\bar{l} \in (0, 1)$, fractions of liquid asset from firm j at date 2, offering to pay a unit price p_{ij} at date 3. The prices can be made contingent on the links. Similar to the simultaneous announcement game in Myerson (1991), each firm simultaneously proposes to contract with other firms.

A contract is signed (a two-sided link is formed) when both firms decide to swap asset claims at the offered prices. Let the matrix L represent the linkage structure; its element satisfies

$$L_{ij} = L_{ji} = \min\{l_{ij}, l_{ji}\}. \quad (1.2)$$

Firms i and j are directly linked ($L_{ij} = \bar{l}$) only if $l_{ij} = l_{ji} = \bar{l}$. This specification ensures that no firms end up being a net asset seller or buyer so each firm still holds one unit of liquid asset. It also captures an important aspect of the OTC derivatives market: firms have large gross notional positions and small net positions. After the asset swaps, each firm holds a non-negative share of its own asset, i.e. $L_{ii} = 1 - \sum_{j \neq i} L_{ij} \geq 0$. As such, L is a symmetric, doubly stochastic matrix by construction.¹⁶ When $L_{ii} = 1$, firm i is isolated.

The set of N firms and the links between them define the network. Depending on the distress level of the two connecting firms, the network is composed of *risk-sharing links* which connect two non-distressed firms, and *distress links* which connect a liquid and a distressed firm.

2.3 Payoffs and Firm Value

Firms' liquid asset holdings, denoted by vector $\tilde{h}_i(\tilde{a}, L)$, depend on not only their direct counterparties, but rather how firms are interconnected. As such, the linkage creates cross-interdependence from the asset side of firms' balance sheets. I model links via asset swaps because prior studies highlight that correlated portfolio exposures are the main source of systemic risk in the financial sector.¹⁷ In addition, asset swaps simplify the calculation of final asset holdings and systemic risk by avoiding kinks in standard cascade models (e.g. Elliott, Golub, and Jackson (2014)).¹⁸

At date 3, firms deliver payment transfers according to the forward swap contracts. Their final payoffs are thus determined by the liquid asset realizations $\tilde{a}$, the network L , and the prices

(2010) argue that liquid banks hoard cash for potential gains from asset sales. This implies that an otherwise endogenous setting would generate even bigger heterogeneity during an aggregate liquidity shortage.

¹⁵From Lemma 1, all results would remain if instead firms have a continuum strategy space, i.e. $l_{ij} \in [0, 1)$.

¹⁶A square matrix is doubly stochastic if all its entries are non-negative and the sum of the entries in each of its rows or columns is 1.

¹⁷See for example Elsinger, Lehar, and Summer (2006) and DeYoung and Torna (2013).

¹⁸The asset swaps may capture in a broad sense cross holdings of deposits in Allen and Gale (2000).

p , given by $\Pi(\tilde{a}, L, p)$,

$$\Pi_i(\tilde{a}, L, p) = \underbrace{\tilde{h}_i(\tilde{a}, L) + R - \tilde{a}_i - 1}_{\text{asset net of debt}} - \underbrace{\mathbb{1}_{(\tilde{h}_i(\tilde{a}, L) < 1)}^c}_{\text{liquidation cost}} - \underbrace{\sum_{j \neq i} (p_{ij} - p_{ji}) L_{ij}}_{\text{net payments from swaps}}, \quad (1.3)$$

Firm value at date 1 is given by taking the expectation of $\Pi_i(\tilde{a}, L, p)$,

$$V_i(z, L, p) = \mathbb{E}_1 \left[\tilde{h}_i(\tilde{a}, L) \right] + R - \nu_i - 1 - \Pr \left(\tilde{h}_i(\tilde{a}, L) < 1 \right) c - \sum_{j \neq i} (p_{ij} - p_{ji}) L_{ij}. \quad (1.4)$$

2.4 Bilateral Prices and Asset Swaps

The key features of a network formation game are the payoff functions and the payment transfers. To further specify these terms in my framework, I next discuss assumptions on bilateral prices and asset swaps process.

Local Contingency Which firms have the power to decide on a link between two firms is crucial to linkage formation. The bilateral prices allow for transfer payments among firms, which in turn define the decision power to form links. Given that a link L_{ij} “alters the payoffs to others, it seems reasonable to suppose that other firms, especially the [direct counterparties of] firms i and j should have some say in the formation of a link between i and j ” (Goyal (2009)). Following this spirit, I assume prices with local contingency.

Assumption 1 (Local Contingency) *The bilateral price p_{ij} is contingent on the direct links entered by the two firms. Let L_i be the i -th row of L , then*

$$p_{ij}(L_i, L_j, L_k) = p_{ij}(L_i, L_j, \hat{L}_k), \quad \forall k, \forall \hat{L}_k \neq L_k. \quad (1.5)$$

Under Assumption 1, firm i offers prices based on its own links L_i and the links of its direct counterparty L_j . Even if firm i foresees that it indirectly connects to a third firm k ($L_{ij} > 0$, $L_{jk} > 0$), the price it offers cannot vary with the links of firm k .

Assumption 1 is the key friction in the model. The motivation lies in an inherent feature of the financial industry: when firms write bilateral contracts in an interconnected setting, it is difficult for institutions to specify in every contract detailed contingencies for every possible network structure. One reason for this is that institutions do not publicly disclose the identities of their counterparties. As in Hart (1993), even if the bilateral relations they form could be foreseeable by other institutions, “they might be difficult to specify in advance in an unambiguous manner. [Hence], a contract that tries to condition on these variables may not be enforceable by a court.” This is essentially one example of *incomplete contracts*.¹⁹

¹⁹An alternative motivation relates to transaction costs à la Williamson (1975). As the size and complexity of the network builds up, it would be prohibitively costly to include all possible structures in each contract for every firm. This is consistent with the fact that we do not observe such types of contracts in practice.

Price Offering Rules In each bilateral contract, what matters for firm payoffs is the net transfer payment $(p_{ij} - p_{ji}) L_{ij}$. The same net payment can be achieved by a continuum of gross payments; hence, to ensure a unique set of equilibrium prices, I assume that buyer i proposes price p_{ij} to j as a *take-it-or-leave-it offer*. The proposed price cannot be lower than firm j 's reservation price p_{jj} . Formally,

$$p_{ij} \geq p_{jj}, \quad \forall i \neq j, \quad (1.6)$$

where p_{jj} equals j 's outside option when it cannot form any links, i.e.

$$p_{jj}(z_j) = V_j(z, L, p | L_j = 0). \quad (1.7)$$

Asset Swap Process I model the cross-interdependence of liquid asset holdings $\tilde{h}(\tilde{a}, L)$ by an iterative asset swap process: firm i swaps liquid asset with its direct counterparties iteratively. Given the linkage matrix L , the vector of asset holdings after the first round of swap is $\tilde{h}^{(1)} = L\tilde{a}$. Applying L to $\tilde{h}^{(1)}$ gives the second round of swap, $\tilde{h}^{(2)} = L\tilde{h}^{(1)} = L^2\tilde{a}$, where L^2 denotes $L \times L$. Specifically, I assume that the iteration goes on for infinitely many rounds.

Assumption 2 (Iterative Swap Process) *Firms swap liquid assets according to the linkage matrix L iteratively for infinite rounds. The final asset holdings $\tilde{h}$ are given by*

$$\tilde{h}(\tilde{a}, L) = \lim_{K \rightarrow \infty} L^K \tilde{a}. \quad (1.8)$$

This iterative process is instantaneous and does not affect the payment of prices. It captures the securitization process such as the origination and trades of asset-backed securities.²⁰

Under Assumption 2, final holdings $\tilde{h}$ depend on the liquid returns of both *direct and indirect* counterparties. Take for instance a network with $N = 3$ and $L_{12} = L_{23} = \bar{l}$, $L_{13} = 0$. After the first round, $\tilde{h}_1^{(1)} = (1 - \bar{l})\tilde{a}_1 + \bar{l}\tilde{a}_2$. After infinite rounds, $\tilde{h}_1 = \tilde{h}_2 = \tilde{h}_3 = \frac{1}{3}\tilde{a}_1 + \frac{1}{3}\tilde{a}_2 + \frac{1}{3}\tilde{a}_3$; hence, firm 1 holds $\frac{1}{3}$ shares of $\tilde{a}_3$ even if it does not directly link with firm 3. The following lemma formalizes this property of the final asset holdings.

Lemma 1 (Complete risk-sharing) *For all L , $\lim_{K \rightarrow \infty} L^K$ is doubly stochastic and coincides with complete risk-sharing among all firms connected in the same component,²¹ i.e. the holdings of each firm are equally weighted by the liquid assets of all firms directly or indirectly connected to it.*

From Lemma 1, it is the linkage structure (whether $L_{ij} = 0$ or $L_{ij} > 0$) rather than the amount of swap that determines the final holdings of each firm. Given Lemma 1, the results still hold if instead $l_{ij} \in [0, 1)$, that is, if we allow firms to make linkage decisions in a continuum space.

²⁰“The possibly iterative procedure through which each firm exchanges assets on its whole array of asset holdings can be viewed as a securitization process of the firm's claims” (Cabrales, Gottardi, and Vega-Redondo (2014)).

²¹A component of a network is a maximally connected collection of firms: each firm in the component can reach any other firm in the same component following one or more links.

This rationalizes the simplification that l_{ij} is a binary variable. Moreover, the holding of own asset $L_{ii} = 1 - \sum_{j \neq i} L_{ij} \geq 0$ implies that the maximum number of links a firm can form is $1/\bar{l}$. If $1/\bar{l}$ is very large, the number of possible network structures increases exponentially with N .²² To maintain tractability, in what follows I restrict the number of links a firm can form.²³

Assumption 3 (Chain Networks) *each firm can form a maximum of two links, i.e. $\bar{l} = \frac{1}{2}$.*

Lemma 1 implies that reducing a circle to a chain structure will not affect firms' final holdings and exposures. Therefore, the possible topology for minimal networks is an arbitrary collection of paths,²⁴ or chain networks. The number of firms here can be interpreted as the largest diameter in an otherwise general network, such as the core-periphery structure empirically observed in the dealers network for municipal bonds (Li and Schrhoff (2014)) and derivative securities (Hollifield, Neklyudov, and Spatt (2014)), and theoretically analyzed in Farboodi (2014).

2.5 The Equilibrium

After observing the distress vector z , firms simultaneously choose linkage decision l and price offerings p to maximize their firm values $V(z, L, p)$. Next I formally define the equilibrium by extending the notion of *pairwise stability* in Jackson and Wolinsky (1996). I embed bilateral prices along the lines of transfer payments in Bloch and Jackson (2007).

Definition 1 *The equilibrium of a network formed by bilateral forward swap contracts is characterized by the linkage structure L^e and the set of bilateral prices p^e , such that*

- *Optimality: each firm i takes as given other firms' strategies $(l_j, p_j), \forall j \neq i$, and chooses its own strategy (l_i, p_i) to optimize its firm value, i.e.*

$$V_i(z, L^e, p^e) = \max_{(l_{ij} \in \{0, \bar{l}\}, p_{ij})_{j \neq i}} V_i(z, L, p), \quad (1.9)$$

subject to (1.2), (1.4), and constraints (1.5) - (1.8).

- *Pairwise stability: denote $L_{-\{ij\}}^e$ as the matrix L^e by deleting L_{ij}^e , and $p_{-\{ij, ji\}}^e$ as the matrix p^e by deleting p_{ij}^e and p_{ji}^e . Then $\forall L_{ij}^e > 0$ and $\forall (\hat{p}_{ij}, \hat{p}_{ji}) \neq (p_{ij}^e, p_{ji}^e)$,*

$$V_i(z, L^e, p^e) \geq V_i(z, L_{-\{ij\}}^e, L_{ij} = 0, p_{-\{ij, ji\}}^e, \hat{p}_{ij}, \hat{p}_{ji}), \quad (1.10)$$

$$V_j(z, L^e, p^e) \geq V_j(z, L_{-\{ij\}}^e, L_{ij} = 0, p_{-\{ij, ji\}}^e, \hat{p}_{ij}, \hat{p}_{ji}); \quad (1.11)$$

²²The number of possible network structures among N heterogeneous firms is $2^{\frac{N(N-1)}{2}}$.

²³A similar assumption on maximum number of links is made in Allen, Babus, and Carletti (2012).

²⁴A path in a network is a sequence of firms and links that start with firm i and end with another firm j .

and $\forall L_{ij}^e = 0$ and $\forall (\hat{p}_{ij}, \hat{p}_{ji}) \neq (p_{ij}^e, p_{ji}^e)$

$$V_i(z, L_{-\{ij\}}^e, L_{ij} = \bar{l}, p_{-\{ij, ji\}}^e, \hat{p}_{ij}, \hat{p}_{ji}) > V_i(z, L^e, p^e), \quad (1.12)$$

$$\Rightarrow V_j(z, L_{-\{ij\}}^e, L_{ij} = \bar{l}, p_{-\{ij, ji\}}^e, \hat{p}_{ij}, \hat{p}_{ji}) < V_j(z, L^e, p^e). \quad (1.13)$$

- *Feasibility:*

$$L \times \mathbb{1}_{N \times 1} = L^\top \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}. \quad (1.14)$$

The pairwise stability concept states that two firms connect only if both decide to connect and prefer no other bilateral prices; two firms do not connect only if, for all possible bilateral prices, at least one firm has no incentive to connect. Pairwise stability naturally applies to this setting as the goal here is to understand which networks are likely to arise and remain stable. Moreover, it eliminates the multiplicity of equilibrium networks due to coordination failures under the standard concept of Nash equilibrium.

2.6 Discussions

Synergy from links The two types of links, risk-sharing links and distress links, generate different sources of synergy. A risk-sharing link always generates a positive surplus by reducing the volatility of liquid assets. For example, a link between two *ex ante* identical non-distressed firms reduces the liquidation probability of each firm.²⁵ In comparison, a distress link has an extra source of synergy from the distress heterogeneity. For example, let $\nu_1 = 1.5$, $\nu_2 = 0.8$. In the forward swap contracts, firm 1 has a claim of $\frac{1}{2}\tilde{a}_2$, and vice versa. Even when $\sigma = 0$, there is gain as the liquidation of firm 2 can be avoided. The surplus from the reduction of total liquidation costs of firms i and j is shown to increase with their distress dispersion $|z_i - z_j|$.²⁶ When $z_j < -1$, the surplus is positive only if $z_i > 0 - z_j > 1$; thus, only firms that are liquid enough are able to profit from such a link.

Distress link as an acquisition relation The price offering rule and the fact that a distressed firm only has one link jointly imply that a distress link establishes an *equity ownership* relation between the liquid and the distressed firm, which can be thought of as an *acquisition*. The reason is that the distressed firm i does not enter other links,²⁷ so $\{p_{ij}, p_{ji}\}$ satisfy $p_{ij} = p_{jj}$ and $V_j(z, L, p_{ij} - p_{ji}) = p_{jj}$. V_j being fixed implies that the liquid firm i is claiming the entire surplus value from the bilateral link. In other words, firm i maximizing V_i is equivalent to maximizing $V_i + V_j$, which resembles an acquisition relation.

²⁵The total expected liquidation costs of two stand alone firms are $2\Pr(\tilde{a}_i < 1)c = 2\Phi(-z_i)c$. That of two connected firms are $2\Phi(-2z_i)c$. The total surplus equals $2(\Phi(-z_i) - \Phi(-2z_i))c > 0$.

²⁶The synergy equals the reduction of liquidation costs of the two firms $\Phi(-z_i)c + \Phi(-z_j)c - 2\Phi(-z_i - z_j)c$. The derivative of synergy with respect to $|z_i - z_j|$, holding the sum $|z_i + z_j|$ fixed, is positive.

²⁷The offered price premium $p_{ij} - p_{jj}$ endogenously responds to the outside option of firm j which is in turn determined by the linkage structure L .

Algorithm for linkage formation There are multiple ways to determine which network emerges given a set of contingent transfer payments (prices). I illustrate the following one. Under rational expectations, firms form a common belief about the equilibrium linkage structure L^b . Based on this belief, firms simultaneously submit strategies $l_i(L^b)$ and $p_i = \left(p_{ij}(z, L_i^b, L_j^b)\right)_{j \neq i}$. Given the strategies, the realized equilibrium network is consistent with the common belief, i.e. $L^e = L^b$. An alternative guess-and-verify approach is described in [Bloch and Jackson \(2007\)](#).

Existence of equilibrium The existence of the pairwise stable equilibrium in Definition 1 follows from a generalization of [Goyal \(2009\)](#) Proposition 7.1 “For any value function and any allocation function, there exists at least one pairwise stable network or a closed cycle of networks.” I refer the reader to [Goyal \(2009\)](#) for more discussion.

Payment seniority The liquid asset obtained from the forward contracts is used to pay debt at date 2, whereas the payments for the forward swap contracts are paid in full at date 3 using yields from the long-term assets. This specification assumes that short-term creditors have seniority over OTC derivative counterparties. The motivation is that derivatives seniority creates an inefficiency in risk-sharing, similar to that illustrated in [Bolton and Oehmke \(2014\)](#). Following the example above, let instead $\nu_1 = 1.2$, $\nu_2 = 0.8$. Suppose further that $\varepsilon_i = \varepsilon_2 = 0$, so there are two units of liquid asset in total. Firm 2 has to incur liquidation cost at date 2 whenever it pays a positive net payment (firm 2 is relatively more distressed) to firm 1. In comparison, when net payment is paid at date 3, both firms avoid liquidation. As such, deferring the payments to the final date helps to isolate the network externality mechanism in my model from other potential inefficiencies associated with the derivatives payments.

3 Network Inefficiency

In this section, I examine the efficiency of the equilibrium network relative to a benchmark that minimizes total liquidation costs. Results show that the equilibrium network is inefficient when the dispersion of financial distress levels is high: there are more distress links and fewer risk-sharing links. Lastly, I discuss the key friction that drives the network inefficiency.

3.1 Optimal Network

Under the model specifications for links and the asset swap process, the social planner chooses the optimal linkage structure that minimizes total liquidation costs (maximizes total bank values).

Definition 2 *The optimal network L^* minimizes total expected liquidation costs, i.e.*

$$L^* = \arg \min_{L_{ij} \in \{0, \bar{l}\}} \sum_i \Pr(\tilde{h}_i < 1) c, \quad (\text{P1})$$

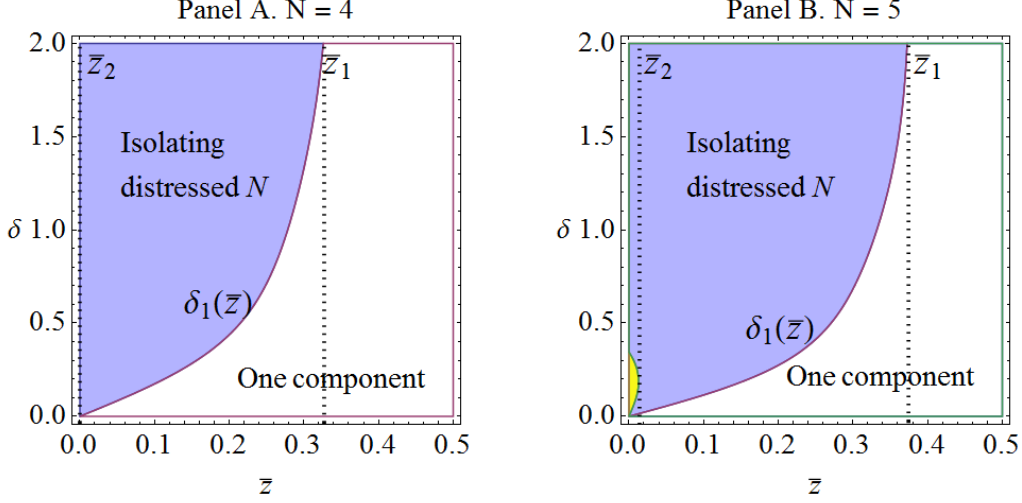


Figure 1. Optimal Network. This figure shows the optimal risk-sharing network characterized in Proposition 1 for $N = 4$ and $N = 5$. The horizontal and vertical axes represent the mean and dispersion of firm distress statistic z . In the white region, all firms are linked in one component. In the dark region ($\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z})$), firm N is isolated.

subject to the conditions of two-sided links $L_{ij} = L_{ji}$, iterative procedure (1.8), and feasibility (1.14).

Based on this definition, next I solve Problem (P1) and characterize the properties of L^* .

Proposition 1 (Optimal Network) $\exists \bar{z}_1, \bar{z}_2, \bar{z}_1 > \bar{z}_2 \geq 0, \exists$ cutoff function $\delta_1(\bar{z}) > 0$ such that

- for $\bar{z} \geq \bar{z}_1, \delta \geq 0$ or $\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta \in [0, \delta_1(\bar{z})]$, all firms are connected in one component; formally, either $L_{ij}^* > 0$ or there exists a path between i and j , i.e. $L_{ik_1}^*, \dots, L_{k_m j}^* > 0$;
- for $\bar{z} \in [\bar{z}_2, \bar{z}_1], \delta > \delta_1(\bar{z})$, the distressed firm N is isolated ($L_{NN}^* = 1$), whereas all other firms are connected in one component.

Proposition 1 states that the optimal network can be characterized by the two moments of the distress distribution, $\{\bar{z}, \delta\}$. All firms fully diversify by connecting in one component in an economy with high enough average $\bar{z}$ (low average distress), or with low $\bar{z}$ and low enough distress dispersion δ . In comparison, when distress dispersion δ is high and $\bar{z}$ is not sufficiently high, the most distressed firm N should be isolated, whereas all other firms are connected in one component. These patterns are shown in Figure 1 for $N = 4$ and $N = 5$.²⁸

The intuition for Proposition 1 is the trade-off between diversification and risks of contagion.²⁹ In an economy with high dispersion δ and low average $\bar{z}$, firm N is heavily distressed

²⁸The cutoff value $\bar{z}_2$ is zero for $N = 4$, and is positive for $N \geq 5$. For $\bar{z} < \bar{z}_2$, there are regions when L^* isolates more than one firm. For instance, in Panel B Figure 1, both firms 4 and 5 are isolated in the hump-shaped region in the lower left corner. As δ increases further, L^* switches from isolating two firms to one firm. This is because the total expected liquidity of firms 1 to $N - 1$ increases with δ which mechanically results from the assumption of symmetric cross-sectional distribution in Equation 1.1. Similar patterns display for $N > 5$.

²⁹The trade-off between risk-sharing and contagion is in line with Cabrales, Gottardi, and Vega-Redondo (2014), who find that, when shock distribution has thin tails, firms should be connected in one component, whereas when

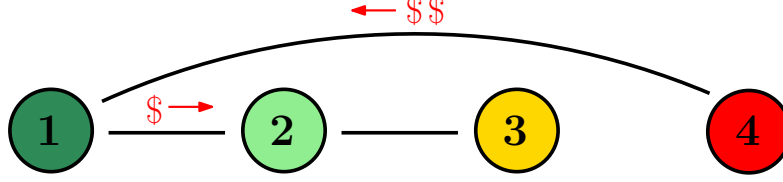


Figure 2. Equilibrium Network ($N = 4$). This figure shows the equilibrium four-firm chain network. Firms are ranked by the level of distress, and firm 4 is distressed. A solid line represents a link between two firms. A \$ arrow indicates the direction of net payment transfers via bilateral prices.

from Equation (1.1). The contamination cost of linking firm N with all other firms dominates the risk-sharing benefit, which rationalizes isolating it.

The model specifications on links and asset swaps do not deviate the optimal network from the best possible risk-sharing outcome. In Appendix A.2, I show that under the iterative swap procedure, the asset holdings implied by the optimal network, $\tilde{h}^* = (L^*)^\infty \tilde{a}$, are equivalent to the optimal allocations when the social planner directly chooses asset holdings for each firm. Hence, total liquidation costs achieve the minimum as long as the network is optimal.

3.2 Excess Distress Link

The question I address next is whether the optimal network can be decentralized in the network formation, and if not, in which ways the equilibrium network is inefficient.

Proposition 2 (Excess Distress Link) *For $N = 4$, all firms are connected in one chain in equilibrium and the distressed firm 4 is linked with the most liquid firm 1; formally, $\forall i, \bar{z}, \delta$, $L_{ii}^e < 1$ and $\forall j \neq i$, either $L_{ij}^e > 0$ or there exists a path between i and j , i.e. $L_{ik_1}^e \dots L_{k_m j}^e > 0$.*

Proposition 2 states that for all parameter values, all firms are connected in one component at equilibrium including the most distressed firm via a distress link. Comparing Propositions 1 and 2, when the average $\bar{z}$ is low and dispersion δ is high, the optimal network has no distress link ($L_{NN}^* = 1$); however, the equilibrium network features over-connection, $\sum_{i \neq N} L_{iN}^e - \sum_{i \neq N} L_{iN}^* > 0$.

The excess distress link implies that equilibrium network is inefficient.

Figure 2 illustrates the intuition. Under reservation prices $p_{ij} = p_{jj}$, firm 1 deviates to link with firm 4 to obtain a large profit (as $p_{14} = p_{44}$). Firm 2 has incentive to sever the 1 – 2 link as the cost of indirectly holding a fraction of $\tilde{a}_4$ is too high. In order to keep firm 2 staying connected, firm 1 offers a premium price p_{12} by sharing part of the profit from L_{14} . This premium price matches the value of firm 2 to the same value that firm 2 gets when it withdraws. This way, there is over-connection at equilibrium: the distressed firm 4 should have been isolated but is linked into the network. Firm 2 cannot afford to pay a premium price high enough to prevent

shock distribution has fat tails, maximum segmentation into small components is optimal.

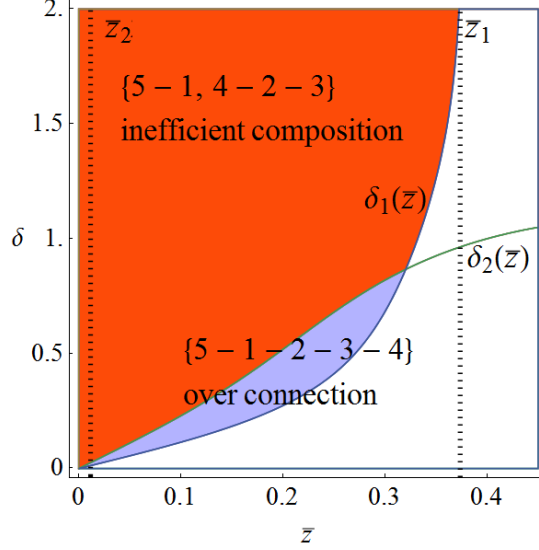


Figure 3. Equilibrium Network ($N = 5$). This figure shows the equilibrium five-firm network. The horizontal and vertical axes represent the mean and dispersion of firm distress statistic z . In colored regions, the optimal network isolates firm 5. Blue (lighter) region denotes over-connection, and orange (darker) region denotes inefficient network composition.

1 from connecting with 4. This is because the benefit of isolating $\tilde{a}_4$ is shared between firms 2 and 3, and so firm 2 would be worse-off paying the required premium fully on its own.

3.3 Risk Sharing Loss

As the chain network gets longer, the excess distress link can crowd out valuable risk-sharing links, thus giving rise to an additional channel of inefficiency from the loss of risk-sharing.

Proposition 3 (Risk Sharing Loss) *For $N = 5$, $\exists$ cutoff function $\delta_2(\bar{z})$ such that when $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta > \delta_1(\bar{z})$, there is excess distress link, $\sum_{i \neq N} L_{iN}^e - \sum_{i \neq N} L_{iN}^* > 0$. In particular,*

- *when $\delta \in [\delta_1(\bar{z}), \delta_2(\bar{z})]$, all firms are connected in one component, so there is over-connection due to the distress link;*
- *when $\delta > \max \{\delta_1(\bar{z}), \delta_2(\bar{z})\}$, the non-distressed firms are not connected in one component: the network has inefficient composition due to both excess distress link and insufficient risk-sharing.*

Proposition 3 formalizes two channels of inefficiency: one from the excess distress link (over-connection), and the other from risk-sharing loss (under-connection). When the average $\bar{z}$ is low and dispersion δ is high, the distressed firm N , which should be isolated, is linked by firm 1 at equilibrium, generating the excess distress link. This result occurs in the colored regions in Figure 3 where $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta > \delta_1(\bar{z})$. Specifically, if the value of dispersion is in a middle

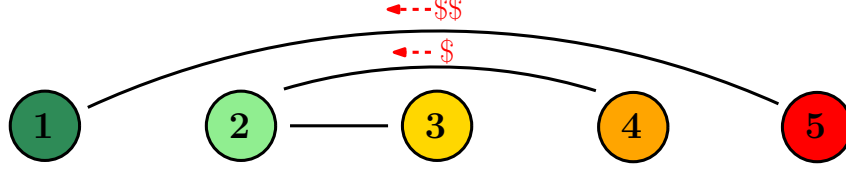


Figure 4. Inefficient Network Composition ($N = 5$). This figure shows the equilibrium connection structure $\{1 - 5, 2 - 3, 4\}$ with inefficient network composition that features both over- and under-connections.

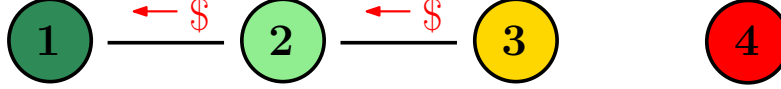


Figure 5. Complete Contingent Contracts ($N = 4$). This figure shows the optimal network is decentralized at equilibrium under complete contingent contracts. Firms are ranked by the level of distress, and firm 4 is distressed. A solid line represents a link between two firms. A \$ arrow indicates the direction of net payment transfers via bilateral prices.

range ($\delta \in [\delta_1(\bar{z}), \delta_2(\bar{z})]$), all firms are linked in one component, so inefficiency only results from over-connection. When the dispersion increases further ($\delta > \max\{\delta_1(\bar{z}), \delta_2(\bar{z})\}$), some risk-sharing link severs: a non-distressed firm becomes isolated or the non-distressed firms separate into multiple components. The externality from the distress link crowds out potential gains from risk-sharing. In this case, the inefficient network features inefficient composition featuring over- and under-connections simultaneously.

Take a $N = 5$ chain network as an example. Without loss of generality, firms start from the chain $1 - 2 - 3 - 4 - 5$. As dispersion δ increases, firm 5 becomes distressed. The $4 - 5$ link terminates and the distress link $1 - 5$ forms: equilibrium network $\overbrace{1 - 2 - 3 - 4, 5}$ generates over-connection. As δ rises further, firm 2 is worse off staying in the network: $1 - 2$ link severs and $2 - 4$ link forms, as shown in Figure 4. Notice that various initial sequences in the stable risk-sharing chain at $\delta = 0$ imply different outside options and deviation incentives for each firm. Different initial sequences therefore lead to different equilibria, all of which share the same inefficiency feature. Detailed analysis is included in Appendix A.3.

3.4 The Key Friction

The inefficiency is caused by network externalities. Due to local contingency specified in Assumption 1, the liquid firm fails to internalize the negative externalities to its direct and indirect counterparties. When Assumption 1 is relaxed, bilateral prices $p_{ij}(z, L)$ can induce the efficient network, which indicates that the incomplete contingency on the network structure is the mere underlying friction.

Recall the $N = 4$ example. When δ is high, linking with the distressed firm 4 by 1 imposes

an externality to both 2 and 3. To prevent this distress link, firms 2 and 3 need to jointly offer incentives to 1. In Appendix A.4, I formally elaborate that there exist unique premium prices p_{21}^* and p_{32}^* such that $L_{14}^e = 0$ if and only if $L_{14}^* = 0$. In particular, p_{32}^* is a function of L_{14} and firm 3 pays a premium price when $L_{14} = 0$. Put differently, the price offered by firm 3 depends not only on the links of 2 and 3, but also on the links of the counterparty's counterparty (see Figure 5).

4 The Distress Dispersion

In this section, I investigate factors that indicate the level of network inefficiency. While prior literature has largely focused on the first moment of financial distress, I show that heterogeneity in firm distress measured by the dispersion δ is a critical indicator for inefficiency. Both inefficiency indicators, value loss and systemic risk, increase with dispersion δ . Using comparative statics, I explain this positive relation by associating the network inefficiency to changes in the network composition.

4.1 Measures of Inefficiency and Dispersion

In the model, I measure network inefficiency by value loss and systemic risk. Define *value loss*, ΔV , as the difference in total expected firm values between the optimal and the equilibrium networks. Then let $\Delta V\%$ be the *percentage value loss*, which is simply the percentage of value loss over total optimal firm values.

$$\Delta V = \sum_{i=1}^N V_i(z, L^*, p^*) - \sum_{i=1}^N V_i(z, L^e, p^e); \quad \Delta V\% = \frac{\Delta V}{\sum_{i=1}^N V_i(z, L^*, p^*)}. \quad (1.15)$$

Under the feasibility condition of asset swaps in Equation (1.14), value loss equals the increment of total liquidation costs. Next, I characterize the properties of value loss as a function of the two moments of firm distress distribution, $(\bar{z}, \delta)$.

Proposition 4 (Value Loss) *Value loss decreases with average $\bar{z}$ and increases with dispersion δ . It increases with δ faster when $\bar{z}$ is lower. Formally, $\frac{\partial \Delta V}{\partial \bar{z}} \leq 0$, $\frac{\partial \Delta V}{\partial \delta} \geq 0$, and $\frac{\partial^2 \Delta V}{\partial \bar{z} \partial \delta} \leq 0$.*

From Proposition 4, value loss is bigger when the average distress is higher or when the dispersion is higher. In such scenarios, firm N is so distressed that linking it with other firms generates large contagion risk. Consequently, the cost from such a distress link causes higher loss in total firm values.

Next I explore an alternative measure for inefficiency: *systemic risk* denoted as $\Pr_{sys}^L$. It is defined as the probability that all firms liquidate at the same time. In a network where all firms are linked in one component, systemic risk equals the liquidation probability of one firm

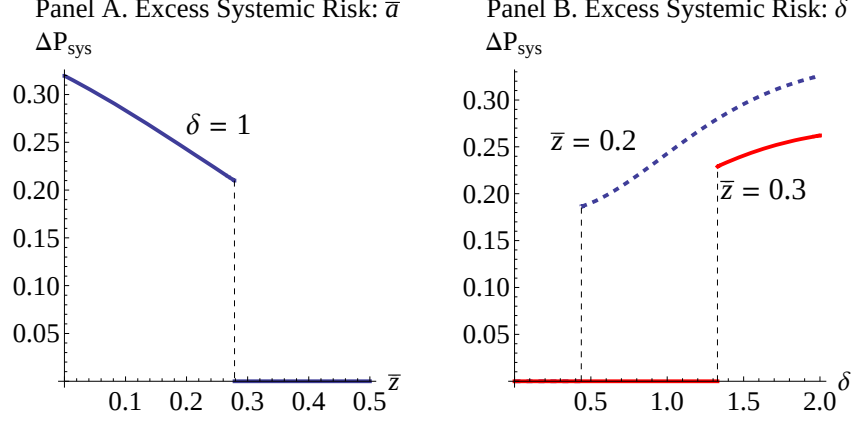


Figure 6. Excess Systemic Risk. This figure plots the excess systemic risk against average $\bar{z}$ and dispersion δ for the equilibrium four-firm chain network.

because all firms hold exactly the same diversified asset, i.e.

$$\Pr_{sys}^{\text{all connect}} = \Pr \left(\frac{1}{N} \sum_{i=1}^N \tilde{a}_i < 1 \right). \quad (1.16)$$

In a network that isolates the distressed firm, systemic risk is the probability that the isolated firm liquidates at the same time when all non-distressed firms in one connected component liquidate,

$$\Pr_{sys}^{\text{isolate N}} = \Pr \left(\frac{1}{N-1} \sum_{i=1}^{N-1} \tilde{a}_i < 1 \right) \times \Pr(\tilde{a}_N < 1). \quad (1.17)$$

Define *excess systemic risk*, $\Delta \Pr_{sys} = \Pr_{sys}^{L^e} - \Pr_{sys}^{L^*}$, i.e. the difference between systemic risk at the equilibrium network compared to the optimal network. In the example of $N = 4$, the excess systemic risk is positive whenever the network is inefficient. That is, $\Delta \Pr_{sys}(N = 4) > 0$ in the inefficient region ($\bar{z} \in [\bar{z}_2, \bar{z}_1]$, $\delta > \delta_1(\bar{z})$).³⁰

Figure 6 plots excess systemic risk as a function of the mean (Panel A) and dispersion of z (Panel B). Excess systemic risk is positive when the average distress is sufficiently high and firm distress is dispersed. $\Delta \Pr_{sys}$ decreases with $\bar{z}$; and as long as the dispersion δ is high enough, it increases with δ at a steeper rate when $\bar{z}$ is lower. The similarity of these patterns with Proposition 4 suggests that excess systemic risk serves as an alternative measure for inefficiency.

4.2 Comparative Statics: dispersion, inefficiency, and network composition

The above analysis shows that firm distress dispersion δ is a key indicator for both measures of inefficiency. To inspect the mechanism, I analyze how the equilibrium network responds to changes in $\bar{z}$ and δ , relative to the optimal network. Especially, I look at the two inefficiency measures, ΔV and $\Delta \Pr_{sys}$, together with changes in the network composition in terms of distress

³⁰For example, when $\bar{z} = 0.2$ and $\delta = 1.5$, $\Delta \Pr_{sys} = 0.34 - 0.05 = 0.29$.

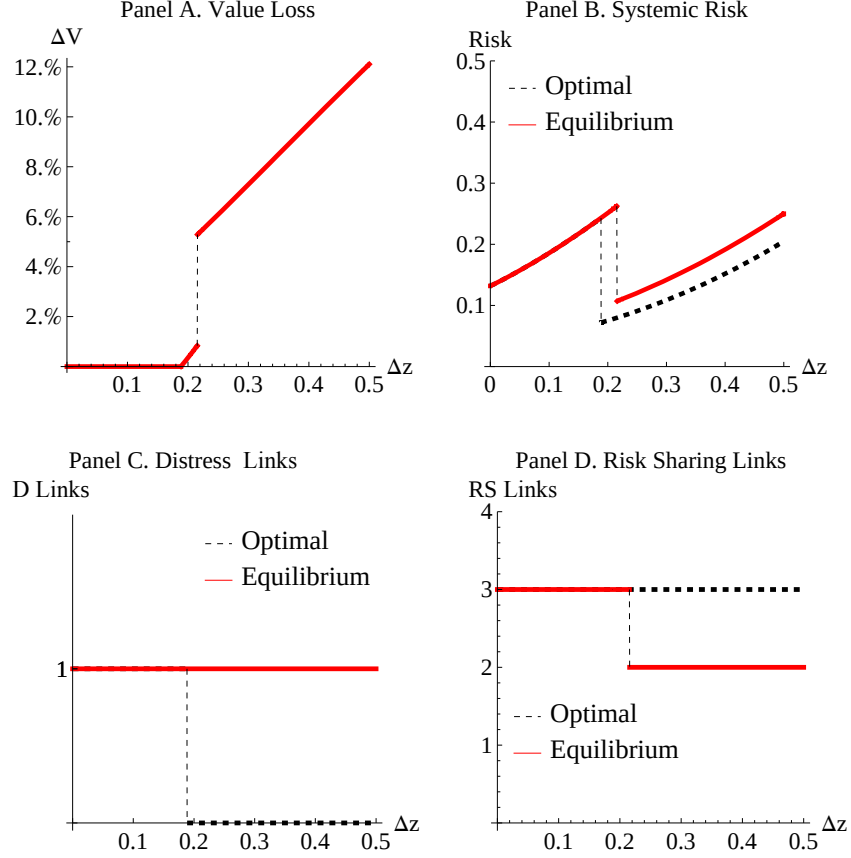


Figure 7. Increase in Average Distress under High Distress Dispersion. This figure shows the properties of the five-firm chain network with high δ when we lower $\bar{z}$. The horizontal axis $\Delta \bar{z}$ is the reduction in $\bar{z}$. I plot the values in the equilibrium network (solid) and the optimal network (dashed).

links and risk-sharing links.

In the first comparative statics, I lower the level of $\bar{z}$ in two cases when δ takes a low and a high value. When firms are similar in financial distress (δ is low), all firms linking in a single component is optimal and pairwise stable. As we lower $\bar{z}$, the optimal network remains unchanged and is also stable. Consequently, both ΔV and ΔPr_{sys} equal zero.

Results are different when firms are dispersed in financial distress (δ is high): a decrease in $\bar{z}$ affects the optimal and the equilibrium network differently. Figure 7 plots the value loss (Panel A), systemic risk (Panel B), distress links (Panel C), and risk-sharing links (Panel D) as functions of the reduction in $\bar{z}$ in a five-firm network, starting from $\delta = 1$ and $\bar{z} = 0.5$.³¹ As $\bar{z}$ reduces, both value loss ΔV and excess systemic risk ΔPr_{sys} (the difference of the solid and the dashed curves in Panel B) rise. Corresponding to where the inefficiency occurs, Panels C and D

³¹I consider a chain network 1 – 2 – 3 – 4 – 5 of which the optimal and equilibrium networks are analyzed in Subsection 3.3 and in Figure 3. In particular, I set $A = 4$, $c = 2$, so that when $\delta = 1$ and $\bar{z} = 0.5$, the average liquidation cost amounts to 8% of total firm value.

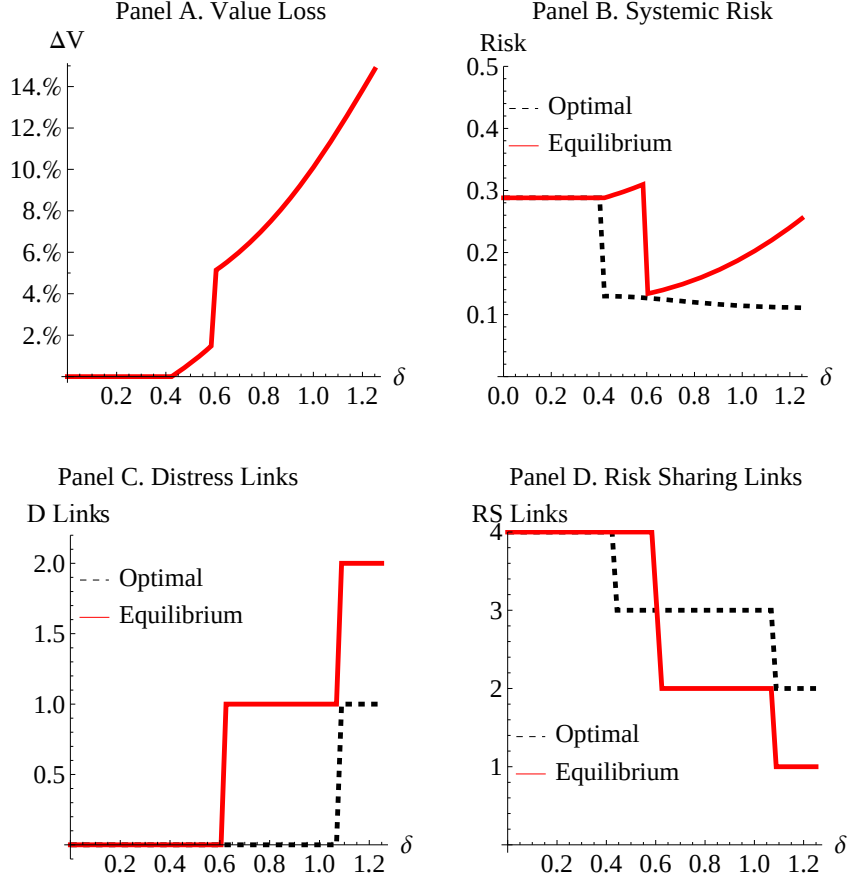


Figure 8. Increase in Dispersion. This figure shows the properties of the five-firm chain network when we raise dispersion δ while adjusting $\bar{z}$ so total firm values at L^* remain constant. I plot the values in the equilibrium network (solid) and the optimal network (dashed).

show that the equilibrium network has one extra distress link between 1 and 5, and one fewer risk-sharing link between 1 and 2. This exercise has two implications. First, comparing the two cases when δ takes a low and a high value, δ only matters to inefficiency when it is sufficiently large. Second, the observed positive relation of inefficiency and δ is associated with changes in the network composition.

In the second comparative statics, I study how the equilibrium network changes with dispersion. However, when firms form links optimally, increasing dispersion alone increases total firm values, as the total liquidation costs decrease monotonically.³² For this reason, in the following exercise, I increase δ while also adjusting $\bar{z}$ such that the total firm values in the optimal network remains constant. This allows me to conduct a “fair” comparison across states when identical firm values can possibly be achieved. Figure 8 plots the inefficiency measures and linkages of the

³²When all firms are linked in a single component, total liquidation costs equal $N\Phi\left[\sqrt{N}(-\bar{z})\right]c$, independent of δ . When the most distressed firm is optimally isolated, total liquidation costs, $(N-1)\Phi\left[\sqrt{N-1}\left(-\bar{z}-\frac{1}{2}\delta\right)\right] + \Phi\left[-\bar{z}-\frac{1-N}{2}\delta\right]$, decrease monotonically with δ . With no linkages, however, liquidation costs increase monotonically with dispersion δ as more firms are distressed.

same chain network as before. From Proposition 4, both measures of inefficiency (see in Panels A and B) increase with dispersion. When δ is large enough, inefficiency becomes positive and increases thereafter. Particularly, systemic risk at equilibrium increases with δ , except for the drop when the 1-2 risk-sharing link severs, which reduces asset correlations.

These patterns are due to over-connection at high values of dispersion and wrong network composition when dispersion gets even higher. This can be seen by comparing the number of distress links and risk-sharing links in Panels C and D. The optimal network isolates firm 5 before it becomes distressed, so the only jump on the dashed curve in Panel C is when firm 4 falls into distress. In comparison, firms 4 and 5 are always connected at equilibrium (see the two steps in the solid curve), which results in over-connection. Shown in Panel D, when δ is high, the optimal network has one more risk-sharing link than the equilibrium network (dashed minus solid curves). The severance of the 1-2 risk-sharing link implies wrong network composition, which creates an extra channel for inefficiency.

To summarize, the above two comparative statics conclude that a decrease in $\bar{z}$ when δ is high, or an increase in δ (together with a decrease in $\bar{z}$) is associated with: (1) *higher value loss and higher systemic risk*, (2) *more distress links*, (3) *fewer risk-sharing links*. In both exercises, the cross-sectional distribution of firm distress has high dispersion.

5 Policy Implications on the Acquisitions of Distressed Firms

In this section, I apply the model to the case where links with distressed firms are interpreted as acquisitions. There are two reasons for this particular application. First, in the data, a major example of the links with distressed firms is acquisitions. [Acharya, Shin, and Yorulmazer \(2010\)](#) and [Almeida, Campello, and Hackbarth \(2011\)](#) provide evidence that liquid firms acquire distressed firms for potential gains from asset sales or advantageous bargaining position. Second, compared with OTC derivative contracts that are challenging to supervise, acquisitions in the financial sector are subject to regulatory approval, which makes it relevant for policy interventions.

Based on the model result, regulations that prevent the inefficient distress links can generate social gains. I begin by proposing one such regulation using an acquisition tax to supervise acquisitions. Then I study an extension of the model that allows for the analysis of optimal government policies both before and after the linkage formation. Results indicate that the too-connected-to-fail problem arises if the excess acquisition is not effectively prevented *ex ante*. In this case, liquidating the distressed firm is too costly due to spillovers to its existing counterparties. Using the extended model, I discuss, respectively, the options of government bailout, subsidized acquisition, and pushed acquisitions. I find that these are *ex post* optimal remedies, thereby rationalizing the government interventions observed during the crisis.

5.1 Acquisition Tax

Current authorities consider acquisition as the primary approach to resolve firm distress as it incurs the least fiscal cost. However, my results imply that acquisitions of distressed firms should rather be regulated accounting for the externalities in the financial linkage formation. If the regulators are able to provide incentives by imposing taxes, then a tax formula that varies with the distress distribution can induce the optimal level of acquisitions and restore the efficient network. Next I formally characterize the tax rate.

Proposition 5 (Acquisition Tax) *In an N -firm chain, the optimal network can be decentralized by a tax τ imposed to firm 1 upon its acquisition of the distressed firm N ,*

$$\tau = \left[N\Phi\left[\sqrt{N}(-\bar{z})\right] - (N-2)\Phi\left(\sqrt{N-1}\left(-\bar{z} - \frac{1}{2}\delta\right)\right) - \Phi(-z_1) - \Phi(-z_N) \right] c. \quad (1.18)$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Furthermore, $\tau > 0 \iff L_{NN}^* = 1$. τ satisfies $\frac{\partial \tau}{\partial \delta} > 0$ and $\frac{\partial \tau}{\partial \bar{z}} < 0$.

Proposition 5 states that the acquisition tax is positive if and only if the most distressed firm should be isolated in the optimal network. Moreover, the acquisition tax increases with the average and dispersion of distress. The intuition is as follows. The acquisition tax equals precisely the negative externalities to all other non-distressed firms $i = 2, \dots, N-1$. Hence, it exactly aligns the individual motivation with the social incentive for acquisition. Accounting for negative spillovers, the acquisition tax is a function of the cross sectional distribution of firm distress in terms of $\{N, \bar{z}, \delta\}$. When dispersion is higher, the negative externalities are bigger; hence, we require bigger incentive to correct for the externality. A similar argument holds for the relation with the average distress. Note that the tax is only imposed conditional on the excess acquisition. Therefore, no tax will be physically collected from the acquirers because the inefficient acquisition is effectively prevented.

The model provides a sharp theoretical guidance on how to regulate acquisitions. In particular, the novel insight of considering firm distress distribution complements the current metrics in regulatory decisions. The “*financial stability*” factor has been included for the first time for processing firm acquisitions by the Dodd-Frank Wall Street Reform and Consumer Protection Act in Section 604(d). This section amended Section 3(c) of the Bank Holding Company Act of 1956 and it requires the Fed to consider “the extent to which a proposed acquisition, merger or consolidation would result in greater or more concentrated risks to the stability of the United States banking or financial system.” In the orders on approving recent acquisitions, for instance Capital One’s acquisition of ING Bank, the Fed illustrates the new financial stability metrics in response to Dodd-Frank’s mandate, including size, substitutability, interconnectedness, complexity, and cross-border activity.³³ The discussion regarding the interconnectedness

³³www.federalreserve.gov/newsevents/press/orders/2012orders.htm

factor, however, only covers the degree of interconnectedness of the resulting firm, rather than considering the entire linkage structure and possible externalities through indirect linkages.

The key issue is how to implement such acquisition tax. From Equation (1.18), the regulators need to account for the distribution of financial distress. One feasible approach detailed in Section 6 is to estimate quarterly Z-scores of all financial firms. Among the limitations of this measurement are the low frequency and the opacity of balance sheets. Using exclusive regulatory data, the banking supervisors can potentially achieve better estimates by using observations with higher frequency or alternative models such as CAMELS ratings.

Once the excess acquisitions are prevented, alternative resolution methods in case of failure include liquidation or the Purchase and Assumption (P&A) transactions. The Federal Deposit Insurance Corporation Improvement Act of 1991 mandates the FDIC to choose the resolution method least costly to the Deposit Insurance Fund. To comply with this mandate, the FDIC chose P&A transactions as the resolution method for a great majority of failing banks (about 95%).³⁴ My results hence indicate that P&As are preferred to relying on private sector solutions which give rise to network externalities and the potential build-up of systemic risk.

5.2 *Ex post* Policies

Several acquisition cases observed during the recent financial crisis render the baseline model counterfactual, including the acquisitions of Bear Stearns, Merrill Lynch, and National City. These cases differ from the baseline setting in several dimensions: links with the target institutions were formed before the distress conditions were fully disclosed. Additionally, government interventions such as bailout or pushed/subsidized acquisitions took place. For example, counterparties did not immediately pull back from trading with Bear Stearns after the failure of its two funds in 2007. When Bear Stearns suffered severe financial distress on March 2008, the Fed provided assistance in the form of a non-recourse loan of \$29 billion to JP Morgan to make the acquisition. To rationalize the observed government interventions of such kind, I next consider extensions of the baseline model, and the key deviation is that the timing of the network formation does not coincide with the observation of distress.

Suppose the linkage cannot be severed once formed at $t = 1$ after ν is learned. Further, assume that the liquid return $\tilde{a}_i$ satisfies

$$\tilde{a}_i = \nu_i + \theta_i + \sigma \varepsilon_i, \quad i = 1, \dots, N, \quad (1.19)$$

where the additional term θ_i is realized after links are formed. Hence, ν_i and θ_i jointly determine the amount of liquid value firm i expects to receive. Let θ be a vector with $\theta_i = 0, \forall i = 1, \dots, N-1$, and $\theta_N = -k\bar{z}\sigma$.³⁵ Further let $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta > \delta_1(\bar{z})$ such that the distress firm N should

³⁴For detailed institutional background on bank failures see White and Yorulmazer (2014), Granja, Matvos, and Seru (2014), and the the Guidance for Developing Effective Deposit Insurance Systems from FDIC, at <http://www.fdic.gov/deposit/deposits/international/guidance/guidance/FailedResolution.pdf>.

³⁵In practice, distress signals are released gradually. The negative θ_N captures persistence in liquidity conditions.

be isolated (Proposition 1). Nonetheless, in the absence of the acquisition tax, all firms are connected at equilibrium (Proposition 2). Now, assume firm N receives a second bad liquidity shock θ_N with $k > N$ such that it drags down the average distress of all firms below zero. In this case, the links do not generate positive risk-sharing surplus, thus total liquidation costs are higher than without any links among firms.

Government Bailout

Next I analyze conditions when government bailout is *ex post* optimal and how total costs compare to those under the *ex ante* optimal policies (imposing acquisition tax). For this purpose, let us enable the option of government bailout in the form of costly liquidity injection. Specifically, let $B\sigma$ denote the amount of government liquidity injection to the heavily distressed firm N . Since all firms are connected and each has the same diversified asset holdings, they share the same probability of liquidation $\Phi \left[-\frac{1}{\sqrt{N}}(N\bar{z} - k\bar{z} + B) \right]$. Here, the total costs incurred include expenses both in liquidation and bailout.³⁶

I find that positive government bailout is *ex post* optimal in an over-connected network as long as the liquidation cost is not very small. The formal analysis is provided in Appendix A.5, Proposition 7. When the liquidation cost satisfies $c > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, a positive government bailout that matches at least the total expected liquid value shortfall ($B^* > (k - N)\bar{z}$) is *ex post* optimal. This lower bound of liquidation cost is smaller when the distressed firm has more counterparties or when asset volatility is lower. Now, suppose the second shock θ_N to firm N is not sufficiently bad, the lower bound of liquidation cost that justifies government bailout will be higher.³⁷ In other words, the worse shock the connected banking system gets, the more likely government bailout is *ex post* optimal. This relation is consistent with the empirical observation that bailout only occurs in rare occasions with severe distress.

Despite the fact that government bailout can be *ex post* optimal, it is likely to be more costly than preventing the excess acquisition *ex ante*. I show that, as long as the bailout cost is not sufficiently low, total costs from *ex post* government bailout is higher than regulating the links *ex ante* using the acquisition tax (see Proposition 8 in Appendix A.5). This result captures one critical aspect of inefficiency in the current policy making: the time-inconsistency problem.³⁸ When a liquid firm observes the distress of some institution, it acquires the distressed target while generating externalities. Precisely owing to the excess acquisition link, liquidation of the distressed firm gets too costly. In consequence, government bailout becomes *ex post* optimal and *ex ante* inefficient.

³⁶The total costs incurred equal $N\Phi \left[-\frac{1}{\sqrt{N}}(N\bar{z} - k\bar{z} + B) \right] + B\sigma$.

³⁷Formally, if $0 \leq k \leq N$ in $\theta_N = -k\bar{z}\sigma$ instead, the average distress $\frac{1}{\sqrt{N}}(N - k)\bar{z}$ is then positive. And the lower bound for liquidation cost is higher than the case of $k > N$, i.e. $c \geq \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} e^{\frac{(N-k)^2\bar{z}^2}{2N}} > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$.

³⁸For other discussions on the time-inconsistency issue, see Acharya and Yorulmazer (2007), Spatt (2009), Chari and Kehoe (2013), and Gimber (2013).

Government Subsidized Acquisition

Back to the Bear Stearns case, instead of injecting capital directly, the Fed provided assistance to the acquirer JP Morgan in the form of a non-recourse loan.³⁹ With a slight variation, the extended framework can explain this behavior. I show that, when there exist healthier institutions currently not connected with the distressed firm, government subsidized acquisition can reduce total liquidation costs.

Consider another group of connected firms that are separate from the existing firms. Suppose there are N firms $i = N+1, \dots, 2N$ with the same average $\bar{z} > 0$ and dispersion $\delta = 0$, such that a complete risk-sharing network optimally emerges.⁴⁰ Let the additional signal θ_i be $\theta_{N+1} = \hat{k}\bar{z}\sigma$ and $\theta_i = 0, \forall i = N+2, \dots, 2N$, so the $(N+1)_{th}$ firm gets a positive shock in the liquid return. The question I address next is whether firm $N+1$ has the incentive to acquire the distressed firm N after the realization of θ , and whether the *ex post* acquisition is socially optimal.

The answer to this question depends on how the liquidity surplus of firm $N+1$ compares with the liquidity shortage of firm N . In Corollary 1 of Appendix A.5, I show that the *ex post* distressed acquisition is efficient and it occurs at equilibrium if and only if the average distress is above zero ($\hat{k} > k - 2N$). However, if the adverse liquidity shock k is considerably large ($k \geq \hat{k} + 2N$), the acquisition has negative surplus, and firm $N+1$ does not have incentive to acquire. In this case, subsidized acquisition in the form of liquidity injection to the acquirer is *ex post* optimal as long as the liquidation cost is not very small ($c > \frac{\sqrt{\pi}\sigma}{\sqrt{N}}$). The intuition is that risk-sharing among the two groups of firms reduces total liquidation costs only when the total expected liquidity is positive. And both acquisition subsidy and government bailout can push the average liquidity above zero. I find that the required optimal government subsidy is lower when the positive liquidity shock of the potential acquirer ($\hat{k}$) is higher. This result rationalizes the observation that the subsidized acquirers during the financial crisis, for instance JP Morgan and PNC (respectively acquirers of Stearns and National City), are relatively more liquid firms.

Comparing the two types of *ex post* policy remedies, the government subsidized acquisition generates lower total costs than government bailout, thus is always preferred. This result holds even when the acquisition alone is socially costly. Nonetheless, if the excess link with the distressed firm was prevented in the first place, liquidation would not be as expensive; hence, neither subsidized acquisition nor bailout would be necessary.

Government Pushed Acquisition

I have shown that when the two groups of firms have the same cardinality, the acquisition link forms at equilibrium if and only if it generates value gains. However, this “if and only if” condition does not hold when the cardinality of the two groups differs. Specifically, if the

³⁹On March 14, 2008, the New York Fed agreed to provide a \$25 billion collateralized loan to Bear Stearns for up to 28 days, but later decided that the loan was unavailable to them.

⁴⁰The results are robust to $\delta > 0$. I leave the robustness on the number of firms in the two groups to the next subsection.

additional healthier group has fewer firms, the acquisition might not occur even if it is *ex post* socially valuable, which motivates direct government interventions.

The relative cardinality of the two groups determines the sign of the bilateral surplus and implies whether the *ex post* acquisition occurs at equilibrium or not. When the potential acquirer in the second group has more counterparties, there are more firms to share the cost of the acquisition than there are in the original distressed group to share the benefit. The bilateral surplus from the acquisition is greater than the social surplus, hence the acquisition link forms *ex post* whenever it is socially valuable. When the cardinality of the two groups are the same, the sign of the bilateral surplus matches that of the social surplus, and we are back to the special case in Section 5.2.

If instead the distressed firm N has more counterparties, the bilateral acquisition surplus is smaller than the social surplus. Especially, the bilateral surplus can be negative even when the social surplus is positive. Hence, the *ex post* socially valuable acquisition does not occur at equilibrium. In such circumstances, government pushed acquisition is socially value improving. For a detailed analysis see Proposition 9, Appendix A.5.

There are many ways in which a government intervention can take place. One approach is by exerting pressure to the potential acquirers. Examples include the Fed pressuring Bank of America to acquire the distressed Merrill Lynch.⁴¹ The regulators can also aim to correct the sign of the bilateral surplus by subsidizing the acquirer using fund collected from the counterparties of the distressed firm. Alternatively, the regulators can provide a coordination device for collective decision making: let the potential acquirer and all the counterparties of the distressed firm bargain over the payments. One such example is the initiation of collective bailout of LTCM by the New York Fed in 1998.⁴²

6 Empirical Evidence

In this section, I document evidence that the distribution of distress across financial institutions provides a novel measure for systemic risk and aggregate failures in the financial sector. I establish this result by first examining how the cross-sectional mean and dispersion of distress correlate with indicators for aggregate systemic risk, liquidation costs, distress links through acquisitions, and interbank risk-sharing. I then confirm the findings using predictive regressions.

⁴¹As discussed in Spatt (2010), “secretary of the Treasury Henry Paulson indicated to [Bank of America CEO] Lewis that banking supervisors would question his suitability to lead Bank of America if BoA backed out of the merger and then needed more federal support, while federal authorities agreed to provide ‘ring-fencing’ of difficult to value Merrill Lynch assets if Bank of America went ahead with the merger.”

⁴²On Sept 23 1998, the New York Fed arranged a meeting for a group of LTCM’s major creditors at one of its conference rooms. During this historic meeting, the creditors worked out a restructuring deal that recapitalized LTCM and avoided its bankruptcy.

6.1 Measurement

The sample of financial institutions I consider includes bank holding companies and all Federal Deposit Insurance Corporation (FDIC) insured commercial banks and savings institutions. The quarterly accounting data of bank holding companies for the period of 1986-2013 are taken from FR Y-9C filings provided by the Chicago Fed. The quarterly accounting data for commercial banks (Call Reports) and savings institutions (Thrift Financial Reports) are taken from the FDIC's Statistics on Depository Institutions, available for 1976-2013. Next, I discuss the method for estimating the distress measure Z-scores and identifying the acquisitions of distressed firms.

Z-score

The quarterly accounting data provide the basis for measuring financial distress and identifying acquisitions of distressed institutions. I measure financial distress by estimating the Z-score, which has been widely used in the recent literature (e.g. [Stiroh \(2004\)](#), [Boyd and De Nicolo \(2005\)](#) and [Laeven and Levine \(2009\)](#)) as an indicator for a institution's distance from insolvency ([Roy \(1952\)](#)). The Z-score is defined as the return on assets plus the capital-asset ratio divided by the standard deviation of return on assets. Simply put, it equals the number of standard deviations that an institution's return on assets has to drop below the expected value before equity is depleted. For this reason, the Z-score provides a good proxy for financial distress, which is denoted by the state variable z_i in my model.

The Z-score combines accounting measures of profitability, leverage and volatility. In particular, it is estimated according to the formula

$$\text{Z-score}_{i,t} = \frac{\frac{1}{T} \sum_{\tau=0}^{T-1} ROA_{i,t-\tau} + \frac{1}{T} \sum_{\tau=0}^{T-1} CAR_{i,t-\tau}}{\sigma_{t-T+1}^t(ROA_i)}, \quad (1.20)$$

where $ROA_{i,t}$ and $CAR_{i,t}$ are respectively the return on assets (net income over total assets) and capital asset ratio (total equity capital over total assets) for firm i in quarter t . In my analysis, the Z-score is computed considering a rolling window of eight observations, i.e. $T = 8$. The estimated Z-score is highly skewed; hence, I follow [Laeven and Levine \(2009\)](#) and [Houston, Lin, Lin, and Ma \(2010\)](#) and adopt the natural logarithm of the Z-score as the distress measure.

The time series of the mean and dispersion of log Z-score are estimated by taking the average and standard deviation across all financial firms in each quarter. Figure 9 plots the quarterly series of dispersion, mean, and the 10-90 percentile range of log Z-score over the period of 1978-2013. For the purpose of visualization, the series are normalized such that both the dispersion and the mean are centered around one. The shaded bars indicate NBER recession dates.

From Figure 9, we can make the following observations. First, relative to the cross-sectional mean, the dispersion of log Z-score displays a fair amount of variation and has an increasing overall trend. Second, the dispersion series demonstrates a countercyclical pattern: it increases during the Savings and Loan crisis, the Dot-com crash and the recession afterwards, as well as

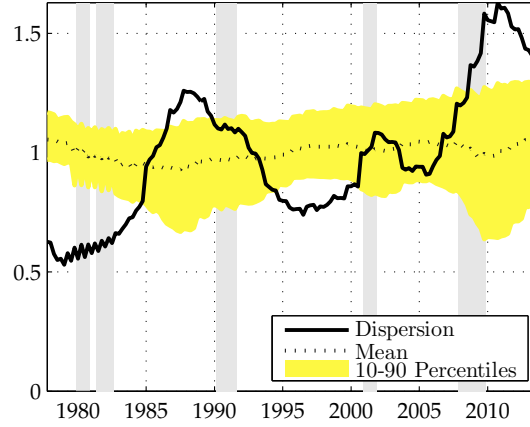


Figure 9. Log Z-score Moments across Financial Institutions. This figure plots the quarterly time series of dispersion, mean, and the 10-90 percentile range of log Z-score across all financial institutions over the period of 1978-2013. The series are normalized such that both the dispersion and the mean are centered around one. Shaded bars indicate NBER recessions.

during the 2007-2009 financial crisis. Based on the comparative statics in Section 4.2, precisely during the crises spell, network inefficiency is more pronounced, which potentially aggravates the crises and increases systemic risk. Finally, the dispersion series appears to lead recessions. Take the most recent crisis for instance, the dispersion starts to increase since 2005, and by the time financial firms enter the crisis in the 3rd quarter of 2007, they already show significant dispersion in financial distress. These features combined suggest that the time series of dispersion can potentially signal economic changes and systemic risk, which I will test at the end of this section.

While the Z-score provides a quantitative measure for distress, it is worth noting a few limitations. First, the quarterly accounting data are an endogenous outcome of certain degrees of risk diversification, thus are not exogenous to firms as assumed in my model. Nonetheless, the Z-score gives the best available proxy for the distress shock in the static framework because it is estimated using past data, which are taken as given by firms to make decisions onwards. The Z-score indicates firm stability well also because, as shown by [Acharya, Shin, and Yorulmazer \(2010\)](#), initially liquid firms tend to hoard liquidity or deleverage for potential gains from asset sales, whereas risk management tools for an initially distressed firm are limited. Hence, the ranks of the estimated Z-score across firms can reflect the ranks of initial distress. The second limitation pertains to the estimation of Z-score using accounting data. It omits off-balance sheet activities, and thus possibly gives a biased measure of firm risk. However, off-balance sheet usages are only relevant for a few institutions, hence do not necessarily affect the entire distribution.

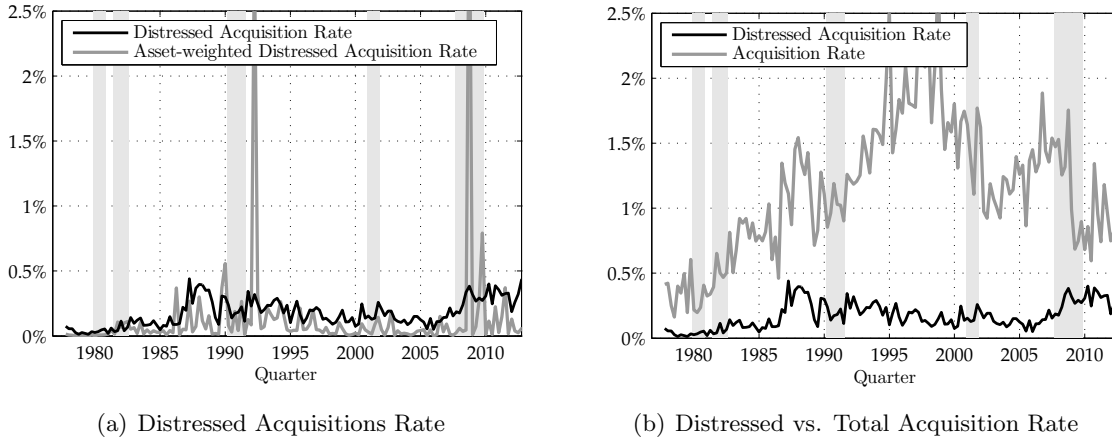


Figure 10. Distressed Acquisitions Rate. This figure plots the quarterly (asset-weighted) distressed acquisition rate for 1978-2013 (left) and compares the distressed acquisition rate to the total acquisition rate (right). Shaded bars indicate NBER recessions.

Acquisitions of Distressed Firms

Based on the above measure, an acquisition of a distressed firm occurs when the target has a low Z-score. This enables us to proxy for the acquisition links with the distressed firms in the model. The acquisition transactions are taken from the Chicago Fed Mergers and Acquisitions dataset. The dataset records all the acquisition transactions of banks and bank holding companies since 1976, keeping track of both the target and acquirer entities at the merger completion date. I drop the observations that are failures or restructurings.⁴³ I then match the dataset with quarterly accounting data using RSSD ID of the target firm two quarters ahead.⁴⁴ Around 86% (17,930) of the observations are matched. Out of the matched sample, I identify a *distressed acquisition* if the target firm reports a negative net income two quarters prior to the acquisition completion date, or if the target firm has a log Z-score of below 2.35 (two standard deviations below the sample mean) at least once, two to four quarters before the acquisition completes.

Using this strategy, around 20% (3,153) of the matched sample acquisitions are classified as distressed acquisitions, whereas the rest mostly took place during the merger wave in the 2000s after the Gramm-Leach-Bliley Act, which enabled mergers among investment banks, commercial banks, and insurance companies. Among the identified distressed acquisitions, some notable examples include Countrywide by Bank of America, Riggs and Sterling by PNC, and Wachovia

⁴³Failures refer to transactions with Termination Reason Code = 5. Restructurings occur when the target entities and the acquirer entities have exactly the same entity name but different Federal Reserve RSSD IDs.

⁴⁴To match as many entities as possible, in this step, I include the FR Y-9LP and FR Y-9SP filings for bank holding companies. However, since these non-consolidated parent banks only report semiannually, I do not include them when computing the Z-score distributions. I match the quarterly accounting dataset two quarters ahead because the merger date in Chicago Fed M&A dataset represents the completion date and is usually later than the last quarter when the non-survivor firm files quarterly report.

Table 1. DISTRESSED ACQUISITION LIKELIHOOD AND LOG Z-SCORE

	Pr(Completing an Acquisition of a Distressed Firm)				
	(1)	(2)	(3)	(4)	(5)
Log Z-score	0.153*	0.145*	0.142*	0.284**	0.317***
	[0.070]	[0.070]	[0.067]	[0.094]	[0.094]
Firm Controls		yes	yes	yes	yes
Year Fixed-Effects			yes		yes
2006-2013				yes	yes
Observations	57,035	57,035	57,035	14,490	14,490
Firm Fixed-Effects	yes	yes	yes	yes	yes

Notes: This table reports the results from a fixed-effects logit regression. The sample includes commercial banks, savings institutions and bank holding companies. The dependent variable Pr(Completing an Acquisition of a Distressed Firm) takes the value of one if institution i completes an acquisition of a distressed firm at time $t + 4$, and zero otherwise. Firm controls include quarterly CAR, ROA, and asset size. Regression coefficients are reported with standard errors in the square bracket. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

by Wells Fargo.

Figure 1.10(a) plots the quarterly percentage of distressed acquisitions over total number of financial institutions as well as the distressed acquisition rate weighted by the asset size of the targets. From the plots, the distressed acquisition rates are countercyclical. Two periods with clustered acquisitions are the Savings and Loan crisis and the 2007-2009 financial crisis. The asset-weighted acquisition rate displays significant spikes (some spikes reach as high as 3%, while the plots are trimmed at 2.5%).⁴⁵ Panel 1.10(b) compares the distressed acquisition rate to the total acquisition rate. The insignificant comovement between the two curves shows that variations in distressed acquisitions are unlikely driven by merger waves.

Model Assumptions on Distressed Acquisitions

To confirm the assumption made in the model that more liquid firms acquire the distressed firms, I match the quarterly firm-level data with the acquisition dataset using the acquirer entities and acquisition completion dates, and perform fixed-effects logit regressions. The dependent variable is a dummy indicating whether a firm conducts a distressed acquisition at a certain quarter. I assume that an acquisition takes on average four quarters to complete, so it starts four quarters prior to the merger completion date recorded in the Chicago Fed dataset. The independent variable of interest is the firm's estimated log Z-score. Results reported in Table 1 confirm that a firm with higher log Z-score has a higher likelihood of acquiring a distressed firm. For a one-standard-deviation increase in log Z-score (.58), the log odds ratio of a distressed acquisition increases by 0.09 ($=0.153 \times 0.58$). The economic and statistical significance of the coefficient

⁴⁵The spikes include one in the 2nd quarter of 1992 due to the acquisition of Security Pacific, one in 2007-2008 mostly due to the acquisitions of Lasalle bank (10/01/2007), Countrywide (01/11/2008), National City (10/24/2008), and Wachovia (12/31/2008).

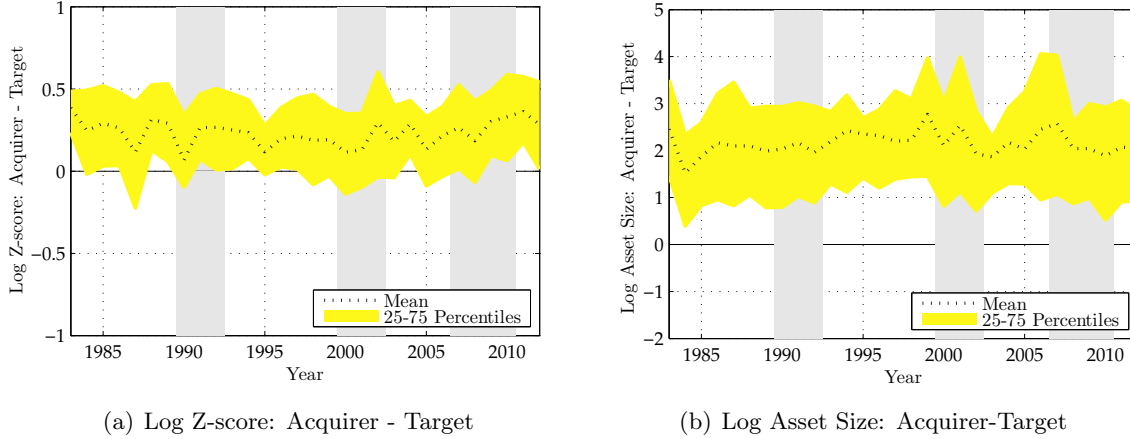


Figure 11. Log Z-score and Asset Size: Acquirer - Target. This figure plots the distribution of log Z-score and log asset size of the acquirer-target wedge for the identified 3,153 distressed acquisitions in 1983-2013. Shaded bars indicate NBER recessions.

is robust to including firm-level controls, year fixed effects, and only considering the post-2006 period.

Among the identified 3,153 distressed acquisitions, a clear pattern emerges among the acquirer-target pairs: the acquirer has higher Z-score and bigger asset size relative to the target. The results are depicted in Figure 11. The plots show the distributions of the acquirer-minus-target log Z-score (Panel 1.11(a)) and log asset size (Panel 1.11(b)). Both distributions are significantly above zero, implying that more stable firms acquire smaller and distressed targets.

In the theoretical analysis, a link with the distressed firm is modeled as a bilateral forward swap contract, which increases the financial distress of the acquirer and thus negatively affects its Z-score. To confirm this assumption, I perform fixed-effects regressions of growth rate in log Z-score on target log Z-score, and the dummy variables representing acquisition and distressed acquisition, controlling for firm-level characteristics. The regression results summarized in Table 2 show strong support for the model assumption. The estimates suggest that the effect of the log Z-score of the targets on the growth rate of Z-score of the acquirers is positive and significant. The economic magnitude of the effect is sizable: a one-standard-deviation decrease in target log Z-score decreases future log Z-score of the acquirer by 0.16, more than four times the magnitude of its average level. Results in columns (3) - (4) show that, while in general completing an acquisition has a positive impact on the future Z-score of the acquirer, completing an acquisition of a distressed target has a significantly negative impact on the future Z-score of the acquirer. These findings are robust to controlling for recession periods, restricting to only top firms with asset size larger than \$1 billion, and including year-quarterly dummy.

Table 2. EFFECT OF TARGET LOG Z-SCORE ON ACQUIRERS' FUTURE Z-SCORE

	$\log z_{i,t+1} - \log z_{i,t}$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Target Log Z-score	0.248*** [0.060]	0.310*** [0.064]			0.326* [0.130]	0.291** [0.095]	0.250*** [0.060]
Acquisition Dummy			0.624*** [0.186]	0.884*** [0.206]			
Distressed Acquisition Dummy		-1.268** [0.447]		-1.373** [0.464]			
Observations	1,326,071	1,326,071	1,326,071	1,326,071	435,635	98,737	1,326,071
Firm Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed-Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
NBER Recessions					Yes		
Top Firms (A>\$1B)						Yes	
Year-quarter Dummy							Yes

Notes: This table reports the coefficients from a fixed-effects regression. The sample includes commercial banks, savings institutions and bank holding companies. The dependent variable $\log z_{i,t+1} - \log z_{i,t}$ is the growth rate of log Z-score for firm i at quarter t . The target log Z-score is the level of log Z-score of the target firm at the acquisition completion date if firm i has an acquisition at quarter t . The dummy variables take 1 (and 0 otherwise) if firm i has an acquisition or a distressed acquisition at quarter t . Firm controls include total assets, total equity, net income, and current level log Z-score. Regression coefficients are reported with standard errors in the square bracket. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

6.2 Model Predictions

As shown in the comparative statics in Section 4.2, an increase in dispersion (together with a decrease in average Z-score) is associated with *higher systemic risk, more liquidations, more (excess) distress links through acquisitions, and fewer risk-sharing links*. Next, I illustrate that patterns in the data provide suggestive evidence for these model-predicted relations.

Aggregate Indicators

The goal is to provide aggregate level evidence that distress dispersion is indicative of economic activity and financial stability. To measure macroeconomic activity, I use the *Chicago Fed National Activity Index (CFNAI)*,⁴⁶ which is adopted in Giglio, Kelly, and Pruitt (2015) to evaluate the predictive power of various systemic risk measures. As an indicator for systemic risk, I take the Chicago Fed's *National Financial Conditions Index (NFCI)*.

Failures are aggregated from the FDIC Failure and Assistance Transaction Reports of all commercial banks and savings institutions in 1976-2013. I append this sample using the failures of bank holding companies, i.e. those in the Chicago Fed Mergers and Acquisitions dataset

⁴⁶The CFNAI is designed to gauge overall economic activity and related inflationary pressure. It includes the following subcomponents: production and income (P&I), sales, orders, and inventories (SO&I), employment, unemployment, and hours (EU&H), and personal consumption and housing (C&H).

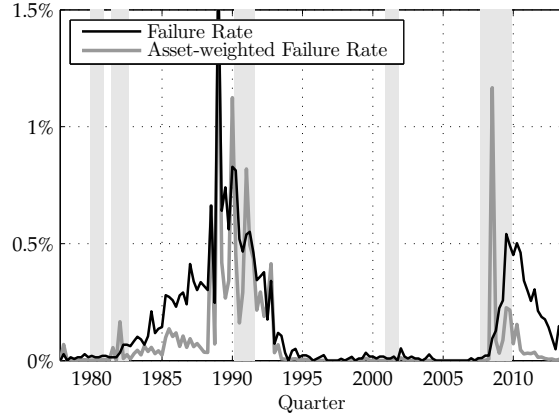


Figure 12. Bank Failure Rates. This figure plots the quarterly failure rate and asset-weighted failure rate of commercial banks and savings institutions for 1978-2013. Shaded bars indicate NBER recession dates.

with Termination Code = 5 (failure). In total, I obtain 3,473 failures with an asset value of 1.84 trillion in 2010 dollars. I construct the quarterly failure rates (numbers of failures over the numbers of total financial institutions) as well as the failure rates weighted by the failing institution's asset size. As depicted in Figure 12, failure rates are strongly countercyclical: the majority of bank failures took place during the Savings and Loan crisis and the 2007-2009 crisis.

Regarding the linkage composition, the model predicts that non-distressed firms that do not engage in distressed acquisitions withdraw from risk-sharing contracts as a consequence of network externalities. Direct evidence on this prediction would be obtained if full information on individual level linkage is available. Instead, I consider the lending and interbank lending behavior of small to medium-sized commercial banks as proxies for risk-sharing contracts since these institutions are more likely to be the non-distressed and non-acquirer firms in the model. In particular, using data from the Fed's H.8 release, I construct the fractions of bank credit and Fed funds and reverse Repos with banks over total assets for small to medium-sized (beyond top 25) commercial banks.

Univariate Correlations

Table 3 provides the summary statistics of the above series as well as their univariate correlation coefficients with the mean and dispersion of financials' log Z-scores. Both the mean and dispersion series are rescaled such that the two series are centered around one. The distress dispersion displays higher variation over time and does not significantly correlate with the mean of distress, thereby confirming that dispersion provides new information not captured by the mean.

Well aligned with the theoretical findings, dispersion series correlate negatively with the

Table 3. SUMMARY STATISTICS AND UNIVARIATE CORRELATIONS

	Mean	StDev	Sacf	Correlations w/ log Z-score	
				Mean	Dispersion
Mean of Log Z-score	1.00	0.03	0.90		
Dispersion of Log Z-score	1.00	0.22	0.97	-0.13	
A. Economic activity and systemic risk					
Chicago Fed National Activity Index (CFNAI)	-0.11	0.72	0.80	-0.03	-0.30**
National Financial Conditions Index (NFCI)	-0.34	0.54	0.84	-0.25**	0.37***
B. Bank failures					
Failure Rate (%)	0.18	0.25	0.72	-0.60***	0.45***
Asset-weighted Failure Rate (%)	0.11	0.25	0.34	-0.38***	0.17*
C. Distressed acquisitions					
Distressed Acquisition Rate (%)	0.21	0.09	0.64	-0.41***	0.60***
Distressed over Total Acquisition Rate	0.19	0.13	0.71	-0.44***	0.68***
D. Lending and interbank lending					
Small Comm. Bk Credit over Assets	0.88	0.02	0.94	-0.26**	-0.73***
Small Comm. Bk Fed Funds Loan over Assets	0.02	0.01	0.85	-0.09	-0.53***

Notes: This table reports summary statistics for the quarterly cross-sectional mean and dispersion of log Z-score, indicators for economic activity and systemic risk (A), bank failures (B), distressed acquisitions (C), and lending and interbank lending (D). Group A series are from FRED. Series in groups B and C are aggregated based on data from the FDIC and the Chicago Fed. Group D series are constructed from the Fed's Z.1 and H.8 release. Data availability on bank holding companies restricts the analysis to 1986-2013. Sacf is the first-order sample autocorrelation coefficient. The last two columns report the correlation coefficients between cross-sectional mean and dispersion of log Z-score and each series in groups A-D. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

economic activity index CFNAI and positively with the systemic risk index NFCI. In other words, high dispersions relate to bad economic times and low financial stability. As the model predicts, the failure rates and distressed acquisition rates are significantly higher when the dispersion is higher or when the average Z-score is lower. Additionally, the distressed acquisitions as a fraction of total acquisitions correlate even more significantly with the log Z-score moments, ruling out the possibility that the variations in distressed acquisitions are due to changes in total acquisition rates. These patterns all corroborate that high dispersion is associated with more distressed acquisitions and consequently, more failures. Last but not least, indicators for lending and interbank lending have negatively significant correlation with dispersion. Small and medium-sized commercial banks reduce interbank lending and exposures with other banks in the Fed Funds and Reverse Repos market, with significance at the 0.001 level. This finding supports that certain risk-sharing contracts terminate as dispersion increases.

Table 4. PREDICTIVE REGRESSIONS USING DISTRESS DISPERSION

Quarters	1	2	3	4	1	2	3	4
Forecasting	A. CFNAI				NFCI			
Dispersion	-2.09***	-4.04***	-5.85***	-7.50***	1.52**	2.77**	3.83**	4.72**
Mean	2.75	6.74	8.66	7.92	-8.95***	-17.80***	-25.73***	-32.32***
R^2	44.85	52.03	54.05	52.48	53.22	53.40	52.24	50.56
R^2 w/o disp	28.15	34.78	36.47	34.74	37.54	39.28	39.42	38.86
Forecasting	B. Failure Rate(%)				Asset-weighted Failure Rate(%)			
Dispersion	0.53***	1.03***	1.56***	2.07***	0.24*	0.48*	0.77*	1.04*
Mean	-3.81***	-7.91***	-12.21***	-17.12***	-2.68**	-5.41**	-7.92**	-11.29**
R^2	58.98	68.10	70.03	71.31	16.97	26.79	32.53	37.64
R^2 w/o disp	50.16	58.46	59.91	60.99	11.07	18.68	22.29	26.14
Forecasting	C. Acquisition Rate(%)				Distressed over Total Acquisition Rate			
Dispersion	0.16*	0.33*	0.50*	0.68*	0.29***	0.63***	1.00***	1.34***
Mean	-1.45**	-2.66**	-3.81**	-4.43**	-1.19*	-2.16*	-3.24**	-3.90*
R^2	47.31	57.25	64.90	67.04	53.90	63.06	72.20	75.26
R^2 w/o disp	41.24	49.52	56.12	57.55	43.64	48.92	54.41	56.11
Forecasting	D. Sml Bk Credit over Assets				Sml Bk Fed Funds over Assets			
Dispersion	-0.04**	-0.08**	-0.12**	-0.16**	-0.01*	-0.02*	-0.03**	-0.04***
Mean	-0.02	-0.01	-0.01	-0.04	0.05	0.08	0.09	0.07
R^2	69.85	70.68	71.18	71.44	57.07	63.49	64.71	63.93
R^2 w/o disp	62.69	63.39	63.83	63.76	54.04	59.60	59.48	56.94

Notes: This table summarizes the ability of distress dispersion to forecast future economic activity, systemic risk, failure rates, distressed acquisition rates, and bank lending behavior. Aggregate indicators in groups A-D are regressed respectively on the cross-sectional dispersion and mean of log Z-score controlling for the term spread, the leverage of financial business and security broker-dealers, and the growth rate of real non-financial corporate liability. Forecasting horizons range from one to four quarters and the data cover the years of 1986-2013. The table reports the regression coefficients of the dispersion and mean of log Z-score, the R^2 , as well as the R^2 when the regressions are run without the dispersion series. *, **, *** denote statistical significance (based on Newey-West standard errors) at the 5%, 1%, and 0.1% level.

Predictive Regressions

Evidence from the univariate correlations provides a strong indication that the distress dispersion comoves with aggregate indicators. However, contemporaneous correlations do not necessarily imply that the distress dispersion is able to forecast systemic risk. Hence, the next goal is to evaluate whether the distress dispersion has predictive power of aggregate indicators by providing additional information beyond what is contained in the average distress and existing systemic risk measures.

To this end, I run forecasting regressions of the above introduced aggregate indicators on the dispersion and mean of log Z-score controlling for moments including the term spread used

in Giglio, Kelly, and Pruitt (2015), the leverage of both financial business and the security broker-dealers as in Adrian, Etula, and Muir (2014), and the growth rate of non-financial corporate liability as a measure of aggregate credit creation. The forecasting horizons range from one to four quarters and the data cover the years 1986-2013. To overcome correlation and autocorrelations in the time series, I calculate Newey-West standard errors.

Table 4 reports the coefficient estimates on the dispersion and mean of log Z-score, the values of R^2 when I run the regressions with and without the dispersion series. The regression results echo those from the correlations and indicate striking predictive power of the dispersion series to forecast economic activity and systemic risk, failures, distressed acquisitions, and interbank lending. The predictive power is evidenced by both the economic significance of the regression coefficients and the differences in the R^2 s with and without dispersion in the regressors. For example, the estimates in the forecasting regression of CFNAI imply that (holding the *mean* fixed) a one-standard-deviation increase in *Dispersion* ($=0.22$) relates to a 0.46 ($= 0.22 \times 2.09$) decrease in CFNAI. Notably, the national activity index CFNAI, the credit and loans and the interbank lending of small and medium-sized commercial banks all respond negatively to an increase in distress dispersion, but not to changes in the mean of distress. Overall, these results paint a clear picture: the second moment of the cross-sectional distress distribution conveys new information about future activities in the financial sector in terms of systemic risk, failures, acquisitions, as well as interbank lending behavior.

7 Conclusion

Given the importance of financial interconnectedness, policies on financial stability and distress resolution should not analyze institutions in isolation. This paper has developed a network formation model to highlight a novel channel of systemic risk due to externalities via financial links.

Adding to the recent literature on financial network formation, this paper embeds firm heterogeneity in financial distress and examines how the linkage formation affects efficiency and systemic risk. I have shown that, when firms display high distress dispersion, the equilibrium network features too many links with the distressed firms and too few risk-sharing links among liquid firms. The reason is that the relatively more liquid firms have incentives to connect with distressed firms for profit while shifting risks away to their direct and indirect counterparties via the links. Particularly, these liquid firms fail to internalize the negative externalities when prices in the bilateral contracts cannot be contingent on the overall network structure. The inefficient link with the distressed firm not only generates risks of contagion but also crowd out valuable risk-sharing links, thereby increasing systemic risk. Notably, this inefficiency is shown to be more severe when institutions are more dispersed in financial distress.

While detailed data on the precise linkages among financial institutions are yet to be col-

lected,⁴⁷ this paper draws a relation between the degree of network inefficiency and the cross-sectional distribution of fundamentals, thus contributing to the measurement and forecast of systemic risk. The test can be extended along the lines of [Giglio, Kelly, and Pruitt \(2015\)](#) by comparing the distress dispersion to existing systemic risk measures such as CoVaR ([Brunnermeier and Adrian \(2011\)](#)) and Marginal and Systemic Expected Shortfall ([Acharya, Pedersen, Philippon, and Richardson \(2010\)](#)). Additionally, my model predicts that links between firms with different distress levels respond differently to an aggregate dispersion increase. With possibly better data access in the future, more work is needed to test these qualitative predictions.

My model provides new insights on policies for financial stability. The links with distressed firms in the model can be interpreted as acquisitions of such firms. In this context, my results call for regulations to eliminate the network inefficiencies associated with acquisitions of distressed firms. The task of the regulators is to oversee the acquisitions of distressed firms, especially those by highly interconnected acquirers when the distress dispersion is high across institutions. Rather than relying on acquisitions as the preferred private sector solution, regulators should instead adopt resolution methods such as purchase and assumption (P&A) for these distressed targets in case of failure.

⁴⁷For current challenges in measuring linkages and systemic risk, see for example [Bisias, Flood, Lo, and Valavanis \(2012\)](#), [Hansen \(2013\)](#), and [Yellen \(2013\)](#).

Appendix

A Technical Appendix

A.1 Notation Summary

General Notation:

- t : event dates, $t = 0, 1, 2, 3$.
- i : firm index, $i \in \{1, \dots, N\}$.
- R : fixed return of asset.
- $\tilde{a}_i$: amount of liquid asset component.
- ν_i : expectation of $\tilde{a}_i$ at $t = 1$.
- ν : the vector of ν_i .
- ε_i : idiosyncratic term in the liquid return, *i.i.d.* standard normal.
- σ : conditional volatility of liquid return.
- c : fixed liquidation cost.
- z_i : the distress statistic of firm i .
- z : the vector of z_i .
- $\bar{z}$: average distance to liquidation.
- δ : distress dispersion across firms.
- l_{ij} : the fraction of firm j 's liquid asset that firm i proposes to buy, $l_{ij} \in \{0, \bar{l}\}$.
- $\bar{l}$: fixed share of bilateral asset swaps.
- l_i : i 's strategy $(l_{i1}, \dots, l_{i,i-1}, l_{i,i+1}, \dots, l_{iN})$
- p_{ij} : the unit price of firm j 's asset offered by buyer firm i .
- p_i : i 's strategy $(p_{i1}, \dots, p_{i,i-1}, p_{i,i+1}, \dots, p_{iN})$.
- p_{jj} : reservation price of firm j .
- L : the matrix representing two-sided links.
- L_i : the i -th row of matrix L .
- $\tilde{h}_i$: the final liquid asset holdings.
- $\Pi_i(z, L, p)$: firm payoff at date 3.
- $V_i(z, L, p)$: firm value under network L and price p at date 1.

Additional notation in Section 3:

- L^* : the optimal network.
- $\tilde{h}^*$: the optimal asset holdings under the optimal network L^* .
- $\bar{z}_1, \bar{z}_2$: the cutoff values of $\bar{z}$.
- $\delta_1(\bar{z})$: the cutoff function of $\delta(\bar{z})$ where the optimal network changes from a single connected component to isolating the distressed firm.
- $\delta_2(\bar{z})$: the cutoff function of $\delta(\bar{z})$ where the equilibrium network changes from over connection to inefficient composition.

Additional notation in Section 4:

- ΔV : the value loss.
- $\Delta V\%$: the percentage value loss.
- $\Pr_{sys}^L$: the systemic risk in network L .
- $\Delta \Pr_{sys}$: the excess systemic risk.

Additional notation in Section 5:

- τ : the acquisition tax.
- θ_i : part of the expected liquid return realized after linkages are formed.
- k : a scalar indicating the negative shock to the distressed firm N , $\theta_N = -k\bar{z}\sigma$.
- $B\sigma$: government liquidity injection after linkages are formed.
- B^* : the optimal government liquidity injection policy.
- $\hat{k}$: a scalar indicating the positive shock to the potential acquirer firm $N + 1$, $\theta_{N+1} = \hat{k}\bar{z}\sigma$.

A.2 Optimal Risk Sharing Allocation

This section provides the technical results for subsection 3.1. I show that under the iterative asset swap procedure, the asset holdings resulting from the optimal network L^* is equivalent to the allocations when the social planner is allowed to choose the asset allocations directly.

Definition 3 Let $\mathcal{H}$ be an asset holding matrix such that firms' liquid asset holdings are $\tilde{h} = \mathcal{H}\tilde{a}$. The optimal asset allocation $\mathcal{H}^*$ is feasible and minimizes total expected liquidation costs,

$$\mathcal{H}^* = \arg \min_{\mathcal{H}} \sum_i^N \mathbb{E} [\tilde{h}_i < 1] c, \quad (\text{P2})$$

subject to

$$\mathcal{H} \times \mathbb{1}_{N \times 1} = \mathcal{H}^\top \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}, \quad (1.21)$$

where Equation (1.21) imposes the feasibility constraint for asset allocation. $\mathcal{H}$ being a doubly stochastic matrix ensures that no assets are created or lost from asset pooling and that each firm still holds one unit of assets. The following lemma states that if it is optimal for a firm to have a diversified asset holding, then it is more likely to be a relatively liquid firm. Moreover, its optimal asset holding is full risk-sharing, i.e. it holds the equally weighted asset composed of assets of all firms that participate in risk-sharing.

Lemma 2 If $\nexists i$ with $\tilde{h}_i^* = \tilde{a}_i$, then $\tilde{h}_i^* = \frac{1}{N} \sum_{j=1}^N \tilde{a}_j, \forall i \leq N$. If $\exists i$ with $\tilde{h}_i^* = \tilde{a}_i$ and $\tilde{h}_{i-1}^* \neq \tilde{a}_{i-1}$,

then $\tilde{h}_j^* = \tilde{a}_j, \forall j \geq i$ and $\tilde{h}_j^* = \frac{1}{i-1} \sum_{k=1}^{i-1} \tilde{a}_k, \forall j \leq i-1$.

Proof The optimal connection minimizes total expected liquidation costs (or equivalently default probabilities). Let the number of firms that participate in risk-sharing and have diversified asset holdings $\tilde{h} = H\tilde{a}$ be M . The total expected liquidation costs equal

$$\sum_{i=1}^M \Pr(\tilde{h}_i \leq 1) c = \sum_{i=1}^M \Phi \left(\frac{-\bar{z} - (1 - \sum_j H_{ij})\nu_i - \sum_j H_{ij}\nu_j}{\sqrt{(1 - \sum_j H_{ij})^2 + \sum_j H_{ij}^2 \sigma}} \right) c. \quad (1.22)$$

The first order condition with respect to H_{ij} is

$$\frac{\partial \sum_{i=1}^M \Pr(\tilde{h}_i \leq 1) c}{\partial H_{ij}} = \frac{\partial \Pr(\tilde{h}_i \leq 1) c}{\partial H_{ij}} + \frac{\partial \Pr(\tilde{h}_j \leq 1) c}{\partial H_{ji}}. \quad (1.23)$$

In particular, the derivative for firm i is

$$\frac{\partial \Pr(\tilde{h}_i \leq 1)c}{\partial H_{ij}} = \Phi' \left(\frac{-\bar{z} - H_{ii}\nu_i - \sum_j H_{ij}\nu_j}{\sqrt{H_{ii}^2 + \sum_j H_{ij}^2}\sigma} \right) c \times \frac{(\nu_i - \nu_j)\sqrt{H_{ii}^2 + \sum_j H_{ij}^2}\sigma + \left(-\bar{z} - H_{ii}\nu_i - \sum_j H_{ij}\nu_j\right)\sigma \left(H_{ii}^2 + \sum_j H_{ij}^2\right)^{-\frac{1}{2}}(H_{ii} - H_{ij})}{H_{ii}^2\sigma^2 + \sum_j H_{ij}^2\sigma^2}.$$

Similarly, write out the symmetric equation for firm j with respect to $H_{ji} = H_{ij}$, and plug into Equation (1.23) we obtain

$$\frac{\partial \sum_{i=1}^M \Pr(\tilde{h}_i \leq 1)c}{\partial H_{ij}} \Big|_{H_{ii}=H_{ij}=\frac{1}{M}} = 0, \quad \forall i \neq j.$$

This implies that the first order conditions with respect to asset holdings equal zero when each element of H is evaluated at $\frac{1}{M}$, thus achieving the optimal allocation. $H_{ij} = \frac{1}{M}$ indicates full risk-sharing. It is worth noting that the above result holds if we relabel Φ as a rather general distribution function even through the above proof explicitly uses normal distribution Φ . The only condition necessarily required is that ε_i is distributed independently across firms.

From Lemma 2, if no firm holds entirely idiosyncratic assets, then all firms share risks fully by having the same holding equally weighted by the liquid assets of all firms. If there exist firms who hold only their original assets, they must be the relatively distressed ones, while all other more liquid firms pool liquid assets equally. As such, the optimal asset holdings boil down to determining who should participate in risk-sharing and who should stay isolated.

Lemma 1 shows that the asset composition matrix H^* implied by the optimal network L^* also coincides with full risk-sharing among all connected firms. In this regard, under the iterative asset swap process, the optimal network H^* in (P1) achieves the best asset allocation matrix $\mathcal{H}^*$ in (P2). Importantly, the iterative feature of the asset swap itself does not deviate equilibrium from the optimal allocation.

A.3 Multiple Equilibria for $N \geq 5$

In Section 3.3, when analyzing the inefficiency in risk-sharing loss, I have focused on one specific equilibrium when firms start from the chain $\{1-2-3-4-5\}$ before distress dispersion increases.

In fact, when $N \geq 5$, different initial sequences in the stable risk-sharing chain at $\delta = 0$ imply different outside options and deviation incentives for each firm. As a result, we can have various equilibrium networks. The four panels in Figure A.I illustrate different equilibrium connection structures in the $(\bar{z}, \delta)$ space. Importantly, across all potential equilibria, a general pattern displays: equilibrium network switches from *optimal connection* to *over-connection* to *over- and under-connections simultaneously* as δ increases.

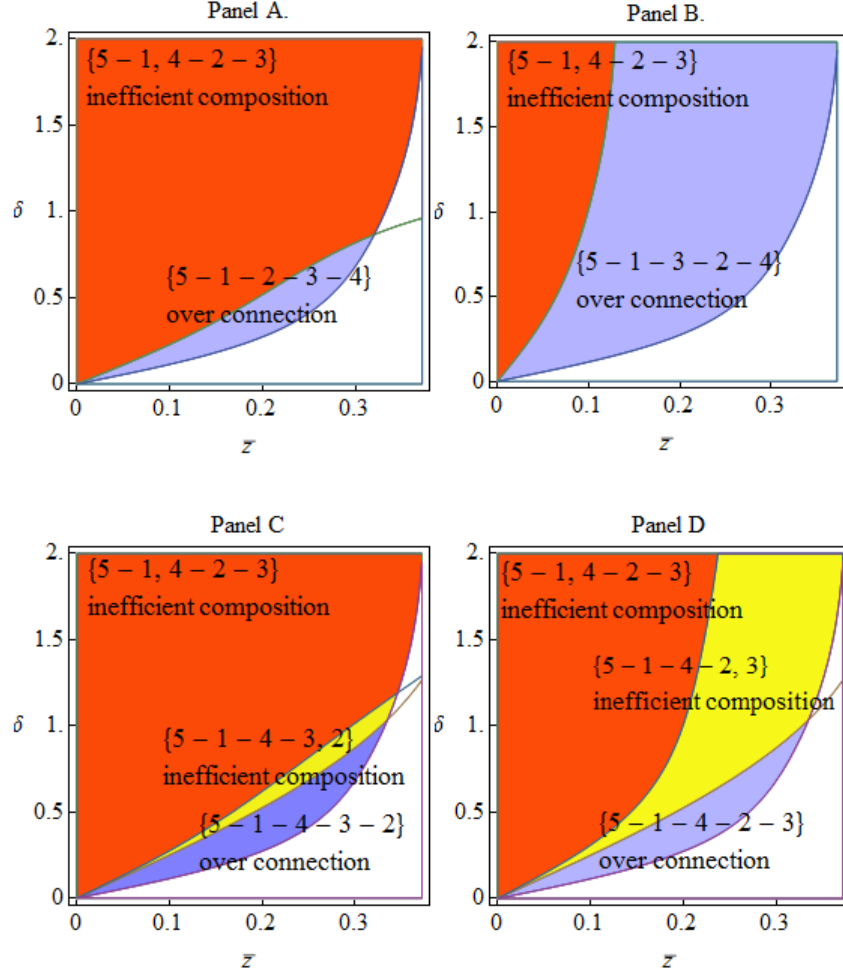


Figure A.I. Equilibria ($N = 5$). This figure shows the equilibrium five-firm network. The horizontal and vertical axes represent the mean and dispersion of firm distress statistic z . The four panels show different stable networks with different initial sequences along the chain. In colored regions, firm 5 is isolated in the optimal network. Blue (lighter) region denotes over-connection; orange (darkest) and yellow (lightest) regions denote inefficient network composition.

A.4 Full Contingent Contracts

This section provides the technical results for subsection 3.4. I use an example of $N = 4$ and show that a complete set of contracts contingent on the entire network structure decentralizes the efficient network.

Proposition 6 *The efficient network is decentralized by bilateral contracts contingent on the entire network structure.*

For $N = 4$, the prices between 1 and 4 are (p_{41}, p_{44}) if 1 links with 2 and (p_{11}, p_{44}) otherwise; prices between 1 and 2 are (p_{11}, p_{12}) if 1 links with 4 and (p_{21}, p_{22}) otherwise; prices between 2

and 3 are (p_{33}, p_{22}) if 1 links with 4 and (p_{33}, p_{32}) otherwise, where

$$\begin{aligned} p_{41} &= p_{11} + \Phi(-z_1)c + \Phi(-z_4)c - 2\Pr(\tilde{h}_4 < 1)c; \\ p_{12} &= p_{22} + \max\left\{\delta + 2\Pr(\tilde{h}_4 < 1)c - 2\Pr\left(\frac{\tilde{a}_2 + \tilde{a}_3}{2} < 1\right)c + \Phi(-z_2)c - \Phi(-z_1)c, 0\right\}; \\ p_{21} &= p_{11} + \delta\sigma + 2\Pr\left(\frac{\tilde{a}_2 + \tilde{a}_3}{2} < 1\right)c + 2\Pr\left(\frac{\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3}{3} < 1\right)c \\ &\quad - 6\Pr(\tilde{h}_4 < 1)c + \Phi(-z_1)c + 2\Phi(-z_4)c - \Phi(-z_2)c; \\ p_{32} &= p_{22} + \delta\sigma + 4\Pr\left(\frac{\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3}{3} < 1\right)c - 6\Pr(\tilde{h}_4 < 1)c + 2\Phi(-z_4)c, \end{aligned}$$

and $\tilde{h}_4 = \frac{\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3 + \tilde{a}_4}{4}$. Under these conditional prices $\{p_{41}, p_{12}, p_{21}, p_{32}\}$, when $\bar{z} \geq \bar{z}_1$ or when $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta < \bar{\delta}(\bar{z})$, all firms are connected in the equilibrium network; when $\bar{z} \in [\bar{z}_2, \bar{z}_1]$ and $\delta > \bar{\delta}(\bar{z})$, distressed firm 4 is isolated in the equilibrium network.

Proof The proof is equivalent to show that under the bilateral contracts $\{(p_{41}, p_{14}), (p_{11}, p_{44})\}$, $\{(p_{11}, p_{12}), (p_{21}, p_{22})\}$, and $\{(p_{33}, p_{22}), (p_{33}, p_{32})\}$, $L^e = \{1-2-3, 4\} \iff L^* = \{1-2-3, 4\}$, and $L^e = \{4-1-2-3\} \iff L^* = \{4-1-2-3\}$. Next I check that firms have no incentive to deviate in the optimal network. In what follows, let $\hat{V}_i^L$ denote the value of firm i in network L under reservation prices $\{p_{ii}\}$, and V_i^L the value of firm i in network L under bilateral prices $\{p_{ji}\}$.

If $L^* = \{4-1-2-3\}$, under bilateral prices, firm values are

$$\begin{aligned} V_1^{4123}((p_{41}, p_{44}), (p_{11}, p_{12})) &= \sum_{i=1,2,4} \hat{V}_i^{4123} - \hat{V}_4^a - \hat{V}_2^{2-3}; \\ V_2^{4123}((p_{41}, p_{44}), (p_{11}, p_{12})) &= \hat{V}_2^{2-3}; \\ V_3^{4123}((p_{41}, p_{44}), (p_{11}, p_{12})) &= \hat{V}_3^{4123}; \\ V_4^{4123}(p_{41}, p_{44}) &= V_4^a. \end{aligned}$$

If $L^* = \{1-2-3, 4\}$, under bilateral prices, firm values are

$$\begin{aligned} V_1^{4,123}((p_{11}, p_{44}), (p_{21}, p_{22})) &= \sum_{i=1,2,4} \hat{V}_i^{4123} - \hat{V}_4^a - \hat{V}_2^{2-3}; \\ V_2^{4,123}((p_{21}, p_{22}), (p_{33}, p_{32})) &= \hat{V}_2^{2-3}; \\ V_3^{4,123}(p_{33}, p_{32}) &= \sum_{i=1}^3 \hat{V}_i^{123} + V_4^a - \sum_{i=1,2,4} \hat{V}_i^{4123}; \\ V_4^{4,123} &= V_4^a. \end{aligned}$$

When $L^* = \{4-1-2-3\}$, no firm would deviate, so $\{4-1-2-3\}$ is an equilibrium. In fact, by checking that all the other possible connection structures are not stable under these bilateral prices, full risk-sharing is the unique, stable, efficient equilibrium. When $L^* = \{1-2-3, 4\}$,

$\sum_{i=1}^3 V_i^{123} + V_4^a - \sum_{i=1,2,4} V_i^{4123} > \hat{V}_3^{4123}$. This implies that firm 3 is willing to pay for the premium price p_{32} . Similarly, by checking that all the other possible connection structures are not stable under these bilateral prices, full risk-sharing is the unique, stable, efficient equilibrium. Therefore, the above bilateral prices are able to decentralize the optimal network structures.

Next I show that under the price offering rules, these are the unique profit maximizing prices to decentralize the optimal networks. If $L^* = \{4 - 1 - 2 - 3\}$, we have $\sum_{i=1}^4 \hat{V}_i^{4123} \geq \sum_{i=1}^2 \hat{V}_i^{123,4} + V_4^a$. Under the outside prices, there is a large region in the $\{\bar{z}, \delta\}$ space in which 2 is better off to withdraw to form a link with 3. If this is the case, $\hat{V}_2^{2-3} > \hat{V}_2^{4123}$, we require that firm 1 pays a premium $\frac{1}{2}(p_{12} - p_{22}) = \hat{V}_2^{2-3} - \hat{V}_2^{4123}$ for $L_{12} = \frac{1}{2}$ share of asset swap. Since firm 1 is offering a take-it-or-leave-it offer to firm 2, the profit maximization behavior of firm 1 implies that

$$p_{12} = p_{22} + \max \left\{ 2 \left(\hat{V}_2^{2-3} - \hat{V}_2^{4123} \right), 0 \right\}.$$

The participation constraint for firm 4 implies that

$$\frac{1}{2}(p_{41} - p_{11}) \leq \hat{V}_4^{4123} - V_4^a. \quad (1.24)$$

For the equilibrium not to have the 1 - 4 link, we require that neither 2 nor 3 be worse off offering price premiums nor 1 is worse off accepting the prices. This means the sum of value of 1 and 2 and 3 is higher with 1 - 4 link than without,

$$\sum_{i=1}^3 \hat{V}_i^{123} \leq \sum_{i=1}^3 \hat{V}_i^{4123} + \frac{1}{2}(p_{41} - p_{11})$$

Combining with $\sum_{i=1}^4 \hat{V}_i^{4123} \geq \sum_{i=1}^2 \hat{V}_i^{123,4} + V_4^a$, we require that

$$\frac{1}{2}(p_{41} - p_{11}) \geq \hat{V}_4^{4123} - \hat{V}_4^a.$$

Combining with Equation (1.24), we obtain

$$\frac{1}{2}(p_{41} - p_{11}) = \hat{V}_4^{4123} - V_4^a.$$

Therefore, the value of firm 1 under $\{4 - 1 - 2 - 3\}$ is

$$V_1^{4123}((p_{41}, p_{44}), (p_{11}, p_{12})) = \hat{V}_1^{4123} + \hat{V}_4^{4123} - V_4^a - \frac{1}{2}(p_{12} - p_{22}) \leq \sum_{i=1,2,4} \hat{V}_i^{4123} - V_4^a - \hat{V}_2^{2-3}.$$

If instead $L^* = \{1 - 2 - 3, 4\}$, $\sum_{i=1}^2 \hat{V}_i^{123,4} + V_4^a \geq \sum_{i=1}^4 \hat{V}_i^{4123}$. It is sufficient to ensure pairwise stability if the following conditions hold: (1) 1 severs link with 4, (2) 2 stays link with

1, (3) 3 stays link with 2.

$$\begin{aligned}\hat{V}_1^{123} + \frac{1}{2}(p_{21} - p_{11}) &\geq \hat{V}_1^{4123} + \hat{V}_4^{4123} - \hat{V}_4^a - \frac{1}{2}(p_{12} - p_{22}) \\ \hat{V}_2^{123} - \frac{1}{2}(p_{21} - p_{11}) + \frac{1}{2}(p_{32} - p_{23}) &\geq \hat{V}_2^{4123} + \frac{1}{2}(p_{12} - p_{22}) \geq \hat{V}_2^{2-3} \\ \hat{V}_3^{123} - \frac{1}{2}(p_{32} - p_{23}) &\geq \hat{V}_3^{4123} \geq V_3^a\end{aligned}$$

Since firm 2 is offering the price premium, the minimum possible p_{21} is

$$\frac{1}{2}(p_{12} - p_{22}) = \hat{V}_1^{4123} + \hat{V}_4^{4123} - V_4^a - \frac{1}{2}(p_{12} - p_{22}) - \hat{V}_1^{123} = \hat{V}_1^{4123} + \hat{V}_4^{4123} - V_4^a - \hat{V}_1^{123} - \hat{V}_2^{2-3} + \hat{V}_2^{4123}.$$

Firm 3 needs to offer price premium p_{32} so that

$$\hat{V}_2^{123} - \frac{1}{2}(p_{21} - p_{11}) + \frac{1}{2}(p_{32} - p_{23}) \geq \hat{V}_2^{4123} + \frac{1}{2}(p_{12} - p_{22}) = \hat{V}_2^{2-3}.$$

So the minimum possible p_{32} is

$$\frac{1}{2}(p_{32} - p_{23}) = \hat{V}_2^{2-3} - \hat{V}_2^{123} + \hat{V}_1^{4123} + \hat{V}_4^{4123} - V_4^a - \hat{V}_1^{123} - \hat{V}_2^{2-3} + \hat{V}_2^{4123}.$$

Based on all the above analysis, the required profit-maximizing prices are uniquely given by

$$\begin{aligned}\frac{1}{2}(p_{21} - p_{11}) &= \hat{V}_1^{4123} + \hat{V}_4^{4123} - V_4^a - \hat{V}_1^{123} - \hat{V}_2^{2-3} + \hat{V}_2^{4123}; \\ \frac{1}{2}(p_{32} - p_{23}) &= \hat{V}_1^{4123} + \hat{V}_4^{4123} + \hat{V}_2^{4123} - \hat{V}_2^{123} - \hat{V}_1^{123} - V_4^a; \\ \frac{1}{2}(p_{12} - p_{22}) &= \max\left\{\left(\hat{V}_2^{2-3} - \hat{V}_2^{4123}\right), 0\right\}; \\ \frac{1}{2}(p_{41} - p_{11}) &= \hat{V}_4^{4123} - V_4^a.\end{aligned}$$

Substituting the values, we recover the prices in Proposition 6.

A.5 Extension with Government Bailout

This section provides the technical results for Section 5.2. I consider slight variations of the baseline model where the timing of the network formation does not coincide with the observation of distress. Under the set up in Section 5.2, if the regulators had optimally isolated the distressed N , the total liquidation cost is

$$C_{\text{iso-N}} = (N-1)\Phi\left[\sqrt{N-1}\left(-\bar{z} - \frac{1}{2}\delta\right)\right]c + \Phi\left[(k-1)\bar{z} + \frac{N-1}{2}\delta\right]c. \quad (1.25)$$

In the absence of the acquisition tax, all firms are connected and the liquidation costs are

$$C = \sum_{i=1}^N \Pr\left(\tilde{h}_i < 1\right)c = N\Phi\left[\frac{k-N}{\sqrt{N}}\bar{z}\right]c. \quad (1.26)$$

When we enable the option of *ex post* government bailout as in 5.2, the costs equal liquidation plus bailout costs,

$$C_{GB} = \sum_{i=1}^N \Pr(\tilde{h}_i < 1) c + B\sigma = N\Phi\left[\frac{(k-N)\bar{z} - B}{\sqrt{N}}\right] c + B\sigma. \quad (1.27)$$

Notice that $C = C_{GB}(B = 0)$. The gain from government bailout is $C - C_{GB}$. The next proposition shows that as long as the fixed liquidation cost c is large enough, a positive government bailout that at least matches the expected liquid value shortfall is *ex post* optimal.

Proposition 7 *If $c > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, $k > N$, the government bailout $B^*\sigma$ generates positive surplus, where*

$$B^* = (k - N)\bar{z} + \sqrt{N}\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}. \quad (1.28)$$

Proof The optimal liquid value injection policy B^* minimizes total costs C_{GB} and thus satisfies the first order condition $\frac{\partial C_{GB}}{\partial B} = 0$, i.e. $N\Phi'\left[\frac{(k-N)\bar{z} - B}{\sqrt{N}}\right] c \left(-\frac{1}{\sqrt{N}}\right) + \sigma = 0$. This gives

$$\Phi'\left[\frac{(k - N)\bar{z} - B^*}{\sqrt{N}}\right] = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(k-N)\bar{z} - B^*}{\sqrt{N}}\right]^2} = \frac{\sigma}{\sqrt{N}c}. \quad (1.29)$$

Solving for B^* gives (1.28). Given $e^{-\frac{1}{2}\left[\frac{(k-N)\bar{z} - B^*}{\sqrt{N}}\right]^2} \leq 1$, (1.29) implies $\frac{\sigma}{\sqrt{N}c} \leq \frac{1}{\sqrt{2\pi}}$. Hence $c \geq \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, i.e. the liquidation cost needs to be large enough.

Further, in order that B^* archives the global minimum of $C_{GB}(B)$, we take the second derivative of B ,

$$\frac{\partial^2 C_{GB}}{\partial B^2} = \Phi''\left[\frac{(k - N)\bar{z} - B^*}{\sqrt{N}}\right] c \geq 0, \quad \forall (k - N)\bar{z} - B^* \leq 0.$$

The second derivative is positive which ensures that B^* archives the global minimum of $C_{GB}(B)$, so the bailout surplus is positive, $C - C_{GB} = C_{GB}(B = 0) - C_{GB}(B = B^*) > 0$. $B^* \geq (k - N)\bar{z}$ requires that B^* at least matches the expected liquid value short fall, $B^*\sigma > (k - N)\bar{z}\sigma$. The extra liquidity injection depends on the uncertainty and cost tradeoff.

From Equation (1.28), $B^*\sigma$ at least matches the expected liquid value shortfall, $B^*\sigma > (k - N)\bar{z}\sigma$. $c > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$ ensures that B^* is non-zero. This requirement is easier to be satisfied when there are more counterparties to the distressed, and when uncertainty is lower. The extra liquidity injection, $\sqrt{N}\sqrt{-2\log\left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c}\right]}$, depends on the trade-off between cost and uncertainty. $\frac{\partial B^*}{\partial c} > 0$ implies that the bigger the liquidation cost is, the higher the optimal government bailout is; from $\frac{\partial B^*}{\partial \sigma} < 0$, optimal government bailout decreases with asset uncertainty.

If instead $0 \leq k \leq N$, the average distress after θ shock is positive. From Equation (1.28), a positive government bailout requires that $c \geq \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} e^{\frac{(N-k)^2 \bar{z}^2}{2N}} > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$. Plugging Equation (1.28)

into (1.27), the total costs under optimal bailout policy B^* is

$$C_{GB}^* = (k - N)\bar{z}\sigma + N\Phi \left[-\sqrt{-2\log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]} \right] c + \sqrt{N}\sigma \sqrt{-2\log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]}. \quad (1.30)$$

Although C_{GB}^* improves upon C , it is important to compare C_{GB}^* with the cost when the acquisition link had been prevented *ex ante*.

Proposition 8 *There exists $\bar{c} > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, such that $C_{GB}^* > C_{isoN}$ for $c \in [\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}, \bar{c}]$ and for all $\delta \geq 0$, where $c = \frac{\sigma B^*}{(N-1)\Phi[-\sqrt{N-1}\bar{z}] + \Phi[(k-1)\bar{z}] - N\Phi[\frac{(k-N)\bar{z} - B^*}{\sqrt{N}}]}$, and B^* is given by Equation (1.28).*

Therefore, when liquidation cost is bounded by $\bar{c}$, C_{GB}^* is more costly than C_{isoN} .

Next I consider the optimal policy when there are healthier institutions currently not connected with the distressed. Denote the existing firms $i = 1, \dots, N$ as group one. Now, consider group two of N other firms $i = N + 1, \dots, 2N$ with the same liquid value structure, $\bar{z} > 0, \sigma > 0$. For simplicity, let the dispersion δ among these firms be zero, so *ex ante* an optimal full risk-sharing network is formed. Let the additional signal θ_i be $\theta_{N+1} = \hat{k}\bar{z}\sigma$ and $\theta_i = 0, \forall i = N + 2, \dots, 2N$, so *ex post* the $N + 1$ th firm gets a positive shock in the liquid value. The next corollary examines whether the *ex post* acquisition of heavily distressed N by the liquid $N + 1$ can reduce total liquidation costs, and if not, whether subsidized acquisition is value increasing.

Corollary 1 *With no subsidy, the liquid firm $N + 1$ acquires the heavily distressed N if and only if $\hat{k} \geq k - 2N$. Government subsidized acquisition is *ex post* optimal if $\hat{k} < k - 2N$ and $c > \frac{\sqrt{\pi}\sigma}{\sqrt{N}}$; the optimal subsidy to the acquirer firm $N + 1$ upon acquisition is $B_A^*\sigma$, where*

$$B_A^* = \left(k - \hat{k} - 2N \right) \bar{z} + \sqrt{2N} \sqrt{-2\log \left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c} \right]}. \quad (1.31)$$

*When there exist healthier institutions, *ex post* subsidized acquisition is always preferred to *ex post* government bailout.*

When the cardinality of the two groups differs, pushed acquisition could be *ex post* optimal. Denote N_1 (instead of N) the number of the group one firms including the heavily distressed $\theta_{N_1} = -k\bar{z}\sigma$. Consider N_2 other firms (group two), with the same $\bar{z} > 0, \sigma > 0$, but $\delta = 0$ for simplicity. *Ex ante* an optimal full risk-sharing network is formed among N_2 firms. The additional signal is $\theta_{N_1+1} = \hat{k}\bar{z}\sigma$, $\theta_i = 0, \forall i = N_1 + 2, \dots, N_1 + N_2$. Hence firm $i = N_1 + 1$ has the highest liquid value *ex post*. Suppose after $t = 1$ when links in each group are formed and prices are exchanged, the most liquid firm can acquire the heavily distressed.

Proposition 9 *The social surplus of the acquisition is positive when the liquidity shocks satisfy*

$$\hat{k} > \max \left[\frac{\sqrt{N_1 + N_2} - \sqrt{N_1}}{\sqrt{N_1 + N_2} - \sqrt{N_2}} (k - N_1) - N_2, k - N_1 - N_2 \right]. \quad (1.32)$$

Under (1.32),

- when $N_2 \geq N_1$ the bilateral surplus is positive;
- when $N_2 < N_1$ the bilateral surplus is negative when

$$2\Phi\left[\frac{k - \hat{k} - (N_1 + N_2)}{\sqrt{N_1 + N_2}}\bar{z}\right]c > \Phi\left[\frac{k - N_1}{\sqrt{N_1}}\bar{z}\right]c + \Phi\left[\frac{-\hat{k} - N_2}{\sqrt{N_2}}\bar{z}\right]c + \frac{(N_2 - N_1)(N_2k + N_1\hat{k})}{N_1N_2(N_1 + N_2)}\bar{z}\sigma. \quad (1.33)$$

As a sufficient condition for a positive social surplus, (1.32) sets a lower bound for the positive liquidity shock $\hat{k}$. The relative cardinality of the two groups of firms is essential in determining the sign of the bilateral surplus. When $N_2 > N_1$, on average, the pair of $i = \{N_1, N_1 + 1\}$ gets bigger surplus than an average bank. When $N_1 = N_2$, we recover the case in subsection 5.2, so the sign of the bilateral surplus matches that of the social surplus. When $N_1 > N_2$, under condition (1.33), bilateral surplus can be negative even if social surplus is positive. (1.33) implies an upper bound for $\hat{k}$, hence is especially relevant when the potential acquirer does not have an abundant supply of liquidity.

B Proofs

B.1 Proof of Lemma 1

Before showing the properties of the asset composition matrix L^∞ , let us first check the features of the initial asset swap matrix L .

Claim 1 *The initial asset swap matrix L is a doubly stochastic matrix. Its largest eigenvalue is 1, and all the other eigenvalues lie within the unit circle.*

Proof By construction, $L \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}$. Thus L is a doubly stochastic matrix, $\lambda = 1$ is its eigenvalue with eigenvector $\mathbb{1}_{N \times 1}$. Suppose for contradiction that there exists an eigenvalue $\lambda > 1$. Then there exists a non-zero vector x such that $Lx = \lambda x > x$. However since the rows of L are non-negative and sum to 1, each element of vector Lx is a convex combination of the components of x . This implies that $\max[Lx] \leq \max[x]$, which contradicts with $\max[\lambda x] > \max[x]$. Hence all the eigenvalues cannot exceed 1 in absolute value.

More formally, we can resort to the properties of self-consistent norm. Let λ be the eigenvalues and x be the corresponding eigenvector. For any self-consistent matrix norm $\|\cdot\|$, we have

$$|\lambda| \times \|x\| = \|\lambda x\| = \|Lx\| \leq \|L\| \|x\|.$$

Because x is non-zero, $|\lambda| \leq \|L\| = \max_j (\sum_i L_{ij}) = 1$.

Lastly, we need to show that $\lambda = -1$ is not an eigenvalue of L . It is equivalent to show that the matrix $L + I$ is non-singular. This can be seen from

$$\det(L + I) = \det \begin{pmatrix} 2 - \sum_{j \neq 1} L_{ij} & L_{12} & \dots & \dots & L_{1M} \\ L_{21} & 2 - \sum_{j \neq 2} L_{ij} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 2 - \sum_{j \neq M-1} L_{ij} & \dots \\ L_{M1} & \dots & \dots & \dots & 2 - \sum_{j \neq M} L_{ij} \end{pmatrix}$$

All the off-diagonal elements are within 0 and 1. All the diagonal elements are within 1 and 2. For any column or row, the largest element is on the diagonal. Hence there are no columns or rows that are zero or linearly dependent. Therefore $\det(L + I) > 0$, and $\lambda = -1$ cannot be an eigenvalue of L .

Next we use the result from Claim 1 to show the limiting properties of $H = L^\infty$. Since L is a symmetric matrix, all the eigenvalues $\{\lambda_1, \lambda_2, \dots, \lambda_M\}$ are real. And there exists an orthogonal matrix Q with $Q' = Q^{-1}$ such that $L^\infty = Q\Lambda Q^{-1}$, where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$. And the columns of Q are eigenvectors of unit length corresponding to the eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_M$.

Without loss of generality, we rank the eigenvalues $\lambda_i \geq \lambda_{i+1}$, then

$$L^\infty = Q\Lambda Q^{-1} \dots Q\Lambda Q^{-1} = Q\Lambda^\infty Q^{-1} = Q \begin{bmatrix} \lambda_1^\infty & 0 & \dots & 0 \\ 0 & \lambda_2^\infty & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \lambda_M^\infty \end{bmatrix} Q^{-1} \rightarrow Q \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 \end{bmatrix} Q^{-1},$$

where the last step follows from $\lambda_i < 1$ and $\lim \lambda_i^\infty = 0, \forall i \neq 1$. Let the first column of Q , which is the unit length eigenvector corresponding to $\lambda_1 = 1$ be x_1 , then

$$\begin{pmatrix} 1 - \sum_{j \neq 1} H_{ij} & H_{12} & \dots & \dots & H_{1M} \\ H_{21} & 1 - \sum_{j \neq 2} H_{ij} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 1 - \sum_{j \neq M-1} H_{ij} & \dots \\ H_{M1} & \dots & \dots & \dots & 1 - \sum_{j \neq M} H_{ij} \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \\ \dots \\ \dots \\ x_{1M} \end{pmatrix} = \begin{pmatrix} x_{11} \\ x_{12} \\ \dots \\ \dots \\ x_{1M} \end{pmatrix}. \quad (1.34)$$

$$\begin{pmatrix} x_{11} & x_{12} & \dots & \dots & x_{1M} \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{12} \\ \dots \\ \dots \\ x_{1M} \end{pmatrix} = 1. \quad (1.35)$$

Combining Equations (1.34) and (1.35), we can solve for the unit length eigenvectors as $x_{11} = x_{12} = \dots = x_{1M} = \frac{1}{\sqrt{M}}$. In this case,

$$L^\infty = Q \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 \end{bmatrix} Q^{-1} = \begin{bmatrix} x_{11}^2 & x_{11}x_{12} & \dots & x_{11}x_{1M} \\ x_{12}x_{11} & x_{12}^2 & \dots & \dots \\ \dots & \dots & \dots & \dots \\ x_{1M}x_{11} & \dots & \dots & x_{1M}^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{M} & \frac{1}{M} & \dots & \frac{1}{M} \\ \frac{1}{M} & \frac{1}{M} & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \frac{1}{M} & \dots & \dots & \frac{1}{M} \end{bmatrix}.$$

Hence L^∞ coincides with full risk-sharing regardless of the initial values of L_{ij} in L .

Finally, since L is a doubly stochastic matrix, $L \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}$, $L^\top \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}$. Then $L^N \times \mathbb{1}_{N \times 1} = L^{N-1} \times L \times \mathbb{1}_{N \times 1} = L^{N-1} \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}$. Similarly $L^{\top N} \times \mathbb{1}_{N \times 1} = \mathbb{1}_{N \times 1}$, so $H = L^\infty$ is also a doubly stochastic matrix. **Q.E.D.**

B.2 Proof of Proposition 1

Let us start by analyzing the risk-sharing decision of $N = 2$ in the following lemma.

Lemma 3 *The risk-sharing surplus for $N = 2$ is positive if and only if $\bar{z} \geq 0$; the risk-sharing surplus increases monotonically with δ .*

Proof The total liquidation cost for two separate firms with distress $\{\bar{z} + \frac{1}{2}\delta, \bar{z} - \frac{1}{2}\delta\}$ is $\Pr(\tilde{a}_1 \leq 1) c + \Pr(\tilde{a}_2 \leq 1) c = \Phi[-\bar{z} - \frac{1}{2}\delta] c + \Phi[-\bar{z} + \frac{1}{2}\delta] c$. The total liquidation cost for the two firms to fully share risk is $\Pr(\tilde{h}_1 \leq 1) c + \Pr(\tilde{h}_2 \leq 1) c = 2\Phi[-\sqrt{2}\bar{z}] c$. The bilateral risk-sharing surplus is given by

$$\Pr(\tilde{a}_1 \leq 1) c + \Pr(\tilde{a}_2 \leq 1) c - 2\Pr(\tilde{h}_1 \leq 1) c = \Phi\left[-\bar{z} - \frac{1}{2}\delta\right] c + \Phi\left[-\bar{z} + \frac{1}{2}\delta\right] c - 2\Phi\left[-\sqrt{2}\bar{z}\right] c.$$

Function $\Phi(x)$ is monotonically increasing for all x , and is convex for $x < 0$ ($\Phi'' > 0, \forall x < 0$). Therefore $\Phi[-\bar{z} - \frac{1}{2}\delta] c + \Phi[-\bar{z} + \frac{1}{2}\delta] c \geq 2\Phi[-\bar{z}] c \geq 2\Phi[-\sqrt{2}\bar{z}] c$, i.e. the surplus is positive whenever $\bar{z} > 0, \delta > 0$. The first derivative with respect to δ is $-\frac{1}{2}\Phi'[-\bar{z} - \frac{1}{2}\delta] c + \frac{1}{2}\Phi'[-\bar{z} + \frac{1}{2}\delta] c = \frac{c}{2} (\Phi'[-\bar{z} + \frac{1}{2}\delta] - \Phi'[-\bar{z} - \frac{1}{2}\delta]) > 0$, for $-\bar{z} + \frac{1}{2}\delta > -\bar{z} - \frac{1}{2}\delta$, i.e. $\delta > 0$. $\square$

Next I show that δ matters for the optimal risk-sharing policy for $N \geq 3$. The total default probability in a full risk-sharing network with N firms is

$$\sum_{i=1}^N \Pr(\tilde{h}_i \leq 1) = N \times \Phi\left[\sqrt{N}(-\bar{z})\right].$$

The total default probability when the best $N - 1$ firms fully share risk, isolating the most distressed firm is

$$\sum_{i=1}^{N-1} \Pr(\tilde{h}_i \leq 1) + \Pr(\tilde{a}_N \leq 1) = (N - 1) \times \Phi\left[\sqrt{N - 1}(-\bar{z} - \frac{1}{2}\delta)\right] + \Phi\left[-\bar{z} - \frac{1 - N}{2}\delta\right].$$

The difference between the above two terms is

$$\sum_{i=1}^{N-1} \Pr(\tilde{h}_i \leq 1) + \Pr(\tilde{a}_N \leq 1) - \sum_{i=1}^N \Pr(\tilde{h}_i \leq 1) \quad (1.36)$$

$$= (N-1) \times \Phi \left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta) \right] + \Phi \left[-\bar{z} - \frac{1-N}{2}\delta \right] - N \times \Phi \left[\sqrt{N}(-\bar{z}) \right]. \quad (1.37)$$

When $\delta \rightarrow \infty$, the limit becomes

$$\lim_{\delta \rightarrow \infty} \sum_{i=1}^{N-1} \Pr(\tilde{h}_i \leq 1) + \Pr(\tilde{a}_N \leq 1) - \sum_{i=1}^N \Pr(\tilde{h}_i \leq 1) = 1 - N \times \Phi \left[\sqrt{N}(-\bar{z}) \right].$$

Then as long as $\bar{z}$ is large enough that $\Phi \left[\sqrt{N}(-\bar{z}) \right] < \frac{1}{N}$, full risk-sharing dominates, i.e. $\bar{z}_1 = -\frac{1}{N}\Phi^{-1}(\frac{1}{N})$. If we consider an upper bound on δ in our analysis, say the upper bound is $\delta \leq 2$, then equating Equation (1.36) to zero and plugging in $\delta = 2$, $\bar{z}_1$ solves

$$(N-1)\Phi \left[\sqrt{N-1}(-\bar{z}) \right] + \Phi \left[N - \bar{z} \right] = N\Phi \left[\sqrt{N}(-\bar{z}) \right].$$

When the best $N-2$ firms fully share risk whereas the two most distressed firms are isolated, we have that the total default probability becomes

$$\begin{aligned} & \sum_{i=1}^{N-2} \Pr(\tilde{h}_i \leq 1) + \Pr(\tilde{a}_{N-1} \leq 1) + \Pr(\tilde{a}_N \leq 1) \\ &= (N-2)\Phi \left[\sqrt{N-2}(-\bar{z} - \delta) \right] + \Phi \left[-\bar{z} - \frac{3-N}{2}\delta \right] + \Phi \left[-\bar{z} - \frac{1-N}{2}\delta \right]. \end{aligned}$$

The difference between isolating one or two distressed firms is

$$\begin{aligned} & \sum_{i=1}^{N-2} \Pr(\tilde{h}_i \leq 1) + \Pr(\tilde{a}_{N-1} \leq 1) + \Pr(\tilde{a}_N \leq 1) - \sum_{i=1}^{N-1} \Pr(\tilde{h}_i \leq 1) - \Pr(\tilde{a}_N \leq 1) \\ &= (N-2)\Phi \left[\sqrt{N-2}(-\bar{z} - \delta) \right] + \Phi \left[-\bar{z} - \frac{3-N}{2}\delta \right] \\ & \quad - (N-1) \times \Phi \left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta) \right]. \end{aligned}$$

When $\delta \rightarrow \infty$, the limit of the function is 1. When $\bar{z} = 0$, the RHS becomes $(N-2)\Phi \left[\sqrt{N-2}(-\delta) \right] + \Phi \left[\frac{N-3}{2}\delta \right] - (N-1) \times \Phi \left[\sqrt{N-1}(-\frac{1}{2}\delta) \right] < 0$ for small values of δ and when $N > 4$. The curve $(N-2)\Phi \left[\sqrt{N-2}(-\bar{z} - \delta) \right] + \Phi \left[-\bar{z} - \frac{3-N}{2}\delta \right] = (N-1)\Phi \left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta) \right]$ is concave with δ and convex with $\bar{z}$. Denote $\bar{z}_2$ the maximum value of $\bar{z}$ on this curve. Then for $\bar{z} > \bar{z}_2$, isolating one distressed firm is preferred to isolating two firms. In this case, the cutoff curve $\delta_1(\bar{z})$ is defined by

$$(N-1) \times \Phi \left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta_1(\bar{z})) \right] + \Phi \left[-\bar{z} - \frac{1-N}{2}\delta_1(\bar{z}) \right] = N \times \Phi \left[\sqrt{N}(-\bar{z}) \right].$$

From the implicit function theorem, the curve is well-defined, and

$$\frac{\partial \delta_1(\bar{z})}{\partial \bar{z}} = - \frac{N\sqrt{N}\Phi' \left[\sqrt{N}(-\bar{z}) \right] - (N-1)\sqrt{N-1}\Phi' \left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta) \right] - \Phi' \left[-\bar{z} - \frac{1-N}{2}\delta \right]}{\frac{N-1}{2} \left(\Phi' \left[-\bar{z} + \frac{N-1}{2}\delta \right] - \sqrt{N-1}\Phi' \left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta) \right] \right)} > 0.$$

Q.E.D.

B.3 Proof of Proposition 2

The proof is equivalent to show that there do not exist bilateral prices (p_{21}, p_{12}) , (p_{41}, p_{14}) , (p_{32}, p_{23}) that can decentralize the optimal network in the parameter region $(\{\bar{z} \in [1, \bar{z}_1], \delta > \delta_1(\bar{z})\})$. In other words, when $L^* = \{4, 1-2-3\}$, there does not exist a feasible premium price p_{21} offered by firm 2 that prevents firm 1 from linking with 4.

In what follows, let $\hat{V}_i^L$ denote the value of firm i in network L under reservation prices $\{p_{ii}\}$, and V_i^L the value of firm i in network L under bilateral prices $\{p_{ji}\}$.

If $L^* = \{4-1-2-3\}$, we have $\sum_{i=1}^4 \hat{V}_i^{4-1-2-3} \geq \sum_{i=1}^3 \hat{V}_i^{1-2-3} + V_4^a$. Under the outside prices, there is a large region in the $\{\bar{z}, \delta\}$ space in which 2 is better off to withdraw and form risk-sharing pair with 3. If this is the case, $\hat{V}_2^{2-3} > \hat{V}_2^{4-1-2-3}$, we require that firm 1 pays at least a premium $\frac{1}{2}(p_{12} - p_{22}) = \hat{V}_2^{2-3} - \hat{V}_2^{4-1-2-3}$ for $L_{12} = \frac{1}{2}$ share of asset swap so that $V_2^{4-1-2-3}(p_{12}, p_{11}) = \hat{V}_2^{2-3}$.

$$V_1^{4-1-2-3}((p_{41}, p_{44}), (p_{22}, p_{12})) = \hat{V}_1^{4-1-2-3} + \frac{1}{2}(p_{41} - p_{11}) - \frac{1}{2}(p_{12} - p_{22}) \geq \hat{V}_1^{1-2-3}; \quad (1.38)$$

$$V_2^{4-1-2-3}((p_{22}, p_{12}), (p_{32}, p_{23})) = \hat{V}_2^{4-1-2-3} + \frac{1}{2}(p_{12} - p_{22}) + \frac{1}{2}(p_{32} - p_{23}) \geq \hat{V}_2^{2-3}; \quad (1.39)$$

$$V_3^{4-1-2-3}(p_{32}, p_{23}) = \hat{V}_3^{4-1-2-3} - \frac{1}{2}(p_{32} - p_{23}) \geq V_3^a. \quad (1.40)$$

$$V_4^{4-1-2-3}(p_{41}, p_{44}) = \hat{V}_4^{4-1-2-3} - \frac{1}{2}(p_{41} - p_{11}) \geq V_4^a. \quad (1.41)$$

From (1.39), the minimum price offered by 1 is

$$\frac{1}{2}(p_{12} - p_{22}) = \hat{V}_2^{2-3} - \hat{V}_2^{4-1-2-3} - \frac{1}{2}(p_{32} - p_{23}). \quad (1.42)$$

Let us pick the upper bound of prices $\frac{1}{2}(p_{32} - p_{23})$ and $\frac{1}{2}(p_{41} - p_{11})$ from the participation constraints (1.40) and (1.41), the value firm 1 gets by linking with 4 is

$$\begin{aligned} & V_1^{4-1-2-3}((p_{41}, p_{44}), (p_{22}, p_{12})) \\ &= \hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \frac{1}{2}(p_{32} - p_{23}) + \frac{1}{2}(p_{41} - p_{11}) - \hat{V}_2^{2-3} \\ &= \hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \hat{V}_4^{4-1-2-3} - V_4^a + \hat{V}_3^{4-1-2-3} - \hat{V}_3^a - \hat{V}_2^{2-3} \\ &> \sum_{i=1}^4 \hat{V}_i^{4-1-2-3} - V_4^a - \hat{V}_3^a - \hat{V}_2^{2-3} > \sum_{i=1}^4 \hat{V}_i^{4-1-2-3} - V_4^a - \hat{V}_3^{2-3} - \hat{V}_2^{2-3} \\ &> \hat{V}_1^{1-4} + \hat{V}_4^{1-4} - V_4^a. \end{aligned}$$

This shows that paying the premium (1.42) to prevent 2 from withdrawing is always a dominating strategy for firm 1. The value of 1 in $L = \{4 - 1 - 2 - 3\}$ is

$$\begin{aligned} V_1^{4-1-2-3}((p_{41}, p_{44}), (p_{22}, p_{12})) \\ = \hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \frac{1}{2}(p_{32} - p_{23}) + \hat{V}_4^{4-1-2-3} - V_4^a - \hat{V}_2^{2-3}. \end{aligned}$$

And equilibrium replicates the optimal connection $L^e = L^* = \{4 - 1 - 2 - 3\}$.

If $L^* = \{4, 1 - 2 - 3\}$, we have $\sum_{i=1}^3 \hat{V}_i^{1-2-3} + V_4^a \geq \sum_{i=1}^4 \hat{V}_i^{4-1-2-3}$. Under the outside prices, for all the region, firm 1 wants to link with 4 and firm 2 wants to withdraw. We require that firm 2 pays at least a premium $\frac{1}{2}(p_{21} - p_{11})$ to prevent 1 from linking with 4.

$$V_1^{4,1-2-3}(p_{11}, p_{21}) = \hat{V}_1^{1-2-3} + \frac{1}{2}(p_{21} - p_{11}) \geq V_1^{4-1-2-3}((p_{41}, p_{44}), (p_{22}, p_{12})); \quad (1.43)$$

$$V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})) = \hat{V}_2^{1-2-3} - \frac{1}{2}(p_{21} - p_{11}) + \frac{1}{2}(p_{32} - p_{23}) \geq \hat{V}_2^{2-3}; \quad (1.44)$$

$$\begin{aligned} V_3^{4,1-2-3}(p_{32}, p_{23}) &= \hat{V}_3^{1-2-3} - \frac{1}{2}(p_{32} - p_{23}) \geq V_3^a. \\ V_4^{4,1-2-3} &= V_4^a. \end{aligned}$$

Notice that (p_{32}, p_{23}) is not contingent on the link of 1 - 4, thus it has the same value in both structures. From (1.43), the minimum required incentive offered by firm 2 to 1 is

$$\frac{1}{2}(p_{21} - p_{11}) = \hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \frac{1}{2}(p_{32} - p_{23}) + \hat{V}_4^{4-1-2-3} - V_4^a - \hat{V}_2^{2-3} - \hat{V}_1^{1-2-3}.$$

Plugging into (1.44), the value of firm 2 then becomes

$$\begin{aligned} V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})) &= \hat{V}_2^{1-2-3} - \frac{1}{2}(p_{21} - p_{11}) + \frac{1}{2}(p_{32} - p_{23}) \\ &= \hat{V}_2^{2-3} + \underbrace{\hat{V}_2^{1-2-3} + \hat{V}_4^a + \hat{V}_1^{1-2-3} - (\hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \hat{V}_4^{4-1-2-3})}_{\text{total surplus of firms 1,2,4}}. \end{aligned}$$

where $\hat{V}_2^{1-2-3} + V_4^a + \hat{V}_1^{1-2-3} - (\hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \hat{V}_4^{4-1-2-3})$ is the group surplus of 1,2,4 in $\{4, 1 - 2 - 3\}$ compared to that in $\{4 - 1 - 2 - 3\}$. The surplus can be expressed as $-\frac{1}{2}\delta - \Phi[-\bar{z} + \frac{3}{2}\delta] - 2\Phi[-\sqrt{3}\bar{z} - \frac{\sqrt{3}}{2}\delta] + 3\Phi[-2\bar{z}]$. When evaluated at $\delta = 0$, the surplus is $-\Phi[-\bar{z}] - 2\Phi[-\sqrt{3}\bar{z}] + 3\Phi[-2\bar{z}] < 0$. Take the derivative of δ , it establishes that

$$\sqrt{3}\Phi'[-\sqrt{3}\bar{z} - \frac{\sqrt{3}}{2}\delta] - \frac{3}{2}\Phi'[-\bar{z} + \frac{3}{2}\delta] - \frac{1}{2} - 6\Phi'[-2\bar{z}] < 0, \forall \delta > 0, \bar{z} > 0.$$

which follows from $\frac{3}{2}\Phi'[-\bar{z} + \frac{3}{2}\delta] + \frac{1}{2} + 6\Phi'[-2\bar{z}] > \Phi'(0) + \Phi'[-2\bar{z}] > 2\Phi'[-\bar{z}] > \sqrt{3}\Phi'[-\bar{z}] >$

$\sqrt{3}\Phi' \left[-\sqrt{3}\bar{z} - \frac{\sqrt{3}}{2}\delta \right]$. Therefore

$$\hat{V}_2^{1-2-3} + \hat{V}_4^a + \hat{V}_1^{1-2-3} < \hat{V}_1^{4-1-2-3} + \hat{V}_2^{4-1-2-3} + \hat{V}_4^{4-1-2-3}.$$

This further implies

$$V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})) < \hat{V}_2^{2-3} = V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})),$$

Firm 2 is worse off providing the required premium price p_{21} than staying in the full connection $\{4-1-2-3\}$. Therefore, the efficient network is not stable. In other words, the equilibrium fails to replicate the optimal connection $L^e = \{4-1-2-3\} \neq L^* = \{4, 1-2-3\}$. This further implies

$$V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})) < \hat{V}_2^{2-3} = V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})).$$

I next show that even if we relax the price offering rule, there still do not exist feasible bilateral prices between 1 and 2 to effectively prevent the $4-1$ link. For $L^* = \{4, 1-2-3\}$ to be stable, we require (1.38), (1.39), (1.43), and

$$V_2^{4,1-2-3}((p_{11}, p_{21}), (p_{32}, p_{23})) = \hat{V}_2^{1-2-3} - \frac{1}{2}(p_{21} - p_{11}) + \frac{1}{2}(p_{32} - p_{23}) \geq V_2^{4-1-2-3}((p_{22}, p_{12}), (p_{32}, p_{23})).$$

Combining all these conditions, we require

$$\hat{V}_2^{1-2-3} + \hat{V}_1^{1-2-3} \geq \hat{V}_2^{4-1-2-3} + \hat{V}_1^{4-1-2-3} + \frac{1}{2}(p_{41} - p_{11}).$$

From (1.41), we require

$$\hat{V}_2^{1-2-3} + \hat{V}_1^{1-2-3} \geq \hat{V}_2^{4-1-2-3} + \hat{V}_1^{4-1-2-3} + \hat{V}_4^{4-1-2-3} - V_4^a. \quad (1.45)$$

Consider the region around cutoff curve $\delta_1(\bar{z})$, where $\sum_{i=1}^3 \hat{V}_i^{1-2-3} + V_4^a = \epsilon + \sum_{i=1}^4 \hat{V}_i^{4-1-2-3}$. The total values under L^* is only slightly greater than that under $L = \{4-1-2-3\}$, but the value difference for firm 3 is big especially when dispersion δ is large,

$$\hat{V}_3^{1-2-3} - \hat{V}_3^{4-1-2-3} = \frac{1}{2}\delta + \Pr(\tilde{h}_4 < 1)c - \Pr\left(\frac{\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3}{3} < 1\right)c > \epsilon,$$

In this case, (1.45) does not hold: there does not exist bilateral price (p_{21}, p_{12}) to prevent the formation of the $4-1$ link. **Q.E.D.**

B.4 Proof of Proposition 3

First consider the case with $N = 5$ firms. The distress vector is $z = \{\bar{z} + 2\delta, \bar{z} + \delta, \bar{z}, \bar{z} - \delta, \bar{z} - 2\delta\}$. I next show that the proposition holds in different equilibrium networks, $\{5-1-2-3-4\}$, $\{5-1-3-2-4\}$, $\{5-1-4-3-2\}$, $\{5-1-4-2-3\}$. In all these structures, I start from the optimal full risk-sharing network and solve for the bilateral prices that decentralize the full

risk-sharing network. Then I fix the contracts between 2, 3, 4 (because these contract should not change with the link 1 – 5), and check whether agents can optimally decentralize the network to isolate the distressed bank, and whether any agent deviates from full connection.

Case 1. I check the stability and efficiency of the chain $\{5 - 1 - 2 - 3 - 4\}$. Firm values under outside prices are given by $\{\hat{V}_1^{51234}, \hat{V}_2^{51234}, \hat{V}_3^{51234}, \hat{V}_4^{51234}, \hat{V}_5^{51234}\}$.

Notice that the bilateral prices between 2 – 3 – 4 are not contingent on L_{15} , whereas those between 1 and 2 ($p_{12}(L_{15})$) is a function of L_{15} . To decentralize the optimal full network, let $p_{43} - p_{34} = \frac{1}{2}\delta\sigma$, $p_{32} - p_{23} = \frac{1}{2}\delta\sigma$, $\frac{1}{2}(p_{12}(L_{15}) - p_{22}) = \sum_{i=2}^4 \hat{V}_i^{234} - V_3^a - V_4^a - \hat{V}_2^{51234}$, so that $V_2^{51234} = \sum_{i=2}^4 \hat{V}_i^{234} - V_3^a - V_4^a$ (outside option of 2 in $\{4 - 2 - 3\}$). As a result, $V_1^{51432} = \hat{V}_1^{51432} + \hat{V}_5^{51432} - V_5^a + \hat{V}_2^{51432} - \sum_{i=2}^4 \hat{V}_i^{234} + V_3^a + V_4^a$; $V_3^{51234} = \hat{V}_3^{51234}$, $V_4^{51234} = \hat{V}_4^{51234} - \frac{1}{2}\delta\sigma$.

I then check that it is not feasible for 2 to offer incentive to 1 not to link with 5, i.e. $L^* = \{5, 1 - 2 - 3 - 4\}$ cannot be decentralized.

Next I check if the full-connection is pairwise stable. Since both 2 and 5 are offered by the contingent contracts exactly their respective outside options, we only need to check the deviation incentives for firm 1, 3, 4. Results show that (1) neither 3 or 4 deviates; (2) there is a region in which 1 is better off severing the linkage with 2 so $L^e = \{5 - 1, 3 - 2 - 4\}$; (3) for the rest regions, full connection is stable so $L^e = \{5 - 1 - 2 - 3 - 4\}$.

Case 2. I check the stability of $\{5 - 1 - 3 - 2 - 4\}$ using the same logic. Firm values under outside prices are given by $\{\hat{V}_1^{51324}, \hat{V}_2^{51324}, \hat{V}_3^{51324}, \hat{V}_4^{51324}, \hat{V}_5^{51324}\}$.

Let the bilateral prices be $p_{23} - p_{32} = \frac{1}{2}\delta\sigma$, $\frac{1}{2}(p_3^1(L_{15}) - p_{33}) = \hat{V}_3^a - \hat{V}_3^{51324} - \frac{1}{2}\delta\sigma$, so that $V_3^{51324} = V_3^a$ (outside option of 3 in $\{4 - 2 - 3\}$). So $V_1^{51324} = \hat{V}_1^{51324} + \hat{V}_5^{51324} - V_5^a + \hat{V}_3^{51324} - V_3^a + \frac{1}{2}\delta\sigma$, $V_2^{51324} = \hat{V}_2^{51324} - \frac{1}{2}\delta\sigma$, $V_3^{51324} = V_3^a$, $V_4^{51324} = \hat{V}_4^{51324}$, $V_5^{51324} = V_5^a$.

In this case, it is not feasible for 3 to offer incentive to 1 not to link with 5, i.e. $L^* = \{5, 1 - 3 - 2 - 4\}$ cannot be decentralized.

I then check if full-connection (all banks connected in one component) is stable. After computing the deviation incentives of 1, 2, 4, results show that (1) neither banks 2 or 4 deviates; (2) there is a region in which 1 is better off severing the linkage with 3 and so $L^e = \{5 - 1, 3 - 2 - 4\}$; (3) for the rest regions, full connection is stable so $L^e = \{5 - 1 - 3 - 2 - 4\}$.

Case 3. I check the stability of $\{5 - 1 - 4 - 3 - 2\}$ using the same logic. Firm values under outside prices are given by $\{\hat{V}_1^{51432}, \hat{V}_2^{51432}, \hat{V}_3^{51432}, \hat{V}_4^{51432}, \hat{V}_5^{51432}\}$.

In order to decentralize the full risk-sharing network, we require that the bilateral prices between 1 and 4 be contingent on L_{15} to prevent 4 from withdrawing, and that the bilateral prices between 4 and 3, 3 and 2 be independent of L_{15} link.

Let $p_{43} - p_{34} = \frac{1}{2}\delta\sigma$, $p_{32} - p_{23} = \frac{1}{2}\delta\sigma$, and $\frac{1}{2}(p_{14}(L_{15}) - p_{44}) = V_4^a - \hat{V}_4^{51432} + \frac{1}{2}\delta\sigma$, so that $V_4^{51432} = V_4^a$ (outside option of 4 in $\{4 - 2 - 3\}$). So $V_1^{51432} = \hat{V}_1^{51432} + \hat{V}_5^{51432} - V_5^a + \hat{V}_4^{51432} - \frac{1}{2}\delta\sigma - V_4^a$, $V_2^{51432} = \hat{V}_2^{51432} + \frac{1}{2}\delta\sigma$, $V_3^{51432} = \hat{V}_3^{51432}$, $V_4^{51432} = V_4^a$, $V_5^{51432} = V_5^a$. Similarly, it is

not feasible for 4 to offer incentive to 1 not to link with 5, i.e. $L^* = \{5, 1-4-2-3\}$ cannot be decentralized.

Next check whether the full-connection is pairwise stable by computing the deviation incentives for 1, 2, 3. It turns out that for a large region, $V_2^{51432} < V_2^a$ and firm 2 withdraws from the end of the chain. Given that 2 withdraws, we need to check if $\{5-1-4-3\}$ is stable, we compare $V_1^{5-1-4-3} = \hat{V}_1^{5-1-4} + \hat{V}_4^{5-1-4} + \hat{V}_5^{5-1-4} - V_5^a - V_4^a$ with $\hat{V}_1^{1-5}$. If $V_1^{5-1-4-3} > \hat{V}_1^{1-5}$, $L^e = \{5-1-4-3, 2\}$; otherwise $L^e = \{5-1, 4-2-3\}$, which has wrong network composition compared to L^* .

Case 4. We move to check the stability of $\{5-1-4-2-3\}$. Firm values under outside prices are given by $\{\hat{V}_1^{51423}, \hat{V}_2^{51423}, \hat{V}_3^{51423}, \hat{V}_4^{51423}, \hat{V}_5^{51423}\}$.

Let $p_{23} - p_{32} = \frac{1}{2}\delta\sigma$, and $\frac{1}{2}(p_{14}(L15) - p_{44}) = V_4^a - V_4^{51432}$, so that $V_4^{51432} = V_4^a$ (outside option of 4 in $\{4-2-3\}$). So $V_1^{51432} = \hat{V}_1^{51432} + \hat{V}_5^{51432} - V_5^a + \hat{V}_4^{51432} - V_4^a$, $V_2^{51432} = \hat{V}_2^{51432} - \frac{1}{2}\delta\sigma$, $V_3^{51432} = \hat{V}_3^{51432} + \frac{1}{2}\delta\sigma$, $V_4^{51432} = V_4^a$, $V_5^{51432} = V_5^a$.

Compared to the previous case, now firm 3 is at the end of the chain. In most of the regions, 3 withdraws when $\hat{V}_3^{51432} - V_3^a < 0$. Given this, the outside option of 2 is to form a pair with 3, i.e. $V_2^{outside} = 1 + \bar{z} + \delta\sigma - \Pr\left(\frac{\tilde{a}_2 + \tilde{a}_3}{2} < 1\right) - \frac{\Phi(-z_2) - \Phi(-z_3)}{2}$. So we then check whether 2 deviates by comparing $\hat{V}_2^{5142}$ and $V_2^{outside}$. If 2 does not withdraw, $L^e = \{5-1-4-2, 3\}$. When 2 withdraws, $\{5-1-4\}$ is not stable, and the equilibrium network becomes $L^e = \{5-1, 4-2-3\}$. [Q.E.D.](#)

B.5 Proof of Proposition 4

I prove this proposition in a four-firm network setting. The inefficiency occurs in the region $(\bar{z} \in [1, \bar{z}_1], \delta > \delta_1(\bar{z}))$, where $L^* = \{4, 1-2-3\}$ and $L^e = \{4-1-2-3\}$. The value loss equals the difference of the total firm values at L^* compared to L^e ,

$$\begin{aligned} V^{loss} &= \sum_{i=1}^3 V_i^{1-2-3} + V_4^a - \sum_{i=1}^4 V_i^{4-1-2-3} \\ &= 4\bar{z} - 3\Pr\left(\frac{\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3}{3} < 1\right)c - \Phi(-z_4) - \left(4\bar{z} - 4\Pr(\tilde{h}_4 < 1)c\right) \\ &= 4 \times \Phi[2(-\bar{z})] - 3\Phi\left[\sqrt{3}\left(-\bar{z} - \frac{1}{2}\delta\right)\right] - \Phi\left[-\bar{z} + \frac{3}{2}\delta\right]. \end{aligned}$$

The value loss has the following properties. First, from Proposition 2, $V^{loss} > 0$, $\bar{z} \in$

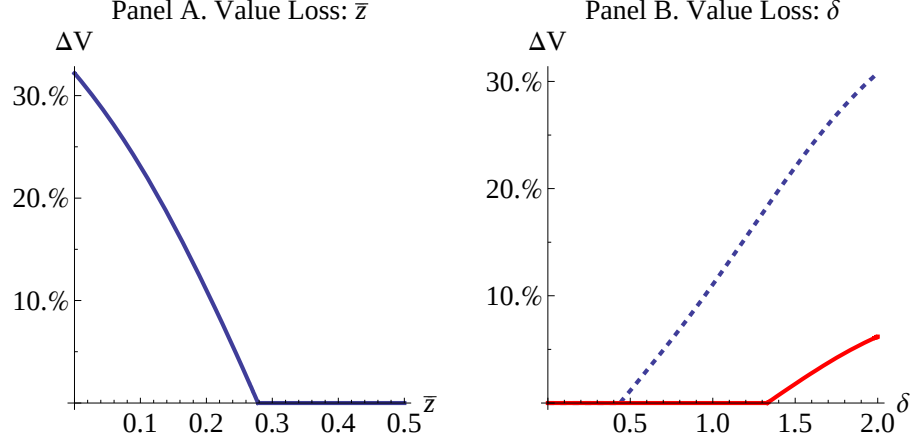


Figure A.II. Percentage Value Loss. This figure plots the percentage value loss against $\bar{z}$ and δ for the equilibrium four-firm chain network.

$[1, \bar{z}_1], \delta > \delta_1(\bar{z})$. Second,

$$\begin{aligned}
 \frac{\partial V^{loss}}{\partial \delta} &= \frac{3\sqrt{3}}{2} \Phi' \left[\sqrt{3} \left(-\bar{z} - \frac{1}{2} \delta \right) \right] - \frac{3}{2} \Phi' \left[-\bar{z} + \frac{3}{2} \delta \right] \\
 &= \frac{3\sqrt{3}}{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{3}{2} \left(-\bar{z} - \frac{1}{2} \delta \right)^2} - \frac{3}{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(-\bar{z} + \frac{3}{2} \delta \right)^2} \\
 &= \frac{3}{2} \frac{1}{\sqrt{2\pi}} \left(\sqrt{3} e^{-\frac{3}{2} \left(-\bar{z} - \frac{1}{2} \delta \right)^2} - e^{-\frac{1}{2} \left(-\bar{z} + \frac{3}{2} \delta \right)^2} \right) > 0, \quad \bar{z} \in [0, \bar{z}_1], \delta > \delta_1(\bar{z}).
 \end{aligned}$$

So V^{loss} increases with δ . Third, the value loss also decreases with average financial distress $\bar{z}$,

$$\frac{\partial V^{loss}}{\partial \bar{z}} = -8\Phi' [2(-\bar{z})] + 3\sqrt{3}\Phi' \left[\sqrt{3} \left(-\bar{z} - \frac{1}{2} \delta \right) \right] + \Phi' \left[-\bar{z} + \frac{3}{2} \delta \right] < 0.$$

Finally, the cross-derivative of value loss with respect $\bar{z}$ and δ is negative

$$\frac{\partial^2 V^{loss}}{\partial \delta \partial \bar{z}} = \frac{3}{2} \Phi'' \left[-\bar{z} + \frac{3}{2} \delta \right] - \frac{9}{2} \Phi'' \left[\sqrt{3} \left(-\bar{z} - \frac{1}{2} \delta \right) \right] < 0,$$

which means that the value loss increases faster with δ when $\bar{z}$ is lower.

Figure A.II illustrates these patterns for $N = 4$. The left panel plots the percentage value loss against $\bar{z}$ when evaluating $\delta = 1$. For $\delta > 0$, the value loss is positive until $\bar{z}$ is large enough. The right panel plots the value loss as a function of the dispersion parameter δ for $\bar{z} = 0.2$ and $\bar{z} = 0.3$. It shows that value loss increases with δ , and the slope is steeper when $\bar{z}$ is smaller. The negative cross-derivative implies that the impact of heterogeneity in network inefficiency is more pronounced during episodes of banking distress. [Q.E.D.](#)

B.6 Proof of Proposition 5

I show that the acquisition tax τ aligns the social incentive for acquisition with that of firm 1. τ equals precisely the negative externality imposed by the acquisition behavior of acquiring firm

1 to all the other non-distressed banks, $i = 2, \dots, N-1$. Under the required tax payment τ , the value of 1 upon acquisition is

$$V_1(\tau) = \hat{V}_1^{12..N} - \tau = \sum_{i=1}^N \hat{V}_i^{12..N} - \sum_{i=2}^{N-1} \hat{V}_i^{12..N-1} - V_N^a.$$

Bank 1 chooses to acquire if and only if $V_1(\tau)$ is larger than the value of not acquiring, i.e.,

$$V_1(\tau) \geq V_1^{12..N-1}.$$

Plugging in τ , this condition is equivalent to

$$\sum_{i=1}^N \hat{V}_i^{12..N} - \sum_{i=2}^{N-1} \hat{V}_i^{12..N-1} - V_N^a \geq \hat{V}_1^{12..N-1},$$

which equals the social surplus function of acquisition. Therefore, $V_1(\tau) \geq V_1^{12..N-1} \iff \sum_{i=1}^N \hat{V}_i^{12..N} \geq \sum_{i=2}^{N-1} \hat{V}_i^{12..N-1} + V_N^a \iff N$ is linked into the network.

Plugging in the following values

$$\begin{aligned} \sum_{i=2}^{N-1} \hat{V}_i^{12..N-1} &= (N-2)(1+\bar{z}) - (N-2)\Phi\left[\sqrt{N-1}\left(-\bar{z} - \frac{1}{2}\delta\right)\right]c; \\ \sum_{i=2}^N \hat{V}_i^{12..N} &= N(1+\bar{z}) - N\Phi\left[\sqrt{N}(-\bar{z})\right]c - \left(1+\bar{z} + \frac{N-1}{2}\delta\sigma - \Phi\left[-\bar{z} - \frac{N-1}{2}\delta\sigma\right]c\right); \\ V_N^a &= 1+\bar{z} + \frac{1-N}{2}\delta\sigma - \Phi\left[-\bar{z} - \frac{1-N}{2}\delta\right]c, \end{aligned}$$

we get that τ is a function of $\{N, \bar{z}, \delta\}$.

$$\tau_1^A = \left(N\Phi\left[\sqrt{N}(-\bar{z})\right] - (N-2)\Phi\left[\sqrt{N-1}\left(-\bar{z} - \frac{1}{2}\delta\right)\right] - \Phi(-z_1) - p_N \right)c.$$

Whenever the most distressed firm should be optimally isolated, we have

$$\sum_{i=1}^{N-1} \hat{V}_i^{12..N-1} + V_N^a - \sum_{i=1}^N \hat{V}_i^{12..N} > 0. \quad (1.46)$$

Combining the optimal condition (1.46) and the acquisition incentive of firm 1, i.e. $\hat{V}_1^{12..N} - \hat{V}_1^{12..N-1} > 0$.

$$\begin{aligned} \tau &= \sum_{i=2}^{N-1} \hat{V}_i^{12..N-1} + V_N^a - \sum_{i=2}^N \hat{V}_i^{12..N} \\ &= \sum_{i=1}^{N-1} \hat{V}_i^{12..N-1} + V_N^a - \sum_{i=1}^N \hat{V}_i^{12..N} + \left(V_1^{12..N} - V_1^{12..N-1} \right) > 0. \end{aligned}$$

$$\begin{aligned}\tau = & N\Phi\left[\sqrt{N}(-\bar{z})\right]c - (N-2)\Phi\left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta)\right]c - \Phi\left[-\bar{z} - \frac{1-N}{2}\delta\right]c \\ & - \Phi\left[-\bar{z} - \frac{N-1}{2}\delta\right]c.\end{aligned}$$

Further, τ increases with dispersion δ , decreases with mean $\bar{z}$. To see this, we take the derivatives of $\bar{z}$, δ , and the cross-derivative of $\bar{z}$ and δ .

$$\begin{aligned}\frac{\partial \tau_1^A}{\partial \bar{z}} &= \sqrt{N-1}(N-2)\Phi'\left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta)\right]c \\ &\quad + \Phi'\left[-\bar{z} - \frac{1-N}{2}\delta\right]c + \Phi'\left[-\bar{z} - \frac{N-1}{2}\delta\right]c - N\sqrt{N}\Phi'\left[\sqrt{N}(-\bar{z})\right]c < 0, \\ \frac{\partial \tau_1^A}{\partial \delta} &= \frac{1}{2}\sqrt{N-1}(N-2)\Phi'\left[\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta)\right]c \\ &\quad + \frac{N-1}{2}\left(\Phi'\left[-\bar{z} - \frac{N-1}{2}\delta\right] - \Phi'\left[-\bar{z} - \frac{1-N}{2}\delta\right]\right)c > 0.\end{aligned}$$

And $\frac{\partial^2 \tau_1^A}{\partial \delta \partial \bar{z}} < 0$. **Q.E.D.**

B.7 Proof of Proposition 8

In this proof, I first show that C_{isoN} decreases monotonically with δ , hence it achieves the maximum at $C_{isoN}(\delta = 0)$. Then I show that C_{GB}^* is a concave function: $C_{GB}^* > C_{isoN}(\delta = 0)$ at the minimum value for cost $c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, C_{GB}^* crosses the linear function $C_{isoN}(\delta = 0)$ at $\bar{c}$. Accordingly, C_{GB}^* is greater than in the region $c \in [\frac{\sqrt{2\pi}\sigma}{\sqrt{N}}, \bar{c})$.

Step 1: $\frac{\partial C_{isoN}}{\partial \delta} < 0$, so C_{isoN} decreases with δ . Take the derivative of C_{isoN} with respect to δ ,

$$\frac{\partial C_{isoN}}{\partial \delta} = \frac{(N-1)c}{2} \left(\Phi' \left[(k-1)\bar{z} + \frac{N-1}{2}\delta \right] - \sqrt{N-1}\Phi' \left[\sqrt{N-1} \left(-\bar{z} - \frac{1}{2}\delta \right) \right] \right). \quad (1.47)$$

Notice that $(k-1)\bar{z} + \frac{N-1}{2}\delta > 0$, $\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta) < 0$, and we can also show that $(k-1)\bar{z} + \frac{N-1}{2}\delta > -\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta)$.⁴⁸ Accordingly, $\Phi'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ implies that

$$\Phi' \left[(k-1)\bar{z} + \frac{N-1}{2}\delta \right] < \Phi' \left[\sqrt{N-1} \left(-\bar{z} - \frac{1}{2}\delta \right) \right] < \sqrt{N-1}\Phi' \left[\sqrt{N-1} \left(-\bar{z} - \frac{1}{2}\delta \right) \right]$$

Plugging into Equation (1.47), we have $\frac{\partial C_{isoN}}{\partial \delta} < 0$. Evaluate C_{isoN} at $\delta = 0$, we obtain a linear function of c ,

$$C_{isoN}(\delta = 0) = (N-1)\Phi\left[-\sqrt{N-1}\bar{z}\right]c + \Phi[(k-1)\bar{z}]c.$$

Step 2: C_{GB}^* is a concave function.

⁴⁸We take the difference of the squares, $((k-1)\bar{z} + \frac{N-1}{2}\delta)^2 - (\sqrt{N-1}(-\bar{z} - \frac{1}{2}\delta))^2 = (k-1)^2\bar{z}^2 + \frac{(N-1)^2}{4}\delta^2 + (k-1)(N-1)\bar{z}\delta - (N-1)\bar{z}^2 - \frac{(N-1)}{4}\delta^2 - (N-1)\bar{z}\delta > (N-1)(N-2)\bar{z}^2 + \frac{(N-1)(N-2)}{4}\delta^2 + (N-2)(N-1)\bar{z}\delta = (N-2)(N-1)(-\bar{z} - \frac{1}{2}\delta)^2 > 0$.

Denote $J = \sqrt{-2 \log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]} > 0$, then $\frac{\partial J}{\partial c} = \frac{1}{Jc}$, and from (1.29), $\Phi'(-J) = \frac{\sigma}{\sqrt{N}c}$. Now sub the expression of J into Equation (1.30), we have

$$C_{GB}^* = N\Phi[-J]c + (k - N)\bar{z}\sigma + \sqrt{N}\sigma J.$$

Take the first derivative of c ,

$$\frac{\partial C_{GB}^*}{\partial c} = N\Phi[-J] + \frac{\sqrt{N}\sigma}{Jc} - \frac{N\Phi'[-J]}{J} = N\Phi[-J] = N\Phi \left[-\sqrt{-2 \log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]} \right].$$

Therefore, $\frac{\partial C_{GB}^*}{\partial c} > 0$. Since $\frac{\partial J}{\partial c} = \frac{1}{Jc} > 0$, $\frac{\partial C_{GB}^*}{\partial c}$ decreases with c , i.e. $\frac{\partial^2 C_{GB}^*}{\partial c^2} < 0$.

Step 3: Establish $C_{GB}^* \left(c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right) > C_{isoN} \left(\delta = 0, c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right)$.

Plugging in $c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$,

$$\begin{aligned} C_{GB}^* \left(c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right) &= \frac{\sqrt{2\pi}\sigma}{2}\sqrt{N} + (k - N)\bar{z}\sigma, \\ C_{isoN} \left(\delta = 0, c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right) &= (N - 1)\Phi \left[-\sqrt{N - 1}\bar{z} \right] \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} + \Phi[(k - 1)\bar{z}] \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}. \end{aligned}$$

Since $k > N$, $\Phi < 1$,

$$C_{isoN} \left(\delta = 0, c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right) < N \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} = \frac{\sqrt{2\pi}\sigma}{2}\sqrt{N} < C_{GB}^* \left(c = \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right).$$

Step 4: Solve for cross point $\bar{c}$.

Equating $C_{GB}^* = C_{isoN}(\delta = 0)$ and solve for c gives $\bar{c}$.

Finally, with the results from Steps 1 - 4, we establish that $C_{GB}^* > C_{isoN}$, $\forall c \in [\frac{\sqrt{2\pi}\sigma}{2}, \bar{c}]$.

Q.E.D.

B.8 Proof of Corollary 1

In this proof, I first analyze conditions for the acquisition link to be *ex post* optimal. Then I examine whether the acquisition link forms at equilibrium, and then move to conditions for the positive subsidy to be optimal. Finally, I conclude that subsidized acquisition is cheaper than government bailout.

Step 1: condition for the acquisition link to be *ex post* optimal. Without acquisition link, total liquidation costs of group one and group two are respectively

$$C_{g1} = N\Phi \left[\frac{-N + k}{\sqrt{N}}\bar{z} \right] c, \quad C_{g2} = N\Phi \left[\frac{-N - \hat{k}}{\sqrt{N}}\bar{z} \right] c. \quad (1.48)$$

With the acquisition link, the total liquidation costs of the two groups become

$$C_{\text{total}} = \sum_{i=1}^{2N} \Pr(\tilde{h}_i < 1) c = 2N\Phi\left[\frac{-2N - \hat{k} + k}{\sqrt{2N}}\bar{z}\right] c. \quad (1.49)$$

The acquisition link generates positive surplus if and only if $C_{g1} + C_{g2} > C_{\text{total}}$. Plugging in (1.48) and (1.49) and applying Lemma 3, we get

$$N\Phi\left[\frac{-N + k}{\sqrt{N}}\bar{z}\right] c + N\Phi\left[\frac{-N - \hat{k}}{\sqrt{N}}\bar{z}\right] c > 2N\Phi\left[\frac{-2N - \hat{k} + k}{\sqrt{2N}}\bar{z}\right] c \iff \hat{k} > k - 2N.$$

Step 2: condition for the acquisition link to be formed ex post at equilibrium. I show that as long as this acquisition is socially optimal, $C_{g1} + C_{g2} > C_{\text{total}}$, the acquisition link will form *ex post* at equilibrium. Since prices are already set between other banks, with only bilateral prices (p_N^{N+1}, p_{N+1}^N) to be contracted. Hence whether the acquisition link can form at equilibrium is equivalent to whether the bilateral surplus between N and $N + 1$ is positive. The value of firm N without the *ex post* acquisition link is⁴⁹

$$\hat{V}_N = 1 + \left(1 - \frac{N-1}{2}\delta - \frac{k}{N}\right)\bar{z}\sigma - \Phi\left[\frac{k-N}{\sqrt{N}}\bar{z}\right] c - \Phi\left(-\bar{z} + \frac{N-1}{2}\delta\right) c + \Phi(-\sqrt{N}\bar{z}) c.$$

Notice that when $k = 0$, $\hat{V}_N = V_N^a$, which matches the outside option of firm N . The value of firm $N + 1$ without the *ex post* acquisition link is

$$\hat{V}_{N+1} = 1 + \frac{\hat{k} + N}{N}\bar{z}\sigma - \Phi\left[\frac{-N - \hat{k}}{\sqrt{N}}\bar{z}\right] c.$$

The bilateral surplus is

$$\Phi\left[\frac{-N - \hat{k}}{\sqrt{N}}\bar{z}\right] c + \Phi\left[\frac{k-N}{\sqrt{N}}\bar{z}\right] c > 2\Phi\left[\frac{-2N - \hat{k} + k}{\sqrt{2N}}\bar{z}\right] c \iff \hat{k} > k - 2N$$

which recovers precisely the condition for positive total acquisition surplus. This shows that if and only if $\hat{k} > k - 2N$, the acquisition link is efficient and forms in equilibrium after θ realizes.

Step 3: the positive acquisition subsidy is optimal if the liquidation cost is large enough. When $\hat{k} \leq k - 2N$, I next show that the positive acquisition subsidy is optimal if the liquidation cost is large enough. Let the positive government subsidy be $B_A\sigma$ given to the acquire $N + 1$. The total cost with subsidized acquisition becomes

$$C_{\text{subA}} = \sum_{i=1}^{2N} \Pr(\tilde{h}_i < 1) c + B_A\sigma = 2N\Phi\left[\frac{(k - \hat{k} - 2N)\bar{z} - B_A}{\sqrt{2N}}\right] c + B_A\sigma.$$

⁴⁹ $\hat{V}_N = \mathbb{E}[\tilde{h}_N] - \Pr(\tilde{h}_N < 1) c + \frac{1}{2}p_N^1 - \frac{1}{2}p_1^N$, $p_N^1 = 1 + (\bar{z} - \frac{N-1}{2}\delta)\sigma - \Phi(-\bar{z} + \frac{N-1}{2}\delta)c$, $p_1^N = 1 + (\bar{z} + \frac{N-1}{2}\delta)\sigma + \Phi(-\bar{z} + \frac{N-1}{2}\delta)c - 2\Phi(-\sqrt{N}\bar{z})c$.

B_A^* satisfies the first order condition

$$\Phi' \left[\frac{(k - \hat{k} - 2N) \bar{z} - B_A^*}{\sqrt{2N}} \right] = \frac{\sigma}{\sqrt{2N}c}. \quad (1.50)$$

Solving for B_A^* gives (1.31), and we require that $c > \frac{\sqrt{\pi}\sigma}{\sqrt{N}}$ and $\hat{k} \leq k - 2N$.

Step 4: subsidized acquisition is preferred to government bailout. I show that the subsidized acquisition is less costly thus preferred to government bailout. Based on Proposition 7, for $c \in \left(\frac{\sqrt{\pi}\sigma}{\sqrt{N}}, \frac{\sqrt{2\pi}\sigma}{\sqrt{N}} \right)$, subsidized acquisition is the only feasible option. For $c > \frac{\sqrt{2\pi}\sigma}{\sqrt{N}}$, costs with government bailout for the two groups are

$$C_{GB}^* = N\Phi \left[-\sqrt{-2 \log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]} \right] c + (k - N)\bar{z}\sigma + \sqrt{N}\sigma \sqrt{-2 \log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]} + N\Phi \left[\frac{-N - \hat{k}}{\sqrt{N}} \bar{z} \right] c.$$

Costs with subsidized acquisition is

$$C_{subA}^* = (k - \hat{k} - 2N) \bar{z}\sigma + \sqrt{2N} \sqrt{-2 \log \left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c} \right]} \sigma + 2N\Phi \left[-\sqrt{-2 \log \left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c} \right]} \right] c.$$

Denote $J = \sqrt{-2 \log \left[\frac{\sqrt{2\pi}\sigma}{\sqrt{N}c} \right]} > 0$, $H = \frac{\hat{k} + N}{\sqrt{N}} \bar{z} > 0$, then

$$C_{GB}^* = \sqrt{N}\sigma J + N\Phi[-J]c + \sqrt{N}\sigma H + N\Phi[-H]c + (k - \hat{k} - 2N) \bar{z}\sigma.$$

From (1.29), $\Phi'(-J) = \frac{\sigma}{\sqrt{N}c}$. Hence, function $f(x) = \sqrt{N}\sigma x + N\Phi[-x]c$, satisfies $f'(J) = 0$, $f''(x) > 0, \forall x > 0$. This implies $C_{GB}^* > 2\sqrt{N}\sigma J + 2N\Phi[-J]c + (k - \hat{k} - 2N) \bar{z}\sigma > \sqrt{2N}\sigma J + 2N\Phi[-J]c + (k - \hat{k} - 2N) \bar{z}\sigma$.

In a similar approach, denote $G = \sqrt{-2 \log \left[\frac{\sqrt{\pi}\sigma}{\sqrt{N}c} \right]} > 0$, then $C_{subA}^* = \sqrt{2N}G\sigma + 2N\Phi[-G]c + (k - \hat{k} - 2N) \bar{z}\sigma$, and from (1.50), $\Phi'[-G] = \frac{\sigma}{\sqrt{2N}c}$. Function $f(x) = \sqrt{2N}\sigma x + 2N\Phi[-x]c$, $x > 0$, achieves global ($x > 0$) minimum at $x = G$. This implies that $C_{GB}^* > C_{subA}^*$ **Q.E.D.**

B.9 Proof of Proposition 9

I first show that condition (1.32) implies positive social surplus from the acquisition link between the liquid $N_1 + 1$ and the distressed firm N_1 . Without acquisition link, total liquidation costs of group one and group two are respectively

$$C_{g1} = N_1\Phi \left[\sqrt{N_1} \left(\frac{k}{N_1} - 1 \right) \bar{z} \right] c, \quad C_{g2} = N_2\Phi \left[\sqrt{N_2} \left(-1 - \frac{\hat{k}}{N_2} \right) \bar{z} \right] c$$

With the acquisition link, the total liquidation costs of the two groups become

$$C_{\text{total}} = \sum_{i=1}^{N_1+N_2} \Pr(\tilde{h}_i < 1) c = (N_1 + N_2) \Phi \left[\sqrt{N_1 + N_2} \left(\frac{k - \hat{k}}{N_1 + N_2} - 1 \right) \bar{z} \right] c.$$

The acquisition link generates positive surplus if and only if $C_{g1} + C_{g2} > C_{\text{total}}$, i.e.

$$\frac{N_1}{N_1 + N_2} \Phi \left[\frac{k - N_1}{\sqrt{N_1}} \bar{z} \right] + \frac{N_2}{N_1 + N_2} \Phi \left[\frac{-\hat{k} - N_2}{\sqrt{N_2}} \bar{z} \right] > \Phi \left[\frac{k - \hat{k} - (N_1 + N_2)}{\sqrt{N_1 + N_2}} \bar{z} \right]. \quad (1.51)$$

Under (1.32), $\hat{k} > \max \left[\frac{\sqrt{N_1 + N_2} - \sqrt{N_1}}{\sqrt{N_1 + N_2} - \sqrt{N_2}} (k - N_1) - N_2, k - N_1 - N_2 \right]$. It follows

$$\begin{aligned} & (N_2 + \hat{k}) \left(\sqrt{N_1 + N_2} - \sqrt{N_2} \right) > (k - N_1) \left(\sqrt{N_1 + N_2} - \sqrt{N_1} \right) \iff \\ & \left(-N_1 \sqrt{N_1} + \sqrt{N_1} k \right) + \left(-\sqrt{N_2} N_2 - \sqrt{N_2} \hat{k} \right) > \sqrt{N_1 + N_2} k - \sqrt{N_1 + N_2} \hat{k} - \sqrt{N_1 + N_2} (N_1 + N_2) \iff \\ & \frac{N_1 \sqrt{N_1} \left(-1 + \frac{k}{N_1} \right) \bar{z}}{N_1 + N_2} + \frac{N_2 \sqrt{N_2} \left(-1 - \frac{\hat{k}}{N_2} \right) \bar{z}}{N_1 + N_2} > \frac{k - \hat{k} - (N_1 + N_2)}{\sqrt{N_1 + N_2}} \bar{z}. \end{aligned} \quad (1.52)$$

Given $\Phi(\cdot)$ is convex when $\frac{k - \hat{k} - (N_1 + N_2)}{\sqrt{N_1 + N_2}} \bar{z} < 0$, by definition

$$\frac{N_1}{N_1 + N_2} \Phi \left[\frac{k - N_1}{\sqrt{N_1}} \bar{z} \right] + \frac{N_2}{N_1 + N_2} \Phi \left[\frac{-\hat{k} - N_2}{\sqrt{N_2}} \bar{z} \right] \geq \Phi \left[\frac{N_1 \sqrt{N_1} \left(-1 + \frac{k}{N_1} \right) \bar{z}}{N_1 + N_2} + \frac{N_2 \sqrt{N_2} \left(-1 - \frac{\hat{k}}{N_2} \right) \bar{z}}{N_1 + N_2} \right].$$

Combining with Equation (1.52), we establish (1.51).

Next I show that under (1.32), the bilateral acquisition surplus is positive when $N_2 \geq N_1$. Since prices are already set between other banks, there are only bilateral prices $(p_{N_1}^{N_1+1}, p_{N_1+1}^{N_1})$ to be contracted. Hence whether the acquisition link can form at equilibrium is equivalent to whether the bilateral surplus between N_1 and $N_1 + 1$ is positive. The value of firm N_1 without acquisition is

$$\hat{V}_{N_1} = 1 + \left(1 - \frac{N_1 - 1}{2} \delta - \frac{k}{N_1} \right) \bar{z} \sigma - \Phi \left[\frac{k - N_1}{\sqrt{N_1}} \bar{z} \right] c - \Phi \left(\frac{N_1 - 1}{2} \delta - \bar{z} \right) c + \Phi \left(-\sqrt{N_1} \bar{z} \right) c.$$

Notice that when $k = 0$, $\hat{V}_{N_1} = V_{N_1}^a$, which matches the outside option of firm N_1 . The value of firm $N_1 + 1$ without the acquisition link is

$$\hat{V}_{N_1+1} = 1 + \left(1 + \frac{\hat{k}}{N_2} \right) \bar{z} \sigma - \Phi \left[\frac{-N_2 - \hat{k}}{\sqrt{N_2}} \bar{z} \right] c.$$

With the acquisition link, the value of firm N_1 , and firm $N_1 + 1$ are respectively

$$\begin{aligned} \hat{V}_{N_1}^A &= 1 + \left(1 - \frac{N_1 - 1}{2} \delta - \frac{k - \hat{k}}{N_1 + N_2} \right) \bar{z} \sigma - \Phi \left[\frac{k - \hat{k} - (N_1 + N_2)}{\sqrt{N_1 + N_2}} \bar{z} \right] c - \Phi \left(-\bar{z} + \frac{N_1 - 1}{2} \delta \right) c + \Phi \left(-\sqrt{N_1} \bar{z} \right) c; \\ \hat{V}_{N_1+1}^A &= 1 + \left(1 - \frac{k - \hat{k}}{N_1 + N_2} \right) \bar{z} \sigma - \Phi \left[\frac{k - \hat{k} - (N_1 + N_2)}{\sqrt{N_1 + N_2}} \bar{z} \right] c. \end{aligned}$$

The bilateral surplus minus the total surplus is

$$\begin{aligned}
& \frac{\hat{V}_{N_1}^A + \hat{V}_{N_1+1}^A - \hat{V}_{N_1} - \hat{V}_{N_1+1}}{2} - \frac{C_{g1} + C_{g2} - C_{\text{total}}}{N_1 + N_2} \\
&= \frac{1}{2} \left(\frac{k}{N_1} - \frac{\hat{k}}{N_2} - 2 \frac{k - \hat{k}}{N_1 + N_2} \right) \bar{z} \sigma + \frac{N_2 - N_1}{2(N_1 + N_2)} \left(\Phi \left[\frac{k - N_1}{\sqrt{N_1}} \bar{z} \right] - \Phi \left[\frac{-\hat{k} - N_2}{\sqrt{N_2}} \bar{z} \right] \right) c \\
&= \frac{(N_2 - N_1) (N_2 k + N_1 \hat{k})}{2 N_1 N_2 (N_1 + N_2)} \bar{z} \sigma + \frac{N_2 - N_1}{2(N_1 + N_2)} \left(\Phi \left[\frac{k - N_1}{\sqrt{N_1}} \bar{z} \right] - \Phi \left[\frac{-\hat{k} - N_2}{\sqrt{N_2}} \bar{z} \right] \right) c \\
&= \frac{N_2 - N_1}{2(N_1 + N_2)} \left[\frac{N_2 k + N_1 \hat{k}}{N_1 N_2} \bar{z} \sigma + \left(\Phi \left[\frac{k - N_1}{\sqrt{N_1}} \bar{z} \right] - \Phi \left[\frac{-\hat{k} - N_2}{\sqrt{N_2}} \bar{z} \right] \right) c \right].
\end{aligned}$$

which is non-negative when $N_2 \geq N_1$. In other words, when $N_2 \geq N_1$, and $C_{g1} + C_{g2} - C_{\text{total}} > 0$,

$$\frac{\hat{V}_{N_1}^A + \hat{V}_{N_1+1}^A - \hat{V}_{N_1} - \hat{V}_{N_1+1}}{2} \geq \frac{C_{g1} + C_{g2} - C_{\text{total}}}{N_1 + N_2} > 0.$$

If $N_1 > N_2$, the average bilateral surplus is smaller than the average social surplus. Under condition (1.33), the bilateral surplus is negative. [Q.E.D.](#)

C Additional Empirical Results

In this Appendix, I provide additional empirical results to supplement the findings in Section 6.

Table A.I presents supplementary univariate correlations to Table 3. I adopt alternative indicators for economic activity and systemic risk, including the *Recession Probability* from [Chauvet and Piger \(2008\)](#), the subcomponents of the Chicago Fed National Activity Index (CFNAI) on personal consumption and housing (C&H) and employment, unemployment, and hours (EU&H). Finally, following [Giglio, Kelly, and Pruitt \(2015\)](#), I take the systemic risk measures relating to liquidity and credit conditions in the financial market: the *Default Spread* (difference between 3-Month BAA bond yields and the Treasury) and the *Term Spread* (difference between 10-Year and 3-Month Treasury).

Table A.I. SUMMARY STATISTICS AND UNIVARIATE CORRELATIONS

	Mean	StDev	Sacf	Correlations w/ log Z-score	
				Mean	Dispersion
A. Economic activity and systemic risk					
Recession Probability	0.08	0.23	0.83	0.00	0.17
CFNAI: Personal Consumption and Housing	-0.03	0.13	0.93	-0.03	-0.78***
CFNAI: Employment, Unemployment, and Hours	-0.06	0.31	0.86	-0.03	-0.20*
Default Spread	4.19	1.54	0.93	-0.14	0.54***
Term Spread	1.87	1.11	0.91	-0.25**	0.37***
B. Lending and interbank lending					
Financial Business Leverage	29.40	5.59	0.94	-0.16	-0.71***
Security Broker-Dealers Leverage	41.11	17.94	0.73	0.51***	-0.18*
$\Delta\%$ Non-financial Corporate Liability	0.01	0.01	0.46	0.14	-0.22*
All Comm. Bank Credit over Assets	0.81	0.03	0.94	-0.13	-0.84***
Small Comm. Interbank Loan over Assets	0.02	0.01	0.87	0.10	-0.51***

Notes: This table supplements to Table 3 and reports the summary statistics for alternative measures of economic activity and systemic risk, lending and interbank lending, as well as their univariate correlation coefficients with the mean and dispersion of financials' log Z-scores. Group A series are taken from FRED. Group B series are constructed from the Fed's Z.1 and H.8 release. Data availability on bank holding companies restricts the analysis to 1986-2013. Sacf is the first-order sample autocorrelation coefficient. The last two columns report the correlation coefficients between the cross-sectional mean and dispersion of log Z-score and each series in groups A-B together with the significance levels. *, **, *** denote statistical significance at the 5%, 1%, and 0.1% level.

The alternative indicators for lending and interbank lending include the leverage of both financial business and the security broker-dealers discussed in [Adrian, Etula, and Muir \(2014\)](#), the growth rate of non-financial corporate liability, the credit and loans of all commercial banks over assets, and the interbank loans over assets of small and medium-sized commercial banks. The correlation coefficients show a clear pattern: the aggregate indicators correlate significantly with dispersion, whereas only the leverage of security broker-dealers comoves strongly with the mean.

Table A.II presents supplementary predictive regression results to Table 4. Using the same method as in Table 4, I run predictive regressions to forecast the alternative measures. The estimates strongly echo the findings from the correlation analysis. Both the significance level of the regression coefficients and the differences in R^2 s with and without dispersion in the regressors suggest the robustness of the predictive power of dispersion series.

The economic magnitude of the predictive power is also sizable. Take the forecasting of Recession Probability for instance, holding the controls fixed, a one-standard-deviation increase in the *Dispersion* ($=0.22$) predicts a 0.095 ($= 0.22 \times 0.43$) increase in the *Recession Probability* in the next quarter, whereas a one-standard-deviation decrease in the *Mean* ($=0.03$) predicts a 0.046 ($= 0.03 \times 1.54$) raise in the future *Recession Probability*.

Table A.II. PREDICTIVE REGRESSIONS USING DISTRESS DISPERSION

Quarters	1	2	3	4	1	2	3	4
Forecasting	A. Recession Probability				B. CFNAI: CH			
Dispersion	0.43**	0.79**	1.07*	1.29*	-0.38***	-0.75***	-1.10***	-1.43***
Mean	-1.54**	-3.28**	-4.83**	-5.12*	0.27	0.70	1.23	1.76
R^2	40.98	43.91	45.95	42.37	70.18	72.88	74.29	75.30
R^2 w/o disp	33.95	37.23	39.95	36.96	53.71	56.23	57.85	59.39
Forecasting	C. Default Spread				D. Term Spread			
Dispersion	0.04***	0.08***	0.13***	0.18***	0.01*	0.02*	0.04*	0.06
Mean	0.00	-0.02	-0.08	-0.12	-0.02	-0.05	-0.13	-0.22
R^2	83.97	79.92	74.97	69.61	87.19	80.07	72.93	66.32
R^2 w/o disp	72.23	65.71	58.38	49.95	86.49	78.48	70.32	62.16
Forecasting	I. Bk Credit over Assets				K. Sml Bk Interbank L over Assets			
Dispersion	-0.09***	-0.18***	-0.28***	-0.37***	-0.01	-0.02	-0.04*	-0.05**
Mean	-0.02	0.07	0.18	0.27	0.06	0.10	0.11	0.09
R^2	80.41	82.41	83.95	85.16	48.13	54.09	56.03	56.46
R^2 w/o disp	64.37	65.53	65.98	65.79	45.00	49.57	49.55	47.33

Notes: This table summarizes the ability of distress dispersion to forecast future economic activity, systemic risk, failure rates, distressed acquisition rates, and bank lending behavior. In A-K, quarterly time series are regressed on the cross-sectional dispersion and mean of log Z-score controlling for the term spread, the leverage of financial business and security broker-dealers, and the growth rate of real non-financial corporate liability. Forecasting horizons range from one to four quarters and the data cover the years 1986-2013. The table reports the predictive regression coefficients on the dispersion and mean of log Z-score, the R^2 , as well as the R^2 when the regressions are run without the dispersion series. *, **, *** denote statistical significance (based on Newey-West standard errors) at the 5%, 1%, and 0.1% level.

Chapter 2

Asset Pricing with Dynamic Labor Contracts

1 Introduction

An average job tenure in the U.S. labor markets is 4.6 years. Why are labor relations long term and how are the compensation schemes designed? What are the impacts on asset prices? In this paper, I consider a key observation of the labor market: Certain attributes of a worker, such as ability, passion, and physical condition, largely affect productivity but cannot be directly observed or measured by employers. Although working hours can be specified in the labor contract, effective effort level is beyond the employers' control.

To capture such features, I build a model of dynamic labor contracts under private information. Firms use long-term compensation schemes, wage payment and promise of future raises, to better provide insurance as well as incentives to workers. This labor relation has unique predictions on the risk-sharing properties of the economy. Quantitative results show that the dynamic contracting feature is important in matching asset prices and business cycle facts.

I model a two-agent production economy in which the worker has private information about her labor productivity. The model features heterogeneous preferences and limited stock market participation: the representative shareholder is less risk averse and is better off holding risky assets, whereas the representative worker earns income by supplying labor. The worker's labor productivity risk has two dimensions: one publicly informed and one privately observed by the worker. The shareholder offers an incentive compatible long-term labor contract, which partially insures the worker against her labor income risk.

Following the recursive formulation in [Spear and Srivastava \(1987\)](#), I solve for the consumption allocations and labor input specified by the optimal incentive compatible labor contract. The equity return and risk-free rate are priced using shareholder's consumption stream as pricing kernel. The model is then calibrated with relatively low risk aversions for both agents. I compare the model's performance to settings with a Walrasian competitive labor market, and with static labor contracts. My model successfully matches both asset returns data and business-

cycle facts, including a countercyclical and high equity premium, a low risk-free rate, procyclical labor input, and countercyclical labor share.

The model generates an equity premium of 6.13% and a risk-free rate 0.98%, matching the observation in [Mehra and Prescott \(1985\)](#). The calibrated fraction of privately observed labor productivity risk is 0.5. The consumption growth volatility matches Consumer Expenditure Survey with that of shareholders within 5.38% $\sim$ 12%, and that of workers within 3% $\sim$ 5.38%.¹ The ratio of the two assumes a value of 1.62, in line with the findings in [Mankiw and Zeldes \(1991\)](#) based on the PSID data. The cyclical feature of all macro variables generated in this model are consistent with data.

The model mechanism is as follows. First, the endogenous labor input is procyclical, thus expanding or contracting aggregate production in response to labor productivity shocks. In good economy (e.g. favorable climate, technology) and when being more productive (e.g. smart, energetic), the worker is given incentive to devote higher effort. In the aggregate, hours worked increase when the economy is good. Hence, consumption volatility is endogenously amplified by the incentive compatible labor contracts.² Second, the risk-sharing feature under the dynamic labor contracts endogenously generates a countercyclical labor share. The less risk averse shareholder offers an incentive compatible long-term labor contract, which partially insures the worker against labor income risk. Essentially the shareholder bears a larger consumption risk in the contract, thus requires to be compensated by a higher equity premium. Finally, the dynamic feature captures the intertemporal risk-sharing in the long-term employment relationship. The promised continuation utility provides shareholders an extra tool to balance insurance and incentive, thus further facilitating risk-sharing. In Section 3, we isolate the effect of intertemporal risk-sharing from intratemporal risk-sharing by observing a higher level of equity premium and risk ratio in the dynamic labor contract model relative to a benchmark static model.

This paper contributes to the large body of literature on equity premium puzzle, which documents the difficulty of using neoclassical models to rationalize the U.S. stock market premiums.³ With plausible constant relative risk aversion, the average consumption growth volatility is too low to rationalize the observed 6.18% equity premium.⁴ [Constantinides and Duffie \(1996\)](#) show that it is possible for incomplete market models to explain the asset pricing anomalies in an economy with uninsurable, persistent, and heteroscedastic labor income shocks. [Kocherlakota \(1996\)](#) also argues that modeling incomplete insurance markets where agents fail to fully insure themselves due to private information leads to a promising direction. As labor income provides the lion's share of consumption for the working class, partial insurance in labor market as explored here is necessarily relevant.

Shareholder's consumption streams are used as pricing kernel. In this sense, this paper

¹See data provided in [Malloy, Moskowitz, and Vissing-Jorgensen \(2009\)](#).

²[Shorish and Spear \(2005\)](#) show in their static agency theoretic model that worker's ability to endogenously control output largely affects asset prices.

³See [Mehra and Prescott \(1985\)](#) for the puzzle, [Kocherlakota \(1996\)](#) for a survey.

⁴[Mehra and Prescott \(1985\)](#) observe for 1889-1978, market return is 6.98% and risk-free rate is 0.80%.

adds to the literature on limited stock market participation.⁵ Mankiw and Zeldes (1991) show evidence that only one fourth of US households own stocks and that stockholders' consumption growth is much more volatile than that of non-stockholders. While most papers rely on a fixed cost of participation or borrowing constraints, agents in my model choose their roles to enter the contracts based on heterogeneous risk aversion. Hence, workers have no need to further seek insurance from the asset market because the labor contract has provided them with maximum possible insurance constrained by private information.

Berk and Walden (2013) explore similar idea that investors provide insurance to wage earners who then optimally choose not to participate in the financial markets. They demonstrate that human capital risk is shared in labor markets through bilateral labor contracts, and investors offload the labor market risk they assumed from workers by participating in financial markets. While they focus on the labor contracts based on working flexibility, I differ by building on the incentive compatible contracts in a world with private information.

My paper is closest to Danthine and Donaldson (2002) who show that *operation leverage* by wage claims magnifies the risk of residual payments to firm owners and the required risk premium. However, in their model, the labor supply is fixed and the optimal risk-sharing formula is exogenously specified. This paper differs by solving the endogenous labor supply and limited risk-sharing determined by labor contracts.

The paper proceeds as follows: Section 2 describes the model. Section 3 presents quantitative results on equilibrium allocations and asset pricing implications. Extension on persistent public shocks is presented in Section 4, and Section 5 concludes.

2 Model

In this section, I describe a production economy with heterogeneous agents, private information, and dynamic labor contracts. The goal is to study the risk-sharing properties and asset pricing implications.

2.1 Environment

This economy has a single, non-durable, consumption good. Time is discrete. A continuum of measure α of shareholders invest in the asset market by holding ownership of the firm. A continuum of measure $1 - \alpha$ of workers supply labor in production and earn consumption from labor income. Heterogeneous preferences in risk aversion and labor disutility sort agents into two groups, thus generating limited market participation. Labor productivity risk contains both publicly informed and privately observed components. A labor contract of a horizon of T periods formalizes the employment relationship between a worker and a shareholder.

⁵See evidence for example in Mankiw and Zeldes (1991), Basak and Cuoco (1998), Cao, Wang, and Zhang (2005), Berk and Walden (2013).

Workers

Assume workers are all alike, so there exists a representative worker. The worker has additively separable period utility defined over consumption C and labor input L . $U(C_t, L_t)$ is increasing in C_t , decreasing in L_t , twice continuously differentiable and concave in both consumption and labor. Workers discount future utility at constant rate $0 < \beta < 1$. Given a sequence of consumption and labor input $\{C_t, L_t\}_{t=1}^T$, the worker's expected discounted utility over the contract horizon is

$$W(\{C_t, L_t\}_{t=1}^T) = \mathbb{E}_0 \sum_{t=1}^T \beta^{t-1} U(C_t, L_t), \quad (2.1)$$

where $\mathbb{E}_0$ denotes expectation at pre-contracting stage.

Shareholders

Shareholders by nature are less risk averse and have stronger labor disutility.⁶ The representative shareholder's period utility $V(D_t)$ is also additively separable over time with discount factor β , and is increasing, twice continuously differentiable and concave in consumption D_t .⁷ Shareholders hold firm shares S_t and risk-free bonds B_t and obtain dividend income D_t as consumption. Let the equilibrium stock price be P_t and bond price be b_t^{t+1} . Shareholder's budget constraint at time t is

$$D_t + P_t S_{t+1} + B_{t+1} b_t^{t+1} \leq (P_t + D_t) S_t + B_t. \quad (2.2)$$

Let ψ be the distribution of shareholders. The asset market clearing condition is given by

$$\int B_t d\psi = 0, \quad \int S_t d\psi = 1. \quad (2.3)$$

Accordingly, a representative shareholder solves

$$\max_{\{D_t, S_t, B_t\}_{t=1}^T} : \mathbb{E}_0 \sum_{t=1}^T \beta^{t-1} V(D_t), \quad (2.4)$$

subject to (2.2) and (2.3).

Production, Risks, and Information

Since the focus is on labor relations, I fix the capital stock as constant for all periods and states, i.e. $K_t = K, \forall z^t, \theta^t$. There are two dimensions of labor productivity risk $\phi_t(z_t, \theta_t)$: publicly informed z_t and privately observed θ_t .⁸ Let $z^t \equiv (z_1, z_2, \dots, z_t) \in Z^t$ and $\theta^t \equiv (\theta_1, \theta_2, \dots, \theta_t) \in \Theta^t$

⁶Sources of heterogeneity include age, wealth, social status, etc.

⁷Labor does not enter the utility function directly because shareholders do not directly supply labor in production.

⁸ θ^t is the systematic component of private labor productivity risk. As in Berk and Walden (2013), the idiosyncratic component of labor productivity risk can be diversified away among workers, even though it is likely

denote the histories of the shocks up to period t . $\phi(z_t, \theta_t)$ is increasing with both arguments. Output is given by Cobb-Douglas form with capital share α ,

$$Y_t = \phi(z_t, \theta_t) L_t^{1-\alpha} K^\alpha. \quad (2.5)$$

Private information of a worker concerns with both private labor productivity shock θ_t and the amount of labor input L_t . We make the following assumptions on the distribution of risks.

Assumption 4 *Labor productivity risks satisfy the following.*

- (i) *The private risk is independent of the public shock.*
- (ii) *The shocks are identically distributed over time and independent of history realizations.*

Under Assumption 4, the probability of drawing shocks $\{z_t, \theta_t\}$ is $\Pi(z_t)P(\theta_t)$. Assumption 4.(i) is to isolate the private information nature of the private risks, so that nothing can be inferred from the realization of public risks. We will relax Assumption 4.(ii) by adding persistence to the public shocks in Section 4. Under 4.(ii), the size of the shocks is time-invariant. Following Ales and Maziero (2010), the fraction of privately observed labor productivity risk is given by

$$\Omega = \frac{Var(\theta_t)}{Var(z_t) + Var(\theta_t)} \in [0, 1], \quad \forall t. \quad (2.6)$$

Dynamic Labor Contract

The timeline of the contract is as follows. At $t = 0$, the shareholder and the worker enter a horizon T exclusive labor contract. This employment relationship promises the worker an expected utility above his initial reservation utility W_0 . We assume two-sided commitment such that both parties commit to stay in the contract once it is signed. Upon observing the realization of both public and private shocks each period, the worker strategically reports to the shareholder about her private labor productivity and exert effort accordingly. At the end of the period, output is realized and consumption is allocated based on the contract.

I solve for the equilibrium allocations and labor input defined by the optimal contract. The revelation principle ensures that we can restrict to direct mechanisms in which workers truthfully report the private labor productivity. Given a worker's initial reservation utility W_0 , the contract specifies consumption allocation, labor input, and required output conditional on the realized history of public shock z^t and the reported history of private shock θ^t , i.e. $\{C(z^t, \theta^t, W_0), Y(z^t, \theta^t, W_0), L(z^t, \theta^t, W_0)\}_{t=1}^T$.

The worker chooses to enter the contract only if the expected discounted utility from the long-term employment is no less than W_0 . A contract satisfies *individual rationality (IR)* if the

to be sizable for an individual worker.

following holds,

$$\sum_{t=1}^T \sum_{z^t, \theta^t} \Pi(z^t) P(\theta^t) \beta^{t-1} U(C(z^t, \theta^t), L(z^t, \theta^t)) \geq W_0. \quad (2.7)$$

In order to induce truth-telling, the contract is such that there are no gains by deviating from truthfully reporting the privately observed shock state. A contract is *incentive compatible (IC)* if it satisfies the following

$$\sum_{t=1}^T \sum_{z^t, \theta^t} \Pi(z^t) P(\theta^t) \beta^{t-1} \left[U(C(z^t, \theta^t), L(z^t, \theta^t)) - U(C(z^t, \tilde{\theta}^t), \hat{L}(z^t, \tilde{\theta}^t)) \right] \geq 0, \forall \tilde{\theta}^t \in \Theta^t, \quad (2.8)$$

where the required effort level if lying is

$$\hat{L}(z^t, \tilde{\theta}^t) = \left[\frac{Y(z^t, \tilde{\theta}^t)}{\phi(z_t, \theta_t) K^\alpha} \right]^{\frac{1}{1-\alpha}} = \left[\frac{\phi(z_t, \tilde{\theta}_t)}{\phi(z_t, \theta_t)} \right]^{\frac{1}{1-\alpha}} L_t(z_t, \tilde{\theta}_t). \quad (2.9)$$

A contract is *feasible* if it satisfies

$$C(z^t, \theta^t) + D(z^t, \theta^t) + B_{t+1} b_t^{t+1}(z^t, \theta^t) \leq \phi(z_t, \theta_t) L_t^{1-\alpha} K^\alpha + B_t, \quad \forall z^t \in Z^t, \theta^t \in \Theta^t. \quad (2.10)$$

For a contract satisfying Conditions (2.8), (2.9), (2.10), the worker truthfully reports his private shock states and exerts optimal labor input in exchange for a compensation profile less volatile than his marginal productivity. Thereby, the labor contract provides partial insurance to the worker against her labor income risk. In other words, the existence of private information induces limited risk-sharing between shareholder and worker.

Financial Market

I consider a setting of incomplete financial markets with two assets: the risky asset and the risk-free asset. Risky asset is the dividend claim from holding the firm share, holdings of which are within the shareholders. Allen (1985) shows that incentive compatibility constraint cannot hold if agents are able to hold assets privately. Hence, I make the following assumption

Assumption 5 *Asset holdings of shareholders and workers are public information.*

Based on Assumption 5, denote the bond holding of a worker as B_t^w , then we have the following characterization on worker's consumption.

Lemma 4 *Equilibrium consumption allocation defined by the optimal contract satisfies*

$$C(z^t, \theta^t | B_t^w) = C(z^t, \theta^t | \tilde{B}_t^w), \quad \forall B_t^w \neq \tilde{B}_t^w. \quad (2.11)$$

Lemma 4 delivers the idea that worker's equilibrium consumption is independent of her bond holdings. When the worker's bond holding is public information, the shareholder will specify labor contract conditional on the bond holdings such that worker's equilibrium consumption

stays the same. This way, any potential insurance effect by participating in the financial market is already covered by the labor contract. Therefore, it is without loss of generality that workers are restricted from asset market participation. I further make the following assumption.

Assumption 6 *Risk-free bonds are in zero net supply, i.e. only private bonds are traded. Shareholders are all alike and behave independently and competitively.*

Lemma 5 *The equilibrium allocations specified by the labor contract depend on the shareholder's bond holding. Under Assumption 6, the equilibrium bond holding of a typical shareholder $B_t = 0$.*

Shareholder's Problem

The representative shareholder maximizes his expected discounted utility by designing the optimal dynamic labor contract and choosing the optimal amount of asset holdings. Hence, the representative shareholder solves the following problem subject to Equations (2.7), (2.8), (2.9), and (2.10).

$$\max_{\{C_t, D_t, L_t, B_t\}_{t=1}^T} : \sum_{t=1}^T \sum_{z^t} \Pi(z^t) \sum_{\theta^t} P(\theta^t) \beta^{t-1} V(D(z^t, \theta^t)). \quad (2.12)$$

Endogenous Stock Market Participation

For each reservation value W_0 , the labor contract specifies a sequence of consumption allocations as a function of the sequence of shocks. The equilibrium reservation value is chosen by shareholders such that workers is better off serving as a worker, that is⁹

$$W_0 \geq \sum_{t=1}^T \sum_{z^t, \theta^t} \Pi(z^t) P(\theta^t) \beta^{t-1} U[D(z^t, \theta^t, W_0), L(z^t, \theta^t) = 0]. \quad (2.13)$$

2.2 Recursive Formulation

To solve (2.12), it is convenient to rewrite the shareholder's problem recursively. Following Spear and Srivastava (1987) and Green (1987), we use the promised utility as a state variable, denoted by W . Provided certain boundary conditions satisfied by our problem, under i.i.d. private shock distribution, temporary incentive compatibility is sufficient to guarantee the general IC condition (2.8).

The shareholder's problem (2.12) can be solved by considering two problems separately: (1) *period T problem*, where the shareholder specifies period T consumption and labor, and (2) *period t problem* where the shareholder chooses current consumption, labor, and promised continuation utility. From here onward, we make the additional assumption that both the public and private labor productivity shocks have two state realizations per period, $z_t \in \{z_h, z_l\}, \theta_t \in \{\theta_h, \theta_l\}$, with $z_h > z_l, \theta_h > \theta_l, \forall t$. The seed values $\{\theta_h, \theta_l\}$ are known to all agents.

⁹There are multiple values of W_0 satisfying Condition (2.13). In the quantitative analysis, I calibrate it to match the level of labor share in dynamic labor contract model with that in the Walrasian RBC model $1 - \alpha = 64\%$.

I consider the relaxed problem: only incentive compatibility constraints to prevent worker with high state realization θ_h from lying are considered.¹⁰ The period T problem is

$$S_T(W_T) = \max_{\{C_T, D_T, L_T\}} \sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) V(D_T) \quad (2.14)$$

subject to

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) U(C_T, L_T) \geq W_T; \quad (2.15)$$

$$U[C(z_T, \theta_h), L(z_T, \theta_h)] \geq U[C(z_T, \theta_l), \hat{L}(z_T, \theta_l)], \quad \forall z_T \in Z_T, \quad (2.16)$$

where $\hat{L}(z_T, \theta_l) = \left[\frac{\phi(z_T, \theta_l)}{\phi(z_T, \theta_h)} \right]^{\frac{1}{1-\alpha}} L(z_T, \theta_l);$

$$C_T + D_T \leq Y_T, \quad \forall z_T \in Z_T, \theta_T \in \Theta_T. \quad (2.17)$$

The period t problem is:

$$S_t(W_t) = \max_{\{C_t, D_t, L_t, W'_t\}} \sum_{z_t} \Pi(z_t) \sum_{\theta_t} P(\theta_t) [V(D_t) + \beta S_{t+1}(W'_t)] \quad (2.18)$$

subject to

$$\sum_{z_t} \Pi(z_t) \sum_{\theta_t} P(\theta_t) [U[C_t, L_t] + \beta W'_t] \geq W_t; \quad (2.19)$$

$$U[C(z_t, \theta_h), L(z_t, \theta_h)] + \beta W'_t(z_t, \theta_h) \geq U[C(z_t, \theta_l), \hat{L}(z_t, \theta_l)] + \beta W'_t(z_t, \theta_l), \quad \forall z_t \in Z_t, \quad (2.20)$$

where $\hat{L}(z_t, \theta_l) = \left[\frac{\phi(z_t, \theta_l)}{\phi(z_t, \theta_h)} \right]^{\frac{1}{1-\alpha}} L(z_t, \theta_l).$

$$C_t + D_t \leq Y_t, \quad \forall z_t \in Z_t, \theta_t \in \Theta_t. \quad (2.21)$$

We adopt the Cobb-Douglas functional form for the worker's utility

$$U(C, L) = \frac{[C^\tau (1-L)^{1-\tau}]^{1-\sigma_w}}{1-\sigma_w}, \quad (2.22)$$

where the consumption share is $\tau \in (0, 1)$ and the curvature parameter is $\sigma_w > 1$. Cobb-Douglas utility implies a constant elasticity of substitution between consumption and leisure; hence, the labor input is constant across states when competitive wage is offered at the marginal product of labor. Shareholder has power utility function with constant relative risk aversion σ_s

$$V(C) = \frac{C^{1-\sigma_s}}{1-\sigma_s}. \quad (2.23)$$

The shareholders are less risk averse than the worker, i.e. $0 < \sigma_s < 1 + \tau(\sigma_w - 1)$. I further

¹⁰The relaxed problem is shown to be equivalent to the original problem in [Green and Oh \(1991\)](#) for a more general setting. In the numerical solutions, I verify that the solution of the relaxed problem satisfies the original problem.

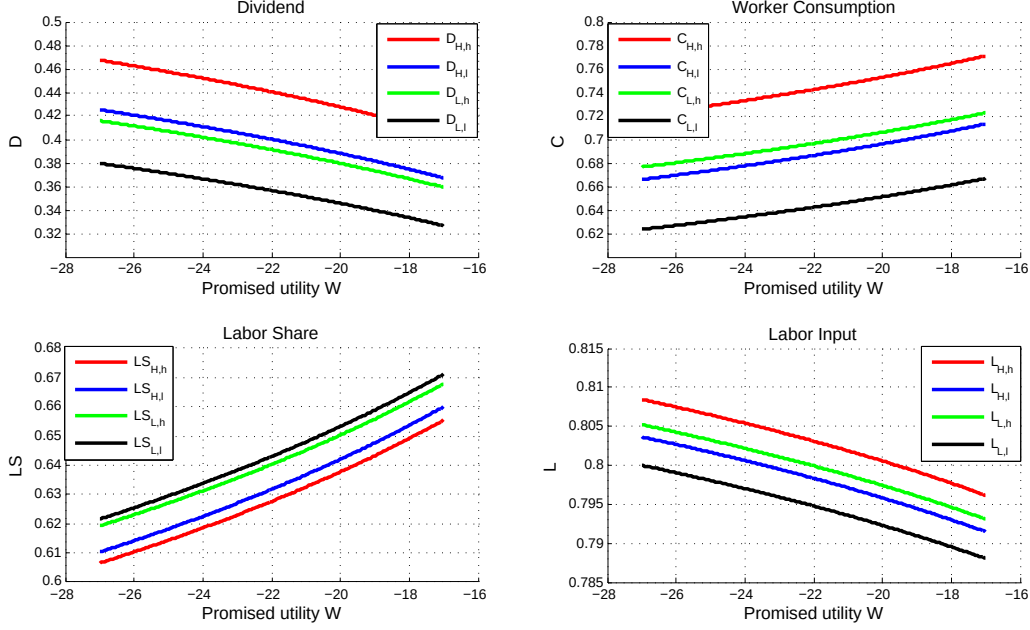


Figure 1. Equilibrium allocations under private information. This figure shows the equilibrium allocations of shareholder dividend, worker consumption, labor share, labor input under private information.

assume the technology shock takes the form

$$\phi(z_t, \theta_t) = z_t \theta_t. \quad (2.24)$$

2.3 Optimality Conditions

I characterize the properties of the equilibrium allocations specified by the dynamic contracts under private information. Figure 1 shows the constrained optimal consumption allocation and labor input at $\Omega = 0.5$. From Panel A and B, both agents' consumption profiles are procyclical with productivity. Shareholder's dividend shows larger percentage deviation than that of the worker. Panel C plots labor share, the percentage of worker's consumption over total output. The countercyclical labor share demonstrates how labor contract provides insurance to the worker. Panel D shows that dynamic labor contract features a procyclical labor input which enlarge the aggregate production risk.

I characterize the optimal solution in the full information case below.

Proposition 10 *The optimal solution under full information features, $\forall(z, \theta)$,*

- (i) *Perfect risk-sharing* $\frac{\partial V(z, \theta)}{\partial D(z, \theta)} = \eta_t \frac{\partial U(z, \theta)}{\partial C(z, \theta)}$;
- (ii) *Countercyclical labor share* $\frac{C(z, \theta)}{Y(z, \theta)} = \frac{\tau(1-\alpha)}{1-\tau} \left(\frac{1}{L(z, \theta)} - 1 \right)$;
- (iii) *Constant promised continuation utility* $W'(z, \theta) = W$;

(iv) *The first best dynamic contracts can be equivalently implemented by a sequence of static contracts.*

2.4 Asset Prices

The limited market participation implies that equilibrium holding of risky asset is Autarky, i.e. $S_t^* = 1$. Given any state realizations $\{z_t, \theta_t\}_{t=1}^T$ and the equilibrium allocation series $\{D_t, C_t, L_t\}_{t=1}^T$, the price of risky asset is given by

$$P_t = \beta \mathbb{E}_t \left[\frac{V'(D_{t+1})}{V'(D_t)} (P_{t+1} + D_{t+1}) \right] = \mathbb{E}_t \left[\sum_{j=1}^{T-t} \beta^{j-1} \frac{V'(D_{t+j})}{V'(D_t)} D_{t+j} \right]. \quad (2.25)$$

and $P_T = 0, \quad \forall z_T, \theta_T$.

Implied equilibrium market return from holding the risky asset is

$$R_t = E_t \left[\frac{P_{t+1} + D_{t+1}}{P_t} \right] - 1. \quad (2.26)$$

Bond price b_t^{t+1} is calculated from the shareholder's intertemporal Euler equation

$$b_t^{t+1} = \beta E_t \left[\frac{V'(D_{t+1})}{V'(D_t)} \right]. \quad (2.27)$$

The implied equilibrium risk-free rate is

$$r_t^f = \frac{1}{b_t^{t+1}} - 1. \quad (2.28)$$

Equity premium is the difference between market return and risk-free rate

$$EP_t = R_t - r_t^f. \quad (2.29)$$

3 Quantitative Results

In this section, I solve the model numerically for equilibrium allocations and asset returns. Then I compare the performance of three different models in matching financial market statistics and macro variable features. The baseline model is the finite horizon dynamic labor contract model under private information. The two reference models are respectively Walrasian real business cycle model (with competitive wage) and a static labor contract model.

Walrasian RBC Model

The canonical Walrasian RBC model features a competitive labor market and is commonly used in the production based asset pricing literature. With a competitive wage $w^* = \frac{\partial Y_{z,\theta}}{\partial L_{z,\theta}} = (1 - \alpha)z\theta L_{z,\theta}^{-\alpha} K^\alpha$, we get constant labor share: $LS = 1 - \alpha$. Worker has constant labor input

$L(z, \theta) = \tau$ and bears her share of income risk $C(z, \theta) = (1 - \alpha)z\theta L_{z,\theta}^{1-\alpha} K^\alpha$.¹¹

Static Agency Model

The second reference model is a static agency model, in which the shareholder and the worker enter a static contract every period. Given the worker's period reservation utility W , the shareholder solves the following problem

$$S(W) = \max_{\{C,D,L\}} \sum_z \Pi(z) \sum_\theta P(\theta) V[D(z, \theta)] \quad (2.30)$$

subject to

$$\sum_z \Pi(z) \sum_\theta P(\theta) U[C(z, \theta), L(z, \theta)] \geq W \quad (2.31)$$

$$U[C(z, \theta_h), L(z, \theta_h)] \geq U\left[C(z, \theta_l), \left[\frac{\phi(z, \theta_l)}{\phi(z, \theta_h)}\right]^{\frac{1}{1-\alpha}} L(z, \theta_l)\right] \quad (2.32)$$

$$C(z, \theta) + D(z, \theta) \leq Y(z, \theta), \quad \forall z \in Z, \theta \in \Theta. \quad (2.33)$$

As shown in Proposition 10.(iv), the first best dynamic contract can be equivalently implemented by a corresponding sequence of static contract under the same private information structure. This enables us to match the static model with the dynamic contract model to the extent that the optimal allocation under private information of the two models are comparable. Given the worker's initial reservation utility W_0 in the dynamic contract, and the corresponding first best solution of the continuation utility as $\{W'_t(z, \theta)\}_{t=1}^{T-1}$, the equivalent static reservation utility at time t is $\bar{W}_t = W'_{t-1} - \beta W'_t$, as in the proof of Proposition 10. Using this transformation, the static first best problem and dynamic first best problem *match exactly* in their equilibrium allocations and asset prices.

The importance of noncompetitive labor market and labor contract framework is shown by comparing labor contract models with the Walrasian RBC model. The comparison of models with dynamic contracts versus models with static contracts shows the intertemporal incentive effect by promised continuation utility. This allows us to separate intertemporal risk-sharing from intratemporal risk-sharing.

3.1 Parameter Calibration

The parameter calibration is shown in Table 1. The model is calibrated at an annual frequency. The contracting time horizon T is parametrized with both empirical and computational consid-

¹¹In competitive labor market, individual worker decides how much labor to input. Given the competitive wage level w^* , each worker solves: $\max \sum_z \Pi_z P_\theta U[C(z, \theta), L(z, \theta)]$, subject to their budget constraint: $C(z, \theta) \leq w^* L(z, \theta)$. First order conditions yield: $-\frac{\partial U[C(z, \theta), L(z, \theta)]}{\partial C(z, \theta)} w^* = \frac{\partial U[C(z, \theta), L(z, \theta)]}{\partial L(z, \theta)}$. From firm's optimization, $w^* = \frac{\partial Y_{z, \theta}}{\partial L_{z, \theta}} = (1 - \alpha)z\theta L_{z, \theta}^{-\alpha} K^\alpha$. Combining the two conditions, the equilibrium labor input is: $\frac{\tau}{L_{z, \theta}} = \frac{1 - \tau}{1 - L_{z, \theta}} \Rightarrow L_{z, \theta} = \tau, \quad \forall z, \theta$.

Table 1. Benchmark Parameter Calibration

Parameter	Symbol	Value
Contracting time horizon	T	8
Capital share	α	0.36
Constant capital stock	k_0	1.257
Probability of a high publicly observed productivity shock	$\Pi(z = H)$	0.5
Probability of a high privately observed productivity shock	$\Pi(\theta = h)$	0.5
Consumption share in worker's utility	τ	0.8
Time preference (discount rate)	β	0.975
Shareholder's risk aversion	σ_s	3.5
Worker's utility function curvature	σ_w	8
Worker's risk aversion	$\tau(\sigma_w - 1) + 1$	6.6
Worker's lifetime reservation utility	W_0	-78
Fraction of labor productivity risk due to private information	Ω	0.5
Total risk of productivity	$Var(z) + Var(\theta)$	0.0033

erations. Despite the lack of formal statistics, several studies have shown that people change jobs every 3 to 10 years in relatively large businesses. The model generated moments tend to be numerically more stable the longer horizon we take. As computation time increases exponentially with the time horizon, $T = 8$ is chosen as a compromise.

Capital share $\alpha = 0.36$ is set following the convention of real business cycle models. Constant capital K^α in the production function is set to 1.257. Both public and private shocks are i.i.d. in the benchmark model with $\Pi(z = H) = \Pi(z = L) = 0.5$, $P(\theta = h) = P(\theta = l) = 0.5$.

The consumption share in worker's utility τ , agents' subjective time preference β , and curvature parameters σ_s , σ_w are free parameters calibrated to match the target moments of asset returns and consumption.¹² We set the shareholder's risk aversion $\sigma_s = 3.5$, and set $\sigma_w = 8$ to yield workers' risk aversion as $\tau(\sigma_w - 1) + 1 = 6.6$, a commonly adopted value in the equity premium literature.¹³

Since the worker's reservation utility W_0 directly determines her bargaining power in the contracting process and her welfare, we calibrate it to match the level of labor share in the dynamic labor contract model with that in the Walrasian RBC model $1 - \alpha = 64\%$.

The fraction of labor productivity risk due to private information Ω is calibrated as $\Omega = 0.5$ in the benchmark model. In Figure 2, we see how the level of equity premium and risk-free rate change in the Walrasian RBC and the dynamic labor contract model when we vary Ω in the range of $[0, 1]$ while keeping the total risk of productivity $Var(z) + Var(\theta)$ fixed. Both the equity premium and risk-free rate decrease as the fraction of private information Ω increases.

¹²Ales and Maziero (2010) estimate in their benchmark model $\tau = 0.69$ using CEX and PSID data.

¹³Guvenen (2009) uses 7, Gomes and Michaelides (2008) select 5, and Danthine and Donaldson (2002) adopt a value around $4 \sim 7$.

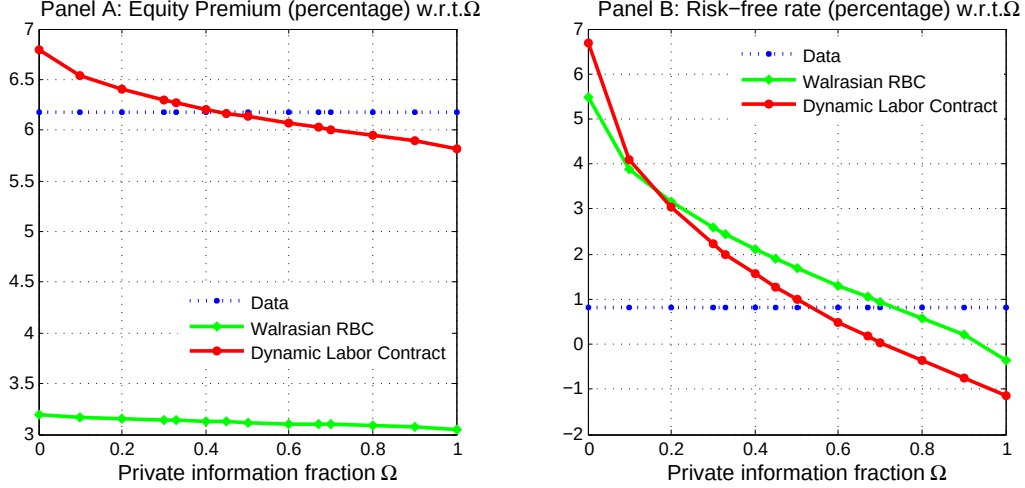


Figure 2. Equity Premium and Risk-free Rate w.r.t. Ω . The figure plots the equity risk premium and risk free rate (in percentage) when we vary the fraction of labor productivity risk due to private information Ω .

The dynamic labor contract model with value $\Omega = 0.5$ gives the two moments in the right ballpark, while Walrasian RBC has equity premium way lower than data.

3.2 Comparison of Model Performance

With the benchmark model parametrized as above, we numerically compare the model generated moments of equilibrium allocation for consumption, labor input, labor share, and the asset returns in the dynamic labor contract model and the two reference models.

Table 2 shows the comparison of model generated *unconditional moments* with real data. historical asset returns data varies greatly depending on the time window. I take the commonly cited data from Table 1 of [Mehra and Prescott \(1985\)](#). For the benchmark calibration with *i.i.d.* shocks and $\Omega = 0.5$, the dynamic labor contract model generates a level of equity premium 6.13% and a risk-free rate 0.98%, which is close to the empirical average during 1889–1978. The dynamic labor contract model clearly performs better in generating higher risky asset returns and lower risk-free rate over the Walrasian RBC and the static contract model.

While the first moments of asset returns and the volatility of market return match the empirical targets, the unconditional volatility of risk-free rate is too high and that of equity premium is too low. This stems from the assumption that shareholders are all alike and their equilibrium bond holdings are zero. In real world, bond trading helps to stabilize risk-free rate. When the volatility of risk-free rate is lowered, we would get a more volatile equity premium.

Using the CEX data available from 1980 to the first quarter of 2005, we get the standard deviation of households' annual (four quarters rather than annualized quarterly data) consumption growth as 3.00%, 5.38%, and 12% respectively for non-shareholders, shareholders, and top shareholders (definitions see [Malloy, Moskowitz, and Vissing-Jorgensen \(2009\)](#)). As our

Table 2. Comparing Model Generated Moments with Data

		Dynamic	Walrasian	Static
Moments	Data	Labor Contract	RBC	Labor Contract
Asset returns				
Equity premium	6.18%	6.13%	3.12%	4.19%
Market return	6.98%	7.12%	4.80%	7.11%
Risk-free rate	0.80%	0.98%	1.69%	2.92%
$\sigma(EP_t)$	16.67%	2.59%	1.09%	1.51%
$\sigma(R_t)$	16.54%	27.03%	17.75%	21.54%
$\sigma(r_t^f)$	5.67%	24.84%	16.85%	20.29%
Consumption risks				
$\sigma(\Delta D_{t+1})$	5.38% - 12%	7.84%	5.66%	7.82%
$\sigma(\Delta C_{t+1})$	3% - 5.38%	4.84%	5.66%	5.95%
$\frac{\sigma(\Delta D_{t+1})}{\sigma(\Delta C_{t+1})}$	1.6	1.6	1	1.3
Average labor share	0.64	0.64	0.64	0.64
Correlations				
$\text{Corr}(EP_t, z_t)$	-	-0.41	-0.41	-0.62
$\text{Corr}(EP_t, \theta_t)$	-	-0.56	-0.53	-0.31
$\text{Corr}(L_t, z_t)$	+	0.35	0	-0.12
$\text{Corr}(L_t, \theta_t)$	+	0.68	0	0.95
$\text{Corr}(D_t, z_t)$	+	0.56	0.54	0.85
$\text{Corr}(D_t, \theta_t)$	+	0.52	0.54	0.12
$\text{Corr}(C_t, z_t)$	+	0.52	0.54	0.23
$\text{Corr}(C_t, \theta_t)$	+	0.56	0.54	0.79
$\text{Corr}(\frac{C_t}{Y_t}, z_t)$	-	-0.60	0	-0.88
$\text{Corr}(\frac{C_t}{Y_t}, \theta_t)$	-	-0.45	0	0.80

Note: Models are compared under the same sequence of state realizations. Sources of data include: (1) Mehra and Prescott (1985); (2) CEX 1980-2005 Q1, see Mankiw and Zeldes (1991).

model assumes shareholders do not work and workers do not hold shares, the consumption growth volatility of the two groups of agents should fall within the range of 3% $\sim$ 5.38% and 5.38% $\sim$ 12%. The dynamic labor contract model generates the shareholder's consumption growth volatility as 7.84% and that of worker as 4.84%, matching the empirical observations.

Define *risk-ratio* to be the ratio of shareholders' consumption growth volatility to that of non-shareholders as a risk-sharing indicator in the economy. Mankiw and Zeldes (1991) document the ratio to be 1.6 using PSID data, although their consumption measure consists of only food expenditures rather than the nondurable goods. The same ratio assumes a value of 1.6 in our benchmark dynamic labor contract model. In comparison, risk-ratio equals 1 in Walrasian RBC model, which means agents in the economy bear risk equally. Under the competitive labor market setting, no risk is shared between agents with different risk aversion. The static labor contract model yields a ratio of 1.3, which indicates risk is not shared enough through the static

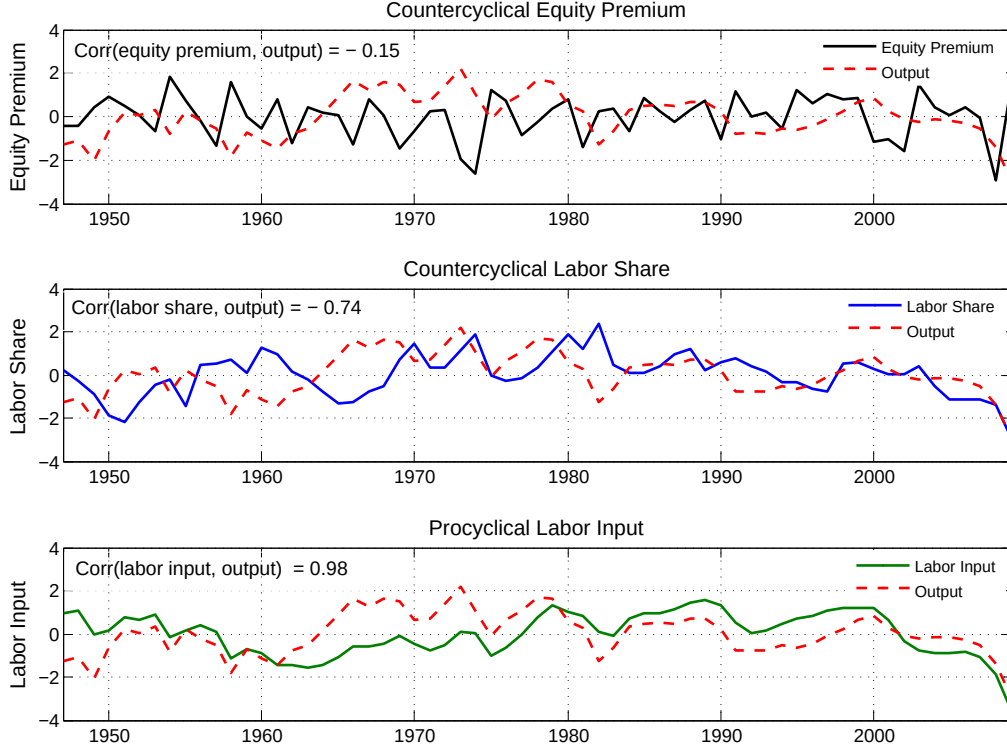


Figure 3. Cyclical Features of Macro Variables. This figure plots the empirical patterns of countercyclical equity premium, countercyclical labor share, and procyclical labor input. The data sources include BLS 1947-2010 and CRSP for asset returns.

labor contract.

Figure 3 shows the countercyclical equity premium and labor share, and procyclical labor input.¹⁴ As in Guvenen (2009), the countercyclical equity premium has been difficult to generate in standard production based asset pricing models with constant labor share. Nonetheless, the countercyclical movement of labor share over the business cycle has been put forth as part of an optimal risk-sharing arrangement between firms and workers.¹⁵ The dynamic labor contract model captures this feature and generates equity premium and labor share negatively correlated with labor productivity shocks. As is shown in the lower half of Table 2, the cyclical features of all macro variables in the model with dynamic labor contracts are consistent with data.

To better understand the risk-sharing properties in long-term employment relationship, and the advantage of dynamic contract model over static ones, we construct the equivalent static contract by matching its first best solution with that of the dynamic case. The key difference between dynamic and static labor contract models is the adoption of promised continuation utility as an instrument to provide incentive and facilitate risk-sharing. The comparison allows us to isolate the effect of intertemporal risk-sharing from intratemporal risk-sharing.

¹⁴To highlight the comovement between series, each data series is detrended, rescaled by its standard deviation. I use log real GDP as output.

¹⁵See for example Gomme and Greenwood (1995), Boldrin and Horvath (1995), Gomme and Rupert (2004).

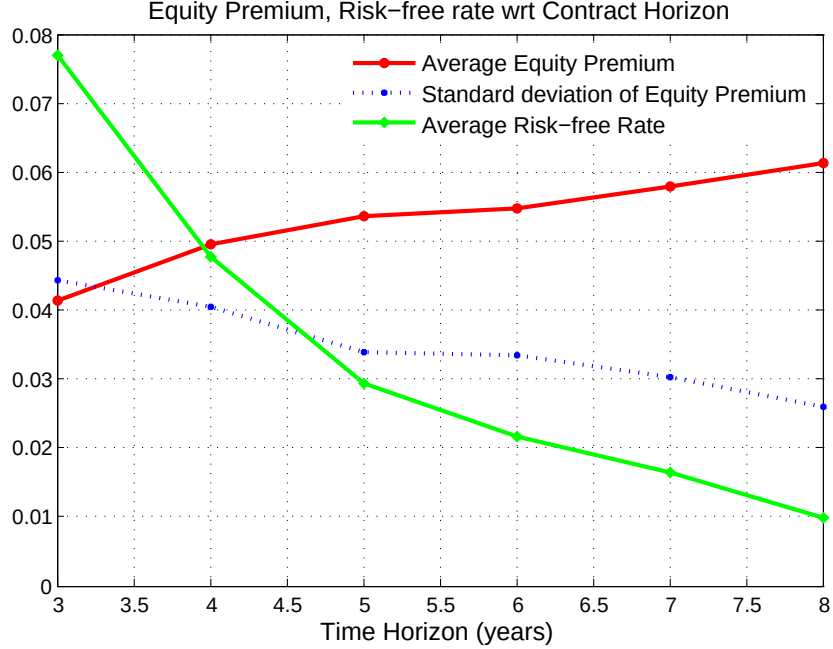


Figure 4. Horizon of Dynamic Labor Contract. This figure plots the asset returns as a function of the horizon of the dynamic labor contract.

Compare Column 3 and 5 in Table 2. Dynamic labor contract model generates a comparable market return but a much lower risk-free rate, compared with static contract model. Not only is the level of equity premium higher, but also is its volatility and countercyclical variation. Labor input is strongly procyclical and labor share is strongly countercyclical in dynamic model, while both the two moments present ambiguous cyclical features in the static model. We observe that $Corr(L_t, \theta_t)_{\text{dynamic}} < Corr(L_t, \theta_t)_{\text{static}}$. This is because part of the second best distortion is transformed to the spread out of continuation utility in the dynamic contract. The risk-sharing property is summarized by an increase in $\sigma(\Delta D_{t+1})$, a decrease in $\sigma(\Delta C_{t+1})$, and a negative $Corr(\frac{C_t}{Y_t}, \theta_t)$ in the dynamic model. Hence we conclude that the intertemporal incentive from dynamic contract facilitates risk-sharing, evidenced from the higher risk ratio, and contributes an additional increase in the equity premium by 1.94%.

To further study how the dynamic effect changes with the contract horizon, in Figure 4 we present the asset returns with respect to parameter T (the rest parameter values are set as in Table 1). We get relatively more plausible and stable asset returns as T increases. In our benchmark case when $T = 8$, levels of equity premium and risk-free rate become closer to data. Besides, the computation results is more stable with respect to state realizations as can be seen by a lower unconditional standard deviation of the excess return.

Simulation results in Figure 5 show that equity premium in dynamic labor contract model is countercyclical, and is significantly higher than the other models. Shareholder's consumption growth volatility is amplified procyclically, while worker's consumption is smoothed under the

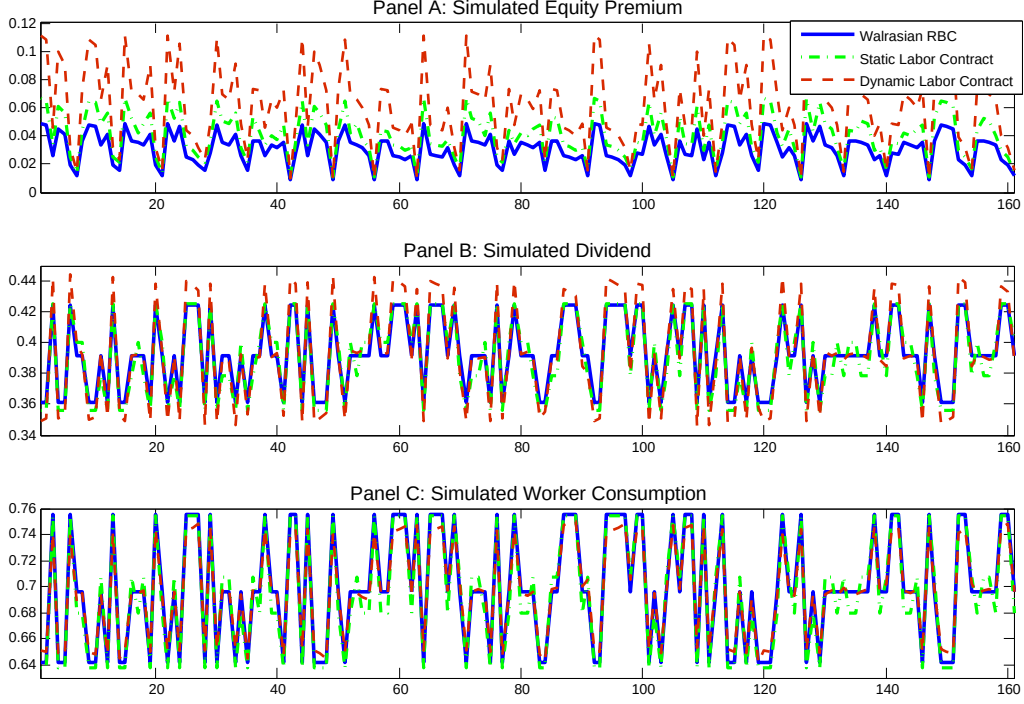


Figure 5. Model Simulated Moments. This figure plots model simulated series of equity premium, dividend (shareholder consumption), and worker consumption for the dynamic labor contract model (red dashed), the static labor contract model (green dot dashed), and a competitive labor market model (green solid). The model is simulated 1000 times and randomly plotted for 160 years of horizon.

labor contract.

4 Persistence of Public Shock

So far the shocks are assumed to be identically distributed, independent of history realizations, and with an equal probability for high and low state. In this section, we relax this assumption by looking at the effect of persistent public shocks.

The persistence of business cycle risk is well understood in the real business cycle literature. As examples of publicly observed common shocks, global climate and technological developments happen gradually. We model the distribution of public shock as a two state first-order Markov process. The transition matrix $\Pi = \begin{bmatrix} \Pi_{HH} & \Pi_{HL} \\ \Pi_{LH} & \Pi_{LL} \end{bmatrix}$ is public information to both agents. Persistence is governed by $\Pi_{HH} \in (0.5, 1)$, and $\Pi_{LH} \in (0, 0.5)$.

The deviation from i.i.d. shock distribution results a difference in the recursive formulation and the numerical procedure. Besides the promised utility W , we now need an extra state variable z_{-1} : the previous period public shock realization. The mapping from W to $S(W)$ is now conditional on z_{-1} since we have different implied shock distributions in the optimization problem depending on whether the previous public realization is z_H or z_L . We compute both

Table 3. Model Generated Moments under Persistent Public Shock

		Dynamic	Walrasian	Static
Moments	Data	Labor Contract	RBC	Labor Contract
Asset returns				
Equity premium	6.18%	5.75%	2.91%	4.37%
Market return	6.98%	6.54%	4.60%	7.00%
Risk-free rate	0.80%	0.79%	1.69%	2.63%
$\sigma(EP_t)$	16.67%	2.28%	0.98%	1.51%
$\sigma(R_t)$	16.54%	23.29%	15.42%	19.63%
$\sigma(r_t^f)$	5.67%	21.47%	14.67%	18.49%
Consumption risks				
$\sigma(\Delta D_{t+1})$	5.38% - 12%	8.59%	6.20%	9.55%
$\sigma(\Delta C_{t+1})$	3% - 5.38%	5.22%	6.20%	5.57%
$\frac{\sigma(\Delta D_{t+1})}{\sigma(\Delta C_{t+1})}$	1.6	1.6	1	1.7
Average labor share	0.64	0.64	0.64	0.64
Correlations				
$\text{Corr}(EP_t, z_t)$	-	-0.40	-0.40	-0.60
$\text{Corr}(EP_t, \theta_t)$	-	-0.51	-0.49	-0.25
$\text{Corr}(L_t, z_t)$	+	0.58	0	0.04
$\text{Corr}(L_t, \theta_t)$	+	0.46	0	0.89
$\text{Corr}(D_t, z_t)$	+	0.77	0.74	0.95
$\text{Corr}(D_t, \theta_t)$	+	0.26	0.30	-0.10
$\text{Corr}(C_t, z_t)$	+	0.71	0.74	0.43
$\text{Corr}(C_t, \theta_t)$	+	0.34	0.30	0.63
$\text{Corr}(\frac{C_t}{Y_t}, z_t)$	-	-0.83	0	-0.88
$\text{Corr}(\frac{C_t}{Y_t}, \theta_t)$	-	-0.13	0	0.79

Note: Models are compared under the same sequence of state realizations. Sources of data include: (1) [Mehra and Prescott \(1985\)](#); (2) CEX 1980-2005 Q1, see [Mankiw and Zeldes \(1991\)](#).

sets of conditional mappings and plug in the corresponding mapping according to the previous state realization.

To construct the equivalent static contract problem, notice that the static reservation utility is also conditional on the previous public state realization z_{-1} . Define $\bar{W}_t/z_{t-1} = W_t/z_{t-1} - \beta \sum_z \Pi_{/z_{t-1}} \sum_\theta P_\theta W'_t(z, \theta)$, $\forall t = 1, 2, \dots, T$, where $W_1/z_0 = W_1 = W_0$, $\Pi_{/H} = \Pi_{/L} = \Pi$ at $t = 1$ (with no prior) and $W'_T(z, \theta) = 0$. The static problem with conditional static reservation utility $\bar{W}_t/z_{t-1}$ matches with the dynamic contract problem in their first best solution. A detailed derivation can be found in the Appendix, proof of Proposition 10.

We parametrize the transition matrix as $\Pi_{HH} = \Pi_{LH} = 0.5$ for $t = 1$, and $\Pi_{HH} = 0.6, \Pi_{LH} = 0.4$, for $t \geq 2$. This corresponds to a quarterly persistence of 0.88 from high state to high state. We keep the other parameter values fixed as in Table 1, and vary Ω . Figure 6 shows how the level of equity premium and risk-free rate change with private shock component. We also plot

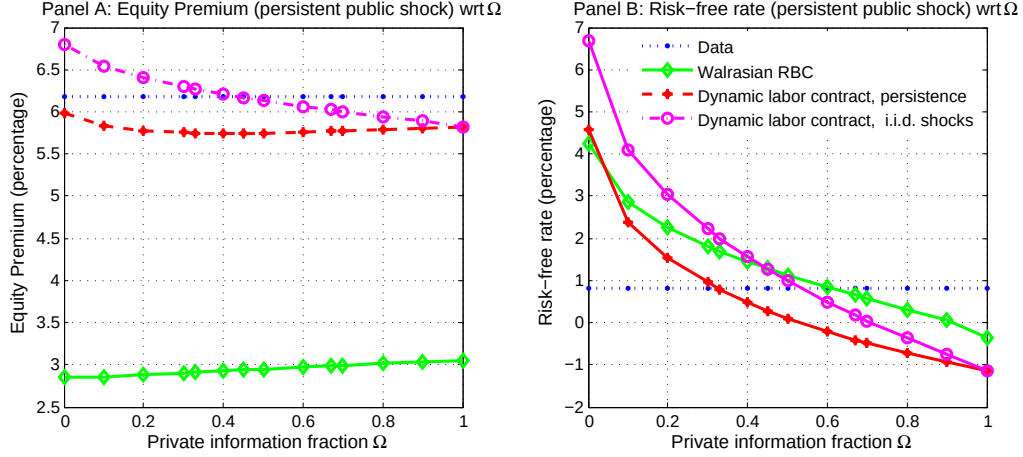


Figure 6. Equity Premium and Risk-free Rate under Persistent Public Shocks. This figure plots the equity risk premium and risk free rate (in percentage) under persistent public shocks when we vary the fraction of labor productivity risk due to private information Ω .

the historical data and i.i.d. case as references.

First, asset returns are lower under persistent public shocks relative to the i.i.d. shock case. This is because being aware of the shock persistence, agents expect a smoother economy. Shareholders now put a smaller probability weight to the event which contributes a larger consumption growth volatility, thus require to be compensated by smaller returns. As Ω increases, equity premium tend to decrease since risk-sharing becomes more limited. Meanwhile, a decrease of the public shock component weakens the persistence effect, which drives the equity premium up. As a result, we observe a first downward and then slight upward trend of the equity premium. At $\Omega = 1$, the persistence effect of public shocks disappears. This is where the results of the two cases coincide.

At the value of $\Omega = \frac{1}{3}$, both levels of equity premium and risk free rate are the closest to their empirical counterparts. Table 3 demonstrates that the advantage of dynamic contract models are robust under persistent public shocks. The size and allocation of consumption risk remains the same, since the persistent shock distribution only alters agents' expectations. Cyclical features of model generated moments are also robust.¹⁶

¹⁶Although adding history dependent private shock is interesting, complexities arise because the shareholder lacks the common prior on the current period shock distribution, which is conditional on the previous privately observed state realization θ_{-1} . Worker may become better off by deviating from truth-telling in the current period in order to send wrong message about the next period's shock distribution. [Fernandes and Phelan \(2000\)](#) show that *temporary incentive compatibility constraint* plus *threat keeping constraint* is equivalent to the incentive compatibility condition. The *threat keeping constraint* is defined as $U[C_t(z, \theta_h), L_t(z, \theta_h)] + \beta W'_t(z, \theta_h | p_h) \geq U[C_t(z, \theta_l), \tilde{L}_t(z, \theta_h)] + \beta \hat{W}'_t(z, \theta_l | p_h)$. where $\tilde{L}_t(z, \theta_h) = \left[\frac{\phi(z, \theta_l)}{\phi(z, \theta_h)} \right]^{\frac{1}{1-\alpha}} L_t(z, \theta_l)$, and $\hat{W}'_t(z, \theta_l | p_h)$ is the promised continuation utility to the worker if he lies under θ_h .

5 Conclusion

Frictions in labor relations are important for understanding risks in the financial markets. In this paper, I demonstrate the potential of modeling labor contracts in a dynamic general equilibrium model to reconcile both financial market and business cycle facts. The model has shown satisfactory quantitative performance in two dimensions. First, with consumption allocations matched to CEX data, the model generates equity premium and risk-free rate comparable with historical average. Second, regarding the cyclical features, this model successfully produces procyclical consumption and labor input, countercyclical equity premium and labor share.

This study adds to the literature by connecting the financial markets with labor market frictions. First, the model generates procyclical labor supply and countercyclical labor share, which are considered challenging for standard production-based asset pricing models. Second, I deviate from previous literature on perfect risk-sharing in labor relations by analyzing the limited risk-sharing implications due to private information. Finally, workers in my model endogenously choose not to participate in the financial markets because the labor contract has insured them to the maximum extent against labor income risk. This rationale provides new insights on the limited asset market participation.

Appendix

A Proofs

A.1 Proof of Lemma 4

We first show that bond holding is not a state variable in the recursive formulation. Suppose for contradiction that the recursive problem has two state variables: W and B , where B is the equilibrium holding of bonds traded between shareholders and workers. At period T , the shareholder solves

$$S_T(W, B) = \max_{\{C_T, D_T, L_T\}} \sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) V[D_T - B] \quad (2.34)$$

subject to

$$(C_T + B) + (D_T - B) \leq Y_T, \quad (2.35)$$

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) U[C_T + B, L_T] \geq W, \quad (\text{IR}) \quad (2.36)$$

$$U[C(z_T, \theta_h) + B, L(z_T, \theta_h)] \geq U[C(z_T, \theta_l) + B, \tilde{L}(z_T, \theta_h)], \quad (\text{IC}) \quad (2.37)$$

where $\tilde{L}(z_T, \theta_h) = \left[\frac{\phi(z, \theta_l)}{\phi(z, \theta_h)} \right]^{\frac{1}{1-\alpha}} L(z_T, \theta_l)$. Now fix W , $\forall \hat{B} \neq B$, the FOCs of this static problem (2.34) stay the same. Hence, $\hat{D}_T - \hat{B} = D_T - B$, $\hat{C}_T + \hat{B} = C_T + B$, $\hat{L} = L$. Essentially, $S_T(W, B) = S_T(W, \tilde{B})$, $\forall B, \tilde{B}$. Therefore, bond holding B is not a state variable for problem (2.34), i.e. bond holding will not affect the final consumption of both agents at period T .

Back to period $T - 1$, if bond holding in period T is irrelevant, then $B'_{z, \theta} = 0$. Then apply the same argument as above, we conclude B is irrelevant as a state variable.

$$S_{T-1}(W, B) = \max_{\{C_{T-1}, D_{T-1}, L_{T-1}, W'\}} \sum_{z_{T-1}} \Pi(z_{T-1}) \sum_{\theta_{T-1}} P(\theta_{T-1}) [V(c_{z, \theta} - B + B'b) + \beta S_T(W')] \quad (2.38)$$

subject to

$$(D_{T-1} - B + B'b) + (C_{T-1} + B - B'b) \leq Y_{T-1} \quad (2.39)$$

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) (U[C_{T-1} + B - B'b, L] + \beta W') = W, \quad (\text{IR}) \quad (2.40)$$

$$U(C_{z, h} + B - B'_{z, h} b_{z, h}, L_{z, h}) + \beta W'_h > U(C_{z, l} + B - B'_{z, l} b_{z, l}, \tilde{L}_{z, l}) + \beta W'_l, \quad (\text{IC}) \quad (2.41)$$

The Lemma is established by applying the same argument through all periods. [Q.E.D.](#)

A.2 Proof of Lemma 5

By assumption 2, shareholders are all alike. Hence at equilibrium, we must have $B_i = B_j$, $\forall i, j$. Because bonds are in zero net supply, equilibrium bond market clearing condition requires: $\int B_{i,t} dB = 0$. Therefore, $B_i = B_j = 0 \forall i, j$.

Now suppose we don't have the condition: $B_i = B_j = 0 \forall i, j$. Following the trading protocol in [Atkeson and Lucas \(1992\)](#), at $t = T$ type W_0 investor solves the problem

$$S_T(W = W_0) = \max_{\{C, D, L\}} \sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) V[D_T - B]$$

subject to the IR, IC, and feasibility constraints.

$$L = \sum \Pi(z_T) \sum P(\theta_T) V[D_T - B] - \lambda_{z, \theta} (C_T + D_T - Y_T) + \eta \left\{ \sum \Pi(z_T) \sum P(\theta_T) U(C_{z, \theta}, L_{z, \theta}) - W \right\} + \delta \left\{ U[C_{z, h}, L_{z, h}] - u[C_{z, l}, \tilde{L}_{z, h}] \right\}. \quad (2.42)$$

Bond market clearing must satisfy: $\sum B(W_0) = 0$.

Compared to the previous problem (2.34) when $B = 0$. The two problems will only coincide if $B_T = 0$; iterate backward, there must be $B_t(W_0) = 0, \forall t, W_0$. Hence, we establish that any nontrivial bond holdings of shareholders will change the original labor contracts and the equilibrium allocations. **Q.E.D.**

A.3 Proof of Proposition 10

Proposition 10.(i), 10.(ii), 10.(iii) are derived from the first order conditions with respect to $C_{z, \theta}$, $L_{z, \theta}$ and $W'_{z, \theta}$ in the first best problem.

Next we prove 10.(iv). Given the initial reservation utility for $t = 1$ is as W_1 , the first best problem for the dynamic contract at $t = 1, 2, \dots, T - 1$ is:

$$S_t(W_t) = \max_{\{C_t, D_t, L_t, W'_t\}} \sum_z \Pi_z \sum_{\theta} P_{\theta} [V(D_{z, \theta}) + \beta S_{t+1}(W'_t)]$$

subject to feasibility and the individual rationality constraint. As $W'_t(z, \theta)$ is independent of state (z, θ) for the i.i.d. shocks from 10.(iii), the IR constraint reduces to

$$\sum_z \Pi_z \sum_{\theta} P_{\theta} U[C_t, L_t] \geq W_t - \beta W'_t. \quad (2.43)$$

Solving the first best dynamic contract problem backward, we obtain $\{W_t\}_{t=1}^T$. Redefine: $\bar{W}_t = W_t - \beta W_{t+1}, \forall t = 1, 2, \dots, T - 1$, and $\bar{W}_T = W_T$. We get the equivalent first best static

contract problem $\forall t = 1, 2, \dots, T$:

$$S_t(W) = \max_{\{C_t, D_t, L_t\}} \sum_z \Pi_z \sum_{\theta} P_{\theta} V(D_{z,\theta})$$

subject to the feasibility constraint and an equivalent static IR constraint

$$\sum_z \Pi_z \sum_{\theta} P_{\theta} U[C_t, L_t] \geq \bar{W}_t.$$

The equivalent static contract under persistent public shocks Under persistent public shocks, mappings $W \rightarrow S(W)$ is conditional on the previous public state realization z_{-1} . The first order condition shows $S'_{/z}(W') + \eta = 0$, where η is the Lagrangian multiplier on the IR constraint.

Given that $S'(\cdot)$ is monotonic, $S'_{/z=H} \neq S'_{/z=L}$ and $S'_{/z=H}(W'_{H,\theta}) = S'_{/z=L}(W'_{L,\theta}) = -\eta$ jointly imply that $W'_{H,\theta} \neq W'_{L,\theta}$. The IR constraint is also conditional on previous public shock realization. For

$$\sum_z \Pi(z/z_{-1} = H) \sum_{\theta} P_{\theta} \{U[C_t, L_t] + \beta W'_t\} \geq W_t(z_{-1} = H) = W'_{t-1}(H, \theta); \quad (2.44)$$

$$\sum_z \Pi(z/z_{-1} = L) \sum_{\theta} P_{\theta} \{U[C_t, L_t] + \beta W'_t\} \geq W_t(z_{-1} = L) = W'_{t-1}(L, \theta). \quad (2.45)$$

Hence the conditional IR constraint reduces to

$$\sum_z \Pi(z/z_{-1} = H) \sum_{\theta} P_{\theta} U[C_t, L_t] \geq W_t(z_{-1} = H) - \beta \sum_z \Pi(z/z_{-1} = H) \sum_{\theta} P_{\theta} W'_t; \quad (2.46)$$

$$\sum_z \Pi(z/z_{-1} = L) \sum_{\theta} P_{\theta} U[C_t, L_t] \geq W_t(z_{-1} = L) - \beta \sum_z \Pi(z/z_{-1} = L) \sum_{\theta} P_{\theta} W'_t. \quad (2.47)$$

Redefine

$$\bar{W}_t/z_{t-1} = W_t/z_{t-1} - \beta \sum_z \Pi(z/z_{-1}) \sum_{\theta} P_{\theta} W'_t, \quad \forall t = 1, 2, \dots, T, \quad (2.48)$$

where $W_1/z_0 = w_1 = w_0$, $\Pi_H = \Pi_L = \Pi$ at $t = 1$ (with no prior) and $W'_T(z, \theta) = 0$. Hence, by defining the conditional static reservation utility $\bar{W}_t/z_{t-1}$, we construct the equivalent static first best problem, of which the equilibrium allocation and asset returns coincide with those of the dynamic first best contract problem. [Q.E.D.](#)

B Numerical Procedure

To solve the model numerically, we adopt the numerical method as in [Ales and Maziero \(2010\)](#).¹⁷ With the recursive formulation of the contract problem, we break the optimization algorithm into end period T problem and period $T - 1$ problem, and solve it using backward induction.

1. Solve the dynamic contracting problem

- (a) In each period, the state variable W (promised utility) is discretized on the appropriate time-variant support interval with grid step 0.05. Given a set of first order equations, we solve the first best problem using Newton's method. Setting the first best solution as the initial guess for the corresponding second best problem greatly helps to improve the computation efficiency and convergence stability.
- (b) Having solved for the optimal policy function, we compute the value function (expected utility) of the shareholder and its numerical derivatives. The first order derivatives are computed using two-sided difference formula, second derivatives using a three-point formula. We repeat the above procedure for period t problem except that the state-contingent promised continuation utility $W'(z, \theta)$ is added to the system of first order equations.
- (c) As the mappings from W to $S(W)$ and its derivatives are well behaved in our setting (see Figure 10), we use cubic spline interpolation as the numerical approximation over the interval of state variable support. The entire dynamic first best and second best mappings are solved by integrating the procedure back to $t = 1$.

2. Simulate the allocations

- (a) Given the period value functions computed above, we now solve the equilibrium allocation forward from $t = 1, 2, \dots, T$ given the initial reservation utility W_0 and a state realization history (z^T, θ^T) .
- (b) W_0 is the promised utility in period $t = 1$ problem. Pick the promised continuation utility according to the state realization (z_1, θ_1) as the promised utility for period $t = 2$, and repeat till $t = T$. This procedure gives us the sequences of the worker's required labor input and consumption streams of both agents in a dynamic contract.

¹⁷I extend their framework to a risk averse shareholder. As a result, I solve for the maximization of expected utility rather than cost minimization.

3. Solve the equivalent static contract problem.

- (a) Take the sequence of promised utility solved in the dynamic contract problem $\{W_t\}_{t=1}^T$ and $\{W'_t\}_{t=1}^{T-1}$ and transform to the redefined static promised utility: $\{\bar{W}_t\}_{t=1}^T = \{W_t - \beta W'_t\}_{t=1}^T$, with $W'_T = 0$.
- (b) Take $\bar{W}_t$ as promised utility for each period t and use the same procedure as in Step 1, period T problem, we get the sequences of the worker's required labor input and consumption streams of both agents in the equivalent static contract.

4. Solve the asset prices.

- (a) With a finite horizon setting tailored to accommodate the dynamic labor contract, the price of risky asset at time t depends on the shareholder's consumption stream from the time t node to all the following possible realizations till end period T . We solve for the contract in all possible state realizations in the event tree, which amounts to a number of 4^T sets of equilibrium allocations.
- (b) Start from the end of the event tree, we calculate period $T - 1$ asset price with the Euler equation (we have 4^{T-1} of them).
- (c) Iterate backward till $t = 1$, we get all the asset prices at each node.
- (d) Picking the sequence of realized asset prices according to the specific state realization (z^T, θ^T) gives equilibrium prices and returns of risky asset.
- (e) Similar procedures are applied to the pricing of bonds.

Chapter 3

A Labor Capital Asset Pricing Model

1 Introduction

Dynamics in the labor market are an integral component of business cycles. More than 10 percent of U.S. workers separate from their employers each quarter. Some move directly to a new job with a different employer, some become unemployed and some exit the labor force. These large flows are costly for firms, because they need to spend resources to search for and train new employees.¹

Building on the seminal contributions of [Diamond \(1982\)](#), [Mortensen \(1982\)](#), and [Pissarides \(1985\)](#), we show that labor search frictions are an important determinant of the cross-section of equity returns. In search models, firms post vacancies to attract workers, and unemployed workers look for jobs. The likelihood of matching a worker with a vacant job is determined endogenously and depends on the congestion of the labor market, which is measured as the ratio of vacant positions to unemployed workers. This ratio, termed *labor market tightness*, is the key variable of our analysis. Intuitively, recruiting new workers becomes more costly when this ratio increases.

We begin by studying the empirical relation between labor market conditions and the cross-section of equity returns. We measure aggregate labor market tightness as the ratio of the monthly vacancy index published by the Conference Board to the unemployed population (cf. [Shimer \(2005\)](#)). To measure the sensitivity of firm value to labor market conditions, we estimate loadings of equity returns on log changes in labor market tightness controlling for the market return. We use rolling firm-level regressions based on three years of monthly data to allow for time variation in the loadings. Using the panel of U.S. stock returns from 1951 to 2012, we show

¹According to the U.S. Department of Labor, the cost of replacing a worker amounts to one-third of a new hire's annual salary. Direct costs include advertising, sign-on bonuses, headhunter fees and overtime. Indirect costs include recruitment, selection, training and decreased productivity while current employees pick up the slack. Similar evidence is contained in [Blatter, Muehlemann, and Schenker \(2012\)](#). [Davis, Faberman, and Haltiwanger \(2006\)](#) provide a review of aggregate labor market statistics.

that loadings on changes in the labor market tightness robustly and negatively predict future stock returns in the cross-section. Sorting stocks into deciles on the estimated loadings, we find an average spread in future returns of firms in the low- and high-loading portfolios of 6% per year. We emphasize that this return differential is not due to mispricing. While it cannot be attributed to differences in loadings on commonly considered risk factors, such as those of the CAPM or the [Fama and French \(1993\)](#) three-factor model, it arises rationally in our theoretical model due to risk associated with labor market frictions as we describe in detail below.

To ensure that the relation between labor search frictions and future stock returns is not attributable to firm characteristics that are known to relate to future returns, we run Fama-MacBeth ([1973](#)) regressions of stock returns on lagged estimated loadings and other firm-level attributes. We include conventionally used control variables such as a firm’s market capitalization and book-to-market ratio as well as recently documented determinants of the cross-section of stock returns that may potentially correlate with labor market tightness loadings, such as asset growth studied by [Cooper, Gulen, and Schill \(2008\)](#) and hiring rates investigated by [Belo, Lin, and Bazdresch \(2014\)](#). The Fama-MacBeth analysis confirms the robustness of results obtained in portfolio sorts. The coefficients on labor market tightness loadings are negative and statistically significant in all regression specifications. The magnitude of the coefficients suggests that the relation is economically important: For a one standard deviation increase in loadings, future annual returns decline by approximately 1.5%.

Our results hold not only when controlling for firm-level characteristics as in Fama-MacBeth regressions but also after accounting for macro variables. For example, labor market tightness and industrial production are correlated and highly procyclical. However, we show that loadings on labor market tightness contain information about future returns, while loadings on industrial production do not. We also find that, unlike many cross-sectional predictors of equity returns that are priced mainly within industries, labor market tightness loadings contain information about future returns when considered both within and across industries. Additional robustness tests confirm our results; for example, excluding micro stocks has a negligible effect on the return spread across labor market tightness portfolios.

To interpret the empirical findings, we propose a labor market augmented capital asset pricing model. Building on the search and matching framework pioneered by Diamond-Mortensen-Pissarides, we develop a partial equilibrium labor search model and study its implications for firm employment policies and stock returns. For tractability, we do not model the supply of labor as an optimal household decision; instead we assume an exogenous pricing kernel. Our model features a cross-section of firms with heterogeneity in their idiosyncratic profitability shocks and employment levels. Given the pricing kernel, firms maximize their value either by posting vacancies to recruit workers or by firing workers to downsize. Both firm policies are costly at proportional rates.

In the model, the fraction of successfully filled vacancies depends on labor market conditions

as measured by labor market tightness (the ratio of vacant positions to unemployed workers). As more firms post vacancies, the likelihood that vacant positions are filled declines, thereby increasing the costs to hire new workers. Since labor market tightness is a function of all firms' vacancy policies, it has to be consistent with individual firm's policies and is thus determined as an equilibrium outcome. In equilibrium, the matching of unemployed workers and firms is imperfect which results in both equilibrium unemployment and rents. These rents are shared between each firm and its workforce according to a Nash bargaining wage rate.

Our model is driven by two aggregate shocks, both of which are priced: a productivity shock and a shock to the efficiency of the matching technology, which was first studied by [Andolfatto \(1996\)](#). The literature has shown that variation in matching efficiency can arise for many reasons, and we are agnostic about the exact source. For example, [Pissarides \(2011\)](#) emphasizes that matching efficiency captures the mismatch between the skill requirements of jobs and the skill mix of the unemployed, the differences in geographical location between jobs and unemployed, and the institutional structure of an economy with regard to the transmission of information about jobs.

Aggregate productivity and matching efficiency are not directly observable in the data. To quantitatively compare the model with the data, we map the aggregate productivity and matching efficiency shocks into the market return and labor market tightness, which are observable in the data. As a result, we show that expected excess returns obey a two-factor structure in the market return and labor market tightness. We call the resulting model the *Labor Capital Asset Pricing Model*. Importantly, a one-factor CAPM does not span all risks and thus implies mispricing, in line with the data.

Our model replicates the negative relation between loadings on labor market tightness and expected returns. Intuitively, firm policies are driven by opposing cash flow and discount rate effects. On the one hand, positive shocks to matching efficiency lower marginal hiring costs. This cash flow channel implies an increase in optimal vacancies postings. On the other hand, positive shocks to matching efficiency are associated with an increase in discount rates. This assumption is consistent with the general equilibrium view that positive efficiency shocks lead to lower consumption as firms incur higher total hiring costs. This discount rate channel implies a reduction in the present value of job creation, and hence a decrease in optimal vacancy postings. As an equilibrium outcome of the labor market, the cash flow channel dominates the discount rate effect at the aggregate level. Thus, labor market tightness is positively related to matching efficiency shocks, so that loadings on labor market tightness are positively related to return exposures to matching efficiency shocks.

The cross-sectional differences in returns arise from frictions and heterogeneity in idiosyncratic productivity. Due to proportional hiring and firing costs, optimal firm policies exhibit regions of inactivity, where firms neither hire nor fire workers. Some firms are hit by low idiosyncratic productivity shocks so that hiring is not optimal when matching efficiency is high.

For these firms, the discount rate channel dominates the cash flow channel, thereby depressing valuations. Their dividends are reduced not only by low idiosyncratic productivity shocks but also by higher wages, arising from tighter labor markets, and by firing costs. Consequently, these firms have countercyclical dividends and valuations with respect to matching efficiency shocks, which renders them more risky. Since labor market tightness loadings and loadings on matching efficiency are positively related, our model can replicate the negative relation between labor market tightness loadings and expected returns.

This paper contributes to the macroeconomic literature by building on the canonical search and matching model of [Mortensen and Pissarides \(1994\)](#). The importance of labor market dynamics for the business cycle has long been recognized, e.g., [Merz \(1995\)](#) and [Andolfatto \(1996\)](#). While the standard model assumes a representative firm, firm heterogeneity has been considered by [Cooper, Haltiwanger, and Willis \(2007\)](#), [Mortensen \(2010\)](#), [Elsby and Michaels \(2013\)](#), and [Fujita and Nakajima \(2013\)](#). These papers have similar model features to ours but do not study asset prices.

Our paper also adds to the production-based asset pricing literature pioneered by [Cochrane \(1991\)](#) and [Jermann \(1998\)](#). Starting with [Berk, Green, and Naik \(1999\)](#), a large literature studies cross-sectional asset pricing implications of firm-level real investment decisions (e.g., [Gomes, Kogan, and Zhang \(2003\)](#), [Carlson, Fisher, and Giammarino \(2004\)](#), [Zhang \(2005\)](#), and [Cooper \(2006\)](#)). More closely related are [Papanikolaou \(2011\)](#) and [Kogan and Papanikolaou \(2014, 2013\)](#) who highlight that investment-specific shocks are related to firm-level risk premia. We differ by studying frictions in the labor market and specifically shocks to the efficiency of the matching technology.

The impact of labor market frictions on the aggregate stock market has been analyzed by [Danthine and Donaldson \(2002\)](#), [Merz and Yashiv \(2007\)](#), [Lochstoer and Bhamra \(2009\)](#), and [Kuehn, Petrosky-Nadeau, and Zhang \(2012\)](#).² A related line of literature links cross-sectional asset prices to labor-related firm characteristics. [Gourio \(2007\)](#), [Chen, Kacperczyk, and Ortiz-Molina \(2011\)](#), and [Favilukis and Lin \(2012\)](#) consider labor operating leverage arising from rigid wages; [Donangelo \(2014\)](#) focuses on labor mobility; [Palacios \(2013\)](#) studies labor intensity as measured by the ratio of wages to revenue; [Ochoa \(2013\)](#) investigates the risk implications of skilled labor; and [Eisfeldt and Papanikolaou \(2013\)](#) study organizational capital embedded in specialized labor input. We differ by exploring the impact of search costs on cross-sectional asset prices.

Closest to our paper is [Belo, Lin, and Bazdresch \(2014\)](#), who also emphasize that firms' hiring policies affect cross-sectional risk premia. They find that hiring growth rates predict returns in the data and explain this finding with a neoclassical Q-theory model with labor and capital adjustment costs. In contrast, we base our analysis on conditional risk loadings rather than firm-

²Whereas we consider labor market frictions, human capital risk is studied by [Jagannathan and Wang \(1996\)](#), [Berk and Walden \(2013\)](#), and [Eiling \(2013\)](#).

level characteristics, and emphasize the risk implications arising in a partial-equilibrium labor search model. Recruiting workers in congested labor market is costly and firms' sensitivity to the tightness of the labor markets affects their valuation.

2 Empirical Results

In this section, we document a robust negative relation between stock return loadings on changes in labor market tightness and future equity returns. We establish this result by studying portfolios sorted by loadings on labor market tightness and confirm it using Fama-MacBeth (1973) regressions. We also show that these loadings forecast industry returns.

2.1 Data

Our sample includes all common stocks (share code of 10 or 11) listed on NYSE, AMEX, and Nasdaq (exchange code of 1, 2, or 3) available from CRSP. Availability of labor market data restricts our analysis to the 1951 to 2012 period. Fama-MacBeth regressions additionally require Compustat data on book equity and other firm-level attributes. Consequently, the analysis based on those data is conducted for the 1960 to 2012 sample. In Appendix A, we list the exact formulas for firm characteristics used in our tests.

2.2 Labor Market Tightness

We obtain the monthly labor force participation and unemployment rates from the Current Population Survey of the Bureau of Labor Statistics for the years 1951 to 2012. The traditionally used measure of vacancies has been the Conference Board's Help Wanted Index, which was based on advertisements in 51 major newspapers. In 2005, Conference Board replaced it with Help Wanted Online, recognizing the importance of online marketing. We follow Barnichon (2010), who combines the print and online data to create a composite vacancy index starting in 1995.³

We define labor market tightness as the ratio of aggregate vacancy postings to unemployed workers. The pool of unemployed workers is the product of the unemployment rate and the labor force participation rate (LFPR). Hence, labor market tightness is given by

$$\theta_t = \frac{\text{Vacancy Index}_t}{\text{Unemployment Rate}_t \times \text{LFPR}_t}. \quad (3.1)$$

Figure 1 plots the monthly time series of θ_t and its components. Labor market tightness is strongly procyclical and persistent as in Shimer (2005). The cyclical nature of θ_t is driven by the pro-cyclicality of vacancies, its numerator, and the counter-cyclicality of the number of unemployed workers, its denominator.

³The data are available on his website, <http://sites.google.com/site/regisbarnichon/>.

We define the labor market tightness factor in month t as the change in logs of the vacancy-unemployment ratio θ_t :

$$\vartheta_t = \log(\theta_t) - \log(\theta_{t-1}). \quad (3.2)$$

Table 1 reports the time series properties of ϑ_t , its components, and other macro variables. We consider changes in the Industrial Production Index (IP) from the Board of Governors, changes in the Consumer Price Index (CPI) from the Bureau of Labor Statistics, the dividend yield of the S&P 500 Index (DY) as computed by Fama and French (1988), the term spread (TS) between 10-year and 3-month Treasury constant maturity yields, and the default spread (DS) between Moody’s Baa and Aaa corporate bond yields.

The labor market tightness factor is more volatile than any of the considered variables. As expected, it is strongly correlated with its components. The factor is also highly correlated with the default spread and changes in industrial production, which motivates us to conduct robustness tests (described below) to confirm that our empirical results are driven by changes in labor market tightness rather than by these other variables.

To study the relation between stock return sensitivity to changes in labor market tightness and future equity returns, we estimate loadings for each stock from a two-factor model based on the market excess return, $R_{M,t}^e$, and labor market tightness, ϑ_t . At the end of each month τ , we run rolling regressions of the form

$$R_{i,t}^e = \alpha_{i,\tau} + \beta_{i,\tau}^M R_{M,t}^e + \beta_{i,\tau}^\theta \vartheta_t + \varepsilon_{i,t}, \quad (3.3)$$

where $R_{i,t}^e$ denotes the excess return on stock i in month $t \in \{\tau - 35, \tau\}$. To obtain meaningful risk loadings at the end of month τ , we require each stock to have non-missing returns in at least 24 of the last 36 months.

2.3 Portfolio Sorts

At the end of each month τ , we rank stocks into deciles by loadings on labor market tightness $\beta_{i,\tau}^\theta$, computed from regressions (3.3). We skip a month to allow information on the vacancy and unemployment rates to become publicly available and hold the resulting ten value-weighted portfolios without rebalancing for one year ($\tau + 2$ through $\tau + 13$, inclusive). Consequently, in month τ each decile portfolio contains stocks that were added to that decile at the end of months $\tau - 13$ through $\tau - 2$. This design is similar to the approach used to construct momentum portfolios and reduces noise due to seasonalities. We show robustness to alternative portfolio formation methods in the next section.

Table 2 presents average firm characteristics of the resulting decile portfolios. Average loadings on labor market tightness (β^θ) range from -0.80 for the bottom decile to 0.91 for the top decile. Firms in the high and low groups are on average smaller with higher market betas than firms in the other deciles, as is often the case when firms are sorted on estimated

loadings. No strong relation emerges between loadings on labor market tightness and any of the other considered characteristics: book-to-market ratios (BM), stock return run-ups (RU), asset growth rates (AG), investment rates (IR), and hiring rates (HN). The lack of a relation between loadings on labor market tightness and hiring rates is of particular interest, as it provides the first evidence that our empirical results are distinct from those of [Belo, Lin, and Bazdresch \(2014\)](#).

For each decile portfolio, we obtain monthly time series of returns from January 1954 until December 2012. Table 3 summarizes returns, alphas, and betas of each decile and of the portfolio that is long the decile with low loadings and short the decile with high loadings on labor market tightness. To control for differences in risk across deciles, we present unconditional alphas from the CAPM, [Fama and French \(1993\)](#) 3-factor model, and [Carhart \(1997\)](#) 4-factor model. We account for possible time variation in betas and risk premiums by calculating conditional alphas following either [Ferson and Schadt \(1996\)](#) (FS) or [Boguth, Carlson, Fisher, and Simutin \(2011\)](#) (BCFS).⁴ The last four columns of the table show market (MKT), value (HML), size (SMB), and momentum (UMD) betas of each decile. Firms in the high decile have somewhat larger size betas and lower momentum loadings.

Both raw and risk-adjusted returns of the ten portfolios indicate a strong negative relation between loadings on the labor market tightness factor and future stock performance. Firms in the low β^θ decile earn the highest average return, 1.12% monthly, whereas the high β^θ decile performs most poorly, generating on average just 0.65% return per month. The difference in performance of the two deciles, at 0.47%, is economically large and statistically significant (t -statistic of 3.41). The corresponding differences in both unconditional and conditional alphas are similarly striking, ranging from 0.41% (t -statistic of 2.99) for Carhart 4-factor alphas to 0.52% (t -statistic of 3.83) for Fama-French 3-factor alphas. Conditional alphas are similar in magnitude to unconditional ones, suggesting negligible time variation in betas.

Results of portfolio sorts thus strongly suggest that loadings on labor market tightness are an important predictor of future returns. To evaluate robustness of this relation over time, we plot the cumulative returns (Panel A) and monthly returns (Panel B) of the long-short β^θ portfolio in Figure 2. The cumulative return steadily increases throughout the sample period, indicating that the relation between loadings on labor market tightness and future stock returns persists over time. Table 4 presents summary statistics for returns on this portfolio and for market, value, size, and momentum factors. The long-short labor market tightness portfolio is

⁴More specifically, we calculate conditional alphas as intercepts from regression

$$R_{j,t}^e = \alpha_j + \beta_j \begin{bmatrix} 1 & Z_{t-1} \end{bmatrix}' R_{M,t}^e + e_{j,\tau}, \quad (3.4)$$

where j indexes portfolios, t indexes months, β_j is a $1 \times (k+1)$ parameter vector, and Z_{t-1} is a $1 \times k$ instrument vector. Ferson and Schadt (1996) conditional alpha is computed using as instruments demeaned dividend yield, term spread, T-bill rate, and default spread. Boguth, Carlson, Fisher, and Simutin (2011) conditional alpha is computed by additionally including as instruments lagged 6- and 36-month market returns and average lagged 6- and 36-month betas of the portfolios.

as volatile as the market and momentum factors and achieves a Sharpe ratio (0.13) comparable to that of the market and the value factors.

We emphasize that although the difference in returns of firms with low and high loadings on labor market tightness cannot be explained by the commonly considered factor models, this difference should not be interpreted as mispricing. It arises rationally in our theoretical framework as compensation for risk associated with labor market frictions. The commonly used factor models such as the CAPM do not capture this type of risk. Consequently, alphas from such models are different for firms with different loadings on labor market tightness.

2.4 Robustness of Portfolio Sorts

We now demonstrate robustness of the relation between stock return loadings on changes in labor market tightness and future equity returns. We use alternative timings of portfolio formation, exclude micro cap stocks, consider modified definitions of the labor market tightness factor, and change regression (3.3) to also include size, value, and momentum factors. Table 5 summarizes the results of the robustness tests.

The portfolio formation design employed in the previous section is motivated by investment strategies such as momentum. It involves holding 12 overlapping portfolios and reduces noise due to seasonalities. We consider two alternatives: forming portfolios only once a year (Panel A) and holding the portfolios for one month (Panel B). Both alternatives ensure that no portfolios overlap. Panels A and B of Table 5 show that each of these approaches results in even more dramatic differences in future performance of low and high β^θ deciles. For example, the difference in average returns of the low and high deciles reaches 0.55% monthly when portfolios are formed once a year, compared to 0.47% reported in Table 3.

We next explore the sensitivity of the results to the length of time between calculating β^θ and forming portfolios. Our base case results in Table 3 are obtained by assuming that all variables needed to compute labor market tightness (vacancy index, unemployment rate, and labor force participation rate) are publicly available within a month. The assumption is well-justified in current markets, where the data for any month are typically available within days after the end of that month. To allow for a slower dissemination of data in the earlier sample, we consider a two-month waiting period. Panel C of Table 5 shows that the results are not sensitive to this change in methodology. The difference in future returns of stocks with low and high loadings on labor market tightness reaches 0.47% per month.

To account for the possibility that the negative relation between stock return loadings on changes in labor market tightness and future equity returns is driven by stocks with extreme loadings, we confirm robustness to sorting firms into quintile rather than decile portfolios. Panel D of Table 5 shows that the difference in future returns of quintiles with low and high loadings is economically and statistically significant.

In Panel E of Table 5 we evaluate robustness to excluding microcaps, which we define as

stocks with market equity below the 20th NYSE percentile. Microcaps on average represent just 3% of the total market capitalization of all stocks listed on NYSE, Amex, and Nasdaq, but they account for approximately 60% of the total number of stocks. Excluding these stocks from the sample does not meaningfully impact the results.⁵

We also evaluate robustness to two alternative definitions of the labor market tightness factor. Table 1 shows that ϑ_t as defined in equation (3.2) is correlated with changes in industrial production and other macro variables. To ensure that the relation between stock return loadings on the labor market tightness factor and future equity returns is not driven by these variables, our first alternative specification involves re-defining the labor market tightness factor as the residual $\tilde{\vartheta}_t$ from a time-series regression

$$\vartheta_t = \gamma_0 + \gamma_1 IP_t + \gamma_2 CPI_t + \gamma_3 DY_t + \gamma_4 TB_t + \gamma_5 TS_t + \gamma_6 DS_t + \tilde{\vartheta}_t, \quad (3.5)$$

where IP_t , CPI_t , DY_t , TB_t , TS_t , and DS_t are changes in industrial production, changes in the consumer price index, the dividend yield, the T-bill rate, the term spread, and the default spread, respectively. For our second alternative definition, we compute the labor market tightness factor as the residual from an ARMA(1,1) specification.

The disadvantage of both of these approaches is that they introduce a look-ahead bias as the entire sample is used to estimate the labor market tightness factor. Yet, the first alternative definition allows us to focus on the component of labor market tightness that is unrelated to macro variables, which may have non-zero prices of risk. The second definition allows us to focus on the unpredictable component of labor market tightness. Panels F and G of Table 5 show that our results are little affected by the changes in the definition of the labor market tightness factor. The difference in future raw and risk-adjusted returns of portfolios with low and high loadings on the factor are always statistically significant and economically important, ranging between 0.41% and 0.51% monthly.

In Table 3, we compute alphas from multi-factor models to ensure that the relation between loadings on labor market tightness and future equity returns is not driven by differences in loadings on known risk factors. For robustness, we also consider modifying regression (3.3) to include size, value and momentum factors. Panel H of Table 5 shows that our results are not sensitive to this alternative method for estimating β^θ .

We provide additional robustness tests in the Internet Appendix. In Tables IA.I and IA.II, we control for the liquidity and profitability factors, and summarize post-ranking β^θ loadings of the decile portfolios. We also evaluate the relation between loadings on labor market tightness and future equity returns conditional on stocks' market betas β^M . Table IA.III shows that, irrespective of whether we consider independent or dependent sorts, stocks with low loadings on labor market tightness significantly outperform stocks with high loadings.

⁵Untabulated results also confirm robustness to imposing a minimum price filter and to excluding Nasdaq-listed stocks.

2.5 Fama-MacBeth Regressions

The empirical evidence from portfolio sorts provides a strong indication of a negative relation between stock return loadings on changes in labor market tightness and subsequent equity returns. However, such univariate analysis does not account for other firm-level characteristics that have been shown to relate to future returns. We compare the loadings on the labor market tightness factor to other well-established determinants of the cross-section of stock returns. Our goal is to evaluate whether the ability of β^θ to forecast returns is subsumed by other firm-level characteristics. To this end, we run annual Fama-MacBeth (1973) regressions

$$R_{i,T+1}^e = \gamma_T^0 + \gamma_T^1 \beta_{i,\tau}^\theta + \sum_{j=1}^K \gamma_T^j X_{i,T}^j + \eta_{i,T}, \quad (3.6)$$

where $R_{i,T+1}^e$ is stock i excess return from July of year T to June of year $T+1$, $\beta_{i,\tau}^\theta$ is the loading from regressions (3.3) with τ corresponding to May of year T , and $X_{i,T}$ are K control variables all measured prior to the end of June of year T . The timing of the variables' measurements in the regression follows the widely accepted convention of [Fama and French \(1992\)](#).

We include in the Fama-MacBeth regressions commonly considered control variables such as the log of a firm's market capitalization (ME), the log of the book-to-market ratio (BM), and the return run-up (RU) ([Fama and French \(1992\)](#) and [Jegadeesh and Titman \(1993\)](#)). We also consider other recently documented determinants of the cross-section of stock returns, including the investment rate (IK) of [Titman, Wei, and Xie \(2004\)](#), asset growth rate (AG) of [Cooper, Gulen, and Schill \(2008\)](#), and the labor hiring rate (HN) of [Belo, Lin, and Bazdresch \(2014\)](#) and [Titman, Wei, and Xie \(2004\)](#). We winsorize all independent variables cross-sectionally at 1% and 99%.

Table 6 summarizes the results of the Fama-MacBeth regressions. The coefficient on β^θ is negative and statistically significant in each considered specification, even after accounting for other predictors of the cross-section of equity returns. The magnitude of the coefficient implies that for a one standard deviation increase in β^θ (0.49), subsequent annual returns decline by approximately 1.5%. Average loadings of firms in the bottom and top decile portfolios are 3.5 standard deviations apart, suggesting that the difference in future stock returns of the two groups exceeds 5% per year, in line with the results presented in Table 3.

Changes in labor market tightness are highly correlated with its components and with changes in industrial production (see Table 1). To ensure that our results are not driven by these macro variables, we estimate loadings from a two-factor regression of stock excess returns on market excess returns and log changes in either labor force participation rate, unemployment rate, vacancy index, or industrial production. Tables IA.IV and IA.V of the Internet Appendix show that none of the considered loadings are robustly related to future equity returns, suggesting that the relation between loadings on the labor market tightness factor and future stock returns is not driven by one particular component of the labor market tightness or by changes

in industrial production.

2.6 Industry-Level Analysis

The ability of commonly considered firm characteristics to predict stock returns is known to be stronger when these characteristics are computed relative to industry averages. In other words, many determinants of the cross-section of stock returns are priced within rather than across industries (e.g., [Cohen and Polk \(1998\)](#), [Asness, Burt, Ross, and Stevens \(2000\)](#), [Simutin \(2010\)](#), [Novy-Marx \(2011\)](#), and [Eisfeldt and Papanikolaou \(2013\)](#)). We now show that unlike many other cross-sectional predictors of stock returns, β^θ contains more information about future returns when considered *across* rather than *within* industries. Our goal in this section is to understand how much of the negative relation between β^θ and future stock returns is due to industry-specific versus firm-specific (non-industry) components.

We begin our analysis by modifying the portfolio assignment methodology used above to ensure that all β^θ decile portfolios have similar industry characteristics. To achieve this, we sort firms into deciles within each of the 48 industries as defined in [Fama and French \(1997\)](#) and then aggregate firms across industries to obtain ten industry-neutral portfolios. Panel A of Table 7 shows that the differences in future performance of firms with low and high loadings on the labor market tightness factor are slightly muted relative to those in Table 3. For example, the return of the long-short β^θ portfolio reaches 0.37% monthly when portfolio assignment is done within industries, whereas the corresponding figure is 0.47% when industry composition is allowed to vary across deciles.

The larger difference in future performance of low and high β^θ stocks when we allow for industry heterogeneity across decile portfolios is particularly interesting given that many known premiums are largely intra-industry phenomena. This result suggests that the labor market tightness factor may be priced in the cross-section of industry portfolios. To investigate this conjecture, we assign 48 value-weighted industry portfolios into deciles on the basis of their loadings on the labor market tightness factor and study future returns of the resulting decile portfolios.⁶ Panel B of Table 7 shows that industries with low loadings outperform industries with high loadings by 0.34% return per month.

3 Model

The goal of this section is to provide an economic model that explains the empirical link between labor market frictions and the cross-section of equity returns. To this end, we solve a partial equilibrium labor market model and study its implications for stock returns. For tractability we do not model endogenous labor supply decisions from households; instead we assume an exogenous pricing kernel.

⁶Industry portfolios are from Ken French's data library. Table IA.VI of the Internet Appendix provides summary statistics for the industry portfolios.

3.1 Revenue

To focus on labor frictions, we abstract from capital accumulation and investment frictions and assume that the only input to production is labor. Firms generate revenue, $Y_{i,t}$, according to a decreasing returns to scale production function

$$Y_{i,t} = e^{x_t + z_{i,t}} N_{i,t}^\alpha, \quad (3.7)$$

where α denotes the labor share of production and $N_{i,t}$ is the size of the firm's workforce. Both the aggregate productivity shock x_t and the idiosyncratic productivity shocks $z_{i,t}$ follow AR(1) processes

$$x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_t^x, \quad (3.8)$$

$$z_{i,t} = \rho_z z_{i,t-1} + \sigma_z \varepsilon_{i,t}^z, \quad (3.9)$$

where $\varepsilon_t^x, \varepsilon_{i,t}^z$ are standard normal i.i.d. innovations. Firm-specific shocks are independent across firms, and from aggregate shocks.

The dynamics of firms' workforce are determined by optimal hiring and firing policies. Firms can expand the workforce by posting vacancies, $V_{i,t}$, to attract unemployed workers. The key friction of labor markets is that not all posted vacancies are filled in a given period. Instead, the rate q at which vacancies are filled is endogenously determined in equilibrium and depends on the tightness of the labor market, θ_t , and an exogenous efficiency shock, p_t , to the matching technology. Firms can also downsize by laying off $F_{i,t}$ workers. Before hiring and firing takes place, a constant fraction s of workers quit voluntarily. Taken together, this implies the following law of motion for the firm workforce size

$$N_{i,t+1} = (1 - s)N_{i,t} + q(\theta_t, p_t)V_{i,t} - F_{i,t}. \quad (3.10)$$

The matching efficiency shock p_t follows an AR(1) process with autocorrelation ρ_p and i.i.d. normal innovations ε_t^p :

$$p_t = \rho_p p_{t-1} + \sigma_p \varepsilon_t^p. \quad (3.11)$$

Matching efficiency innovations are uncorrelated with aggregate productivity innovations. The matching efficiency shock is common across firms and thus represents aggregate risk. This shock was first studied by [Andolfatto \(1996\)](#) who argues that it can be interpreted as a reallocation shock, distinct from disturbances that affect production technologies. In search models, the efficiency of the economy's allocative mechanism is captured by the technological properties of the aggregate matching function. Changes in this function can be thought of as reflecting mismatches in the labor market between the skills, geographical location, demography or other dimensions of unemployed workers and job openings across sectors, thereby causing a shift in the so-called aggregate Beveridge curve.

Several recent studies empirically analyze sources of changes in matching efficiency. Using micro-data [Barnichon and Figura \(2015\)](#) show that fluctuations in matching efficiency can be related to the composition of the unemployment pool, such as a rise in the share of long-term unemployed or fluctuations in participation due to demographic factors, and to dispersion in labor market conditions; [Herz and van Rens \(2015\)](#) and [Sahin, Song, Topa, and Violante \(2014\)](#) highlight the role of skill and occupational mismatch between jobs and workers; [Sterk \(2015\)](#) focuses on geographical mismatch exacerbated by house price movements; and [Fujita \(2011\)](#) analyzes the role of reduced worker search intensity due to extended unemployment benefits.

3.2 Matching

Labor market tightness affects how easily vacant positions can be filled. It is a function of aggregate vacancy postings and employment. The aggregate number of vacancies, $\bar{V}$, and aggregate employment, $\bar{N}$, are simply the sums of all firm-level vacancies and employment, respectively, that is,

$$\bar{V}_t = \int V_{i,t} d\mu_t \quad \bar{N}_t = \int N_{i,t} d\mu_t, \quad (3.12)$$

where μ_t denotes the time-varying distribution of firms over the firm-level state space $(z_{i,t}, N_{i,t})$.

The mass of firms is normalized to one. The labor force with mass L is defined as the sum of employed and unemployed. Hence, the unemployment rate is given by $(L - \bar{N})/L$. The mass of the labor force searching for a job includes workers who have just voluntarily quit, $sN_{i,t}$, and is given by

$$\bar{U}_t = L - (1 - s)\bar{N}_t. \quad (3.13)$$

Labor market tightness can now be defined as the ratio of aggregate vacancies to the mass of the labor force who are searching for a job, that is, $\theta_t = \bar{V}_t/\bar{U}_t$.

Following [den Haan, Ramey, and Watson \(2000\)](#), vacancies are filled according to a constant returns to scale matching function

$$\mathcal{M}(\bar{U}_t, \bar{V}_t, p_t) = \frac{e^{p_t} \bar{U}_t \bar{V}_t}{(\bar{U}_t^\xi + \bar{V}_t^\xi)^{1/\xi}}, \quad (3.14)$$

and the rate q at which vacancies are filled per unit of time can be computed from

$$q(\theta_t, p_t) = \frac{\mathcal{M}(\bar{U}_t, \bar{V}_t, p_t)}{\bar{V}_t} = e^{p_t} \left(1 + \theta_t^\xi\right)^{-1/\xi}. \quad (3.15)$$

The matching rate is decreasing in θ , meaning that an increase in the relative scarcity of unemployed workers relative to job vacancies makes it more difficult for firms to fill a vacancy. It is increasing in p , as a positive efficiency shock makes finding a worker easier.

3.3 Wages

In equilibrium, the matching of unemployed workers and firms is imperfect, which results in both equilibrium unemployment and rents. These rents are shared between each firm and its workforce according to a Nash bargaining wage rate. Following [Stole and Zwiebel \(1996\)](#), we assume Nash bargaining wages in multi-worker firms with decreasing returns to scale production technology. Specifically, firms renegotiate wages every period with its workforce based on individual (and not collective) Nash bargaining.

In the bargaining process, workers have bargaining weight $\eta \in (0, 1)$. If workers decide not to work, they receive unemployment benefits b , which represent the value of their outside option. They are also rewarded the saving of hiring costs that firms enjoy when a job position is filled, $\kappa_h \theta_t$, where κ_h is the unit cost of vacancy postings. As a result, wages are given by

$$w_{i,t} = \eta \left[\frac{\alpha}{1 - \eta(1 - \alpha)} \frac{Y_{i,t}}{N_{i,t}} + \kappa_h \theta_t \right] + (1 - \eta)b. \quad (3.16)$$

Firms benefit from hiring the marginal worker not only through an increase in output by the marginal product of labor but also through a decrease in wage payment to its current workers, $Y_{i,t}/N_{i,t}$. The term $\alpha/(1 - \eta(1 - \alpha))$ represents a reduction in wages coming from decreasing returns to scale. At the same time, workers can extract higher wages from firms when the labor market is tighter. Unemployment benefits provide a floor to wages.⁷

3.4 Firm Value

We do not model the supply side of labor coming from households. This would require to solve a full general equilibrium model. Instead, following [Berk, Green, and Naik \(1999\)](#), we specify an exogenous pricing kernel and assume that both the aggregate productivity shock x_t and efficiency shock p_t are priced. The log of the pricing kernel is given by

$$\ln M_{t+1} = \ln \beta - \gamma_x(\sigma_x \varepsilon_{t+1}^x + \phi x_t) - \gamma_{p,t}(\sigma_p \varepsilon_{t+1}^p + \phi p_t), \quad (3.17)$$

where β is the time discount rate, γ_x the constant price of risk of aggregate productivity shocks, $\gamma_{p,t} = \gamma_{p,0} e^{\gamma_{p,1} p_t}$ the time-varying price of risk of efficiency shocks, and ϕ measures the sensitivity of interest rates with respect to aggregate shocks.

The objective of firms is to maximize their value $S_{i,t}$ either by posting vacancies $V_{i,t}$ to hire workers or by firing $F_{i,t}$ workers to downsize. Both adjustments are costly at rate κ_h for hiring and κ_f for firing. Firms also pay fixed operating costs f . Dividends to shareholders are given by revenues net of operating, hiring, firing, and wages costs

$$D_{i,t} = Y_{i,t} - f - \kappa_h V_{i,t} - \kappa_f F_{i,t} - w_{i,t} N_{i,t}. \quad (3.18)$$

⁷The same wage process is used in [Elsby and Michaels \(2013\)](#) and [Fujita and Nakajima \(2013\)](#). See the first paper for a proof.

The firm's Bellman equation solves

$$S_{i,t} = \max_{V_{i,t} \geq 0, F_{i,t} \geq 0} \{D_{i,t} + \mathbb{E}_t[M_{t+1}S_{i,t+1}]\}, \quad (3.19)$$

subject to equations (7)–(18). Notice that the firms' problem is well-defined given labor market tightness θ_t and expectations about its dynamics. Given optimal cum-dividend firm value $S_{i,t}$, expected excess returns are given by

$$\mathbb{E}_t[R_{i,t+1}^e] = \frac{\mathbb{E}_t[S_{i,t+1}]}{S_{i,t} - D_{i,t}} - \frac{1}{\mathbb{E}_t[M_{t+1}]}. \quad (3.20)$$

3.5 Equilibrium

In search and matching models, optimal firm employment policies depend on the dynamics of the aggregate labor market. This is typically not the case for models with labor adjustment costs based on the Q-theory. Rather, in our setup firms have to know how congested labor markets are when they decide about optimal hiring policies as next period's workforce, Equation (3.10), depends on aggregate labor market tightness θ via the vacancy filling rate q . At the same time, labor market tightness depends on the distribution of vacancy postings implied by the firm-level distribution μ_t and the aggregate shocks.

Equilibrium in the labor market requires that the beliefs about labor market tightness are consistent with the realized equilibrium. Consequently, the firm-level distribution enters the state space, which is given by $\Omega_{i,t} = (N_{i,t}, z_{i,t}, x_t, p_t, \mu_t)$, and labor market tightness θ_t at each date is determined as a fixed point satisfying

$$\theta_t = \frac{\int V(\Omega_{i,t}) d\mu_t}{\bar{U}_t}. \quad (3.21)$$

This assumes that each individual firm is atomistic and takes labor market tightness as exogenous.

Let Γ be the law of motion for the time-varying firm-level distribution μ_t such that

$$\mu_{t+1} = \Gamma(\mu_t, x_{t+1}, x_t, p_{t+1}, p_t). \quad (3.22)$$

The recursive competitive equilibrium is characterized by: (i) labor market tightness θ_t , (ii) optimal firm policies $V(\Omega_{i,t})$, $F(\Omega_{i,t})$, and firm value function $S(\Omega_{i,t})$, (iii) a law of motion Γ of the firm-level distribution μ_t , such that: (a) Optimality: Given the pricing kernel (3.17), Nash bargaining wage rate (3.16), and labor market tightness θ_t , $V(\Omega_{i,t})$ and $F(\Omega_{i,t})$ solve the firm's Bellman equation (3.19) where $S(\Omega_{i,t})$ is its solution; (b) Consistency: θ_t is consistent with the labor market equilibrium (3.21), and the law of motion Γ of the firm-level distribution μ_t is consistent with the optimal firm policies $V(\Omega_{i,t})$ and $F(\Omega_{i,t})$.

3.6 Approximate Aggregation

The firm's hiring and firing decisions trade off current costs and future benefits, which depend on the aggregation and evolution of the firm-level distribution μ_t . Rather than solving for the high dimensional firm-level distribution exactly, we follow [Krusell and Smith \(1998\)](#) and approximate it with one moment. In search models, labor market tightness θ_t is a sufficient statistic to solve the firm's problem (3.19) and thus enters the state vector replacing μ_t , i.e., the approximate state space is $\tilde{\Omega}_{i,t} = (N_{i,t}, z_{i,t}, x_t, p_t, \theta_t)$.

To approximate the law of motion Γ , Equation (3.22), we assume a log-linear functional form

$$\log \theta_{t+1} = \tau_0 + \tau_\theta \log \theta_t + \tau_x \sigma_x \varepsilon_{t+1}^x + \tau_p \sigma_p \varepsilon_{t+1}^p. \quad (3.23)$$

Under rational expectations, the perceived labor market outcome equals the realized one at each date of the recursive competitive equilibrium. In equilibrium, we can express the labor market tightness factor ϑ as the log changes in labor market tightness

$$\vartheta_{t+1} = \tau_0 + (\tau_\theta - 1) \log \theta_t + \tau_x \sigma_x \varepsilon_{t+1}^x + \tau_p \sigma_p \varepsilon_{t+1}^p. \quad (3.24)$$

This definition is consistent with our empirical exercise in Section 2.

Our application of [Krusell and Smith \(1998\)](#) differs from [Zhang \(2005\)](#) along two dimensions. First, future labor market tightness θ_{t+1} is a function of the firm distribution at time $t+1$; hence, it is not in the information set of date t . The forecasting rule (3.23) at time t does not enable firms to learn θ_{t+1} perfectly, but rather to form a rational expectation about θ_{t+1} . In contrast, Zhang (2005) assumes that firms can perfectly forecast next period's industry price given time t information. If firms could perfectly forecast next period's labor market tightness, it would not carry a risk premium. Second, at each period of the simulation, we impose labor market equilibrium by solving θ_t as the fixed point in Equation (3.21). Hence, there is no discrepancy between the forecasted and the realized θ_{t+1} .

3.7 Equilibrium Risk Premia

The model is driven by two aggregate shocks: productivity and matching efficiency. To test the model's cross-sectional return implications on data, it is convenient to derive an approximate log-linear pricing model. Based on the Euler equation for expected excess returns, we can apply a log-linear approximation to the pricing kernel (3.17) implying

$$\mathbb{E}_t[R_{i,t+1}^e] \approx \beta_{i,t}^x \lambda^x + \beta_{i,t}^p \lambda_t^p, \quad (3.25)$$

where $\beta_{i,t}^x$ and $\beta_{i,t}^p$ are loadings on aggregate productivity and matching efficiency shocks and λ^x and λ_t^p are their respective factor risk premia. All proofs of this section are contained in Appendix B.

Both aggregate productivity and matching efficiency are not directly observable in the data.

Since we would like to take the model to the data, it is necessary to express expected excess returns in terms of observable variables such as the return on the market and labor market tightness. To this end, we also approximate the excess return on the market as an affine function of the aggregate shocks

$$R_{M,t+1}^e = \nu_0 + \nu_\theta \log \theta_t + \nu_{x,0} x_t + \nu_{p,0} p_t + \nu_{x,1} x_{t+1} + \nu_{p,1} p_{t+1}. \quad (3.26)$$

As a result, we can show that expected excess returns obey a two-factor structure in the market excess return and log-changes in labor market tightness, which is summarized in the following proposition.

Proposition 1 *Given a log-linear approximation of the pricing kernel (3.17) and laws of motion (3.24) and (3.26), the log pricing kernel satisfies*

$$m_{t+1} = -\gamma_{M,t} R_{M,t+1}^e - \gamma_{\theta,t} \vartheta_{t+1}, \quad (3.27)$$

where the prices of market risk $\gamma_{M,t}$ and labor market tightness $\gamma_{\theta,t}$ are given by

$$\gamma_{M,t} = \frac{\tau_p \gamma_x - \tau_x \gamma_{p,t}}{\tau_p \nu_x - \tau_x \nu_p} \quad \gamma_{\theta,t} = \frac{\nu_x \gamma_{p,t} - \nu_p \gamma_x}{\tau_p \nu_x - \tau_x \nu_p}. \quad (3.28)$$

The pricing kernel (3.27) implies a linear pricing model in the form of

$$\mathbb{E}_t[R_{i,t+1}^e] = \beta_{i,t}^M \lambda_t^M + \beta_{i,t}^\theta \lambda_t^\theta, \quad (3.29)$$

where $\beta_{i,t}^M$ and $\beta_{i,t}^\theta$ are the loadings on the market excess return and log-changes in labor market tightness

$$\beta_{i,t}^M = \frac{\tau_p}{\tau_p \nu_x - \tau_x \nu_p} \beta_{i,t}^x + \frac{-\tau_x}{\tau_p \nu_x - \tau_x \nu_p} \beta_{i,t}^p \quad (3.30)$$

$$\beta_{i,t}^\theta = \frac{-\nu_p}{\tau_p \nu_x - \tau_x \nu_p} \beta_{i,t}^x + \frac{\nu_x}{\tau_p \nu_x - \tau_x \nu_p} \beta_{i,t}^p \quad (3.31)$$

and λ_t^M and λ_t^θ are the respective factor risk premia given by

$$\lambda_t^M = \nu_x \lambda^x + \nu_p \lambda_t^p \quad \lambda_t^\theta = \tau_x \lambda^x + \tau_p \lambda_t^p. \quad (3.32)$$

We call relation (3.29) the *Labor Capital Asset Pricing Model*.⁸ The goal of the model is to endogenously generate a negative factor risk premium of labor market tightness, λ_t^θ . We will explain the intuition behind Proposition 1 after the calibration in Section III.C.

In the data, the CAPM cannot explain the returns of portfolios sorted by loadings on labor market tightness, $\beta_{i,t}^\theta$. To replicate this failure of the CAPM in the model, we can compute a misspecified one-factor CAPM and compare the CAPM-implied alphas with the data. The

⁸Note that the risk loadings (3.30) and (3.31) are not univariate regression betas because the market return and labor market tightness are correlated.

following proposition summarizes this idea.

Proposition 2 *Given a log-linear approximation of the pricing kernel (3.17) and laws of motion (3.24) and (3.26), the CAPM implies a linear pricing model in the form of*

$$\mathbb{E}_t[R_{i,t+1}^e] = \alpha_{i,t}^{CAPM} + \beta_{i,t}^{CAPM} \lambda_t^{CAPM}, \quad (3.33)$$

where the CAPM mispricing alphas are given by

$$\alpha_{i,t}^{CAPM} = \beta_{i,t}^\theta \gamma_{\theta,t} \frac{(\tau_x \nu_p - \nu_x \tau_p)^2 \sigma_p^2 \sigma_x^2}{\nu_x^2 \sigma_x^2 + \nu_p^2 \sigma_p^2}, \quad (3.34)$$

CAPM loadings on the market return by

$$\beta_{i,t}^{CAPM} = \frac{\nu_x \sigma_x^2}{\nu_x^2 \sigma_x^2 + \nu_p^2 \sigma_p^2} \beta_x + \frac{\nu_p \sigma_p^2}{\nu_x^2 \sigma_x^2 + \nu_p^2 \sigma_p^2} \beta_p, \quad (3.35)$$

and the CAPM factor risk premium $\lambda_t^{CAPM} = \lambda_t^M = \nu_x \lambda^x + \nu_p \lambda_t^p$.

Intuitively, this proposition states that CAPM betas are independent of the price of risk of labor market tightness $\gamma_{\theta,t}$, whereas the CAPM mispricing alphas are inversely related to labor market tightness loadings when $\gamma_{\theta,t}$ is negative. These insights are qualitatively in line with the empirical findings above and are confirmed quantitatively next.

4 Quantitative Results

In this section, we first describe our calibration strategy and present the numerical results of the equilibrium forecasting rules. Given the equilibrium dynamics for the labor market, we then calculate loadings on labor market tightness and show that the model is consistent with the inverse relation between loadings and future stock returns in the cross-section. We solve the competitive equilibrium numerically in the discretized state space $\tilde{\Omega}_{i,t}$ using an iterative algorithm described in Appendix C.

4.1 Calibration

Table 8 summarizes the parameter calibration of the benchmark model. Labor and equity market data are available monthly and we choose this frequency for the calibration.

The labor literature provides several empirical studies to calibrate labor market parameters. Following [Elsby and Michaels \(2013\)](#) and [Fujita and Nakajima \(2013\)](#), we scale the size of labor force L to match the average unemployment rate. The elasticity of the matching function determines the responsiveness of the vacancy filling rate to changes in labor market tightness. Based on the structural estimate in [den Haan, Ramey, and Watson \(2000\)](#), we set the elasticity ξ at 1.27.

The bargaining power of workers η determines the rigidity of wages over the business cycle. As emphasized by [Hagedorn and Manovskii \(2008\)](#) and [Gertler and Trigari \(2009\)](#), aggregate wages are half as volatile as labor productivity. We follow their calibration strategy and set $\eta = 0.125$ to match the relative volatility of wages to output.⁹ It is important to highlight that our model is not driven by sticky wages as proposed by [Hall \(2005\)](#) and [Gertler and Trigari \(2009\)](#). In our model, wages are less volatile than productivity but, conditional on productivity, they are not sticky. This is consistent with [Pissarides \(2009\)](#), who argues that Nash bargaining wage rates are in line with wages for new hires.

If workers decide not to work, they receive the flow value of unemployment activities b . [Shimer \(2005\)](#) argues that the outside option for rejecting a job offer are unemployment benefits and thus sets $b = 0.4$. [Hagedorn and Manovskii \(2008\)](#), on the other hand, claim that unemployment activities capture not only unemployment benefits but also utility from home production and leisure. They calibrate b close to one. As in the calibration of [Pissarides \(2009\)](#), we follow [Hall and Milgrom \(2008\)](#) and set the value of unemployment activities at 0.71.¹⁰

The labor share of income, which [Gomme and Rupert \(2007\)](#) estimate to be around 0.72, is highly affected by the value of unemployment activities b and the output elasticity of labor α . Since the value of unemployment activities is close to the labor share of income, we can easily match the labor share by setting α to 0.735. We assume less curvature in the production function than, for instance, [Cooper, Haltiwanger, and Willis \(2007\)](#). They, however, do not model wages as the outcome of Nash bargaining.

Motivated by [Davis, Faberman, and Haltiwanger \(2006\)](#), we use the flows in the labor market as measured in the Job Openings and Labor Turnover Survey (JOLTS) collected by the Bureau of Labor Statistics to calibrate the monthly separation rate s as well as the proportional hiring κ_h and firing κ_f costs. JOLTS provides monthly data on the rates of hires, separations, quits, and layoffs.

The total separation rate captures both voluntary quits and involuntary layoffs. As firms in our model can optimize over the number of worker to be laid off, we calibrate the separation rate only to the voluntary quit rate, which captures workers switching jobs, for instance, for reasons of career development, better pay or preferable working conditions. As such, we set the monthly exogenous quit rate s to 2.2%.

The proportional costs of hiring and firing workers, κ_h and κ_f , determine both the overall costs of adjusting the workforce as well as the behavior of firm policies. Since the literature provides little guidance on estimates of hiring costs, we set κ_h to 0.75 to match the aggregate hiring rate of workers, defined as the ratio of aggregate filled vacancies to employed labor force, $q_t \bar{V}_t / \bar{N}_t$. As hiring costs increase, firms post fewer vacancies so that the hiring rate rises. Our

⁹[Hagedorn and Manovskii \(2008\)](#) set the bargaining power of workers at 0.054 and [Lubik \(2009\)](#) estimates it to be 0.03.

¹⁰Similarly, [Lubik \(2009\)](#) estimates that unemployment activities amount to 0.74 relative to unit mean labor productivity.

parameter choice is close to [Hall and Milgrom \(2008\)](#), who account for both the capital costs of vacancy creation and the opportunity cost of labor effort devoted to hiring activities.

Employment protection legislatures are a set of rules and restrictions governing the dismissals of employees. Such provisions impose a firing cost on firms along two dimensions: a transfer from the firm to the worker to be laid off (e.g., severance payments), and a tax to be paid outside the job-worker pair (e.g., legal expenses). As the labor search literature does not provide guidance on the magnitude of this parameter, we set the flow costs of firing workers κ_f to 0.35 to match the aggregate layoff rate, defined as the ratio of total laid off workers to employed labor force, $\bar{F}_t/\bar{N}_t$. As firing costs increase, firms lay off fewer workers so that the firing rate drops.

The last cost parameter is fixed operating costs f . Without these costs, the model would overstate the net profit margin of firms. Consequently, we target the aggregate profit to aggregate output ratio to calibrate f .

We calibrate the two aggregate shocks following the macroeconomics literature. Since labor is the only input to production, aggregate productivity is typically measured as aggregate output relative to the labor hours used in the production of that output. As such, labor productivity is more volatile than total factor productivity. Similar to [Gertler and Trigari \(2009\)](#), we set $\rho_x = 0.95^{1/3}$ and $\sigma_x = 0.005$. Shocks to the matching efficiency tend to be less persistent but more volatile than labor productivity shocks. For instance, [Andolfatto \(1996\)](#) estimates matching shocks to have persistence of 0.85 with innovation volatility of 0.07 at quarterly frequency. We follow more recent estimates by [Cheremukhin and Restrepo-Echavarria \(2014\)](#) and set $\rho_p = 0.88^{1/3}$ and $\sigma_p = 0.025$.¹¹

For the persistence ρ_z and conditional volatility σ_z of firm-specific productivity, we choose values close to those used by [Zhang \(2005\)](#), [Gomes and Schmid \(2010\)](#), and [Fujita and Nakajima \(2013\)](#) to match the cross-sectional properties of firm employment policies.

The pricing kernel is calibrated to match financial moments. We choose the time discount rate β and the pricing kernel parameters γ_x , $\gamma_{p,0}$, $\gamma_{p,1}$, and ϕ so that the model approximately matches the first and second moments of the risk-free rate and market return. This requires that β equals 0.994, $\gamma_x = 1$, $\gamma_{p,0} = -4.7$, $\gamma_{p,1} = 3.6$, and $\phi = -0.0214$. Importantly, shocks to matching efficiency carry a negative price of risk and are pro-cyclical. A small parameter value of ϕ allows for a time-varying but smooth interest rate.

[Berk, Green, and Naik \(1999\)](#) provide a motivation for $\gamma_x > 0$ in an economy with only aggregate productivity shocks. The assumption of $\gamma_p < 0$ can be motivated as follows. In a general equilibrium economy with a representative household, a positive matching efficiency shock increases the probability that vacant jobs are filled and thereby lowers the expected unit hiring cost. As a result, job creation becomes more attractive and firms spend more resources on hiring workers, thus depressing aggregate consumption.¹²

¹¹Similar structural estimates are contained in [Furlanetto and Groshenny \(2012\)](#) and [Beauchemin and Tasci \(2014\)](#).

¹²The same intuition is shown to hold in general equilibrium for investment-specific shocks by [Papanikolaou](#)

4.2 Aggregate and Firm-Level Moments

Table 9 summarizes aggregate and firm-level moments computed on simulated data of the model and compares them with the data. The model closely matches firm-level and aggregate employment quantities as well as financial market moments. In equilibrium, the unemployment rate is 5.8%, the aggregate hiring rate is 3.6%, and the layoff rate is 1.4% on average, close to what we observe in the JOLTS dataset for the years 2001 to 2012.

Davis, Faberman, and Haltiwanger (2006) illustrate that the net change in employment over time can be decomposed into either worker flows, defined as the difference between hires and separations, or job flows, defined as the difference between job creation and destruction. While a single firm can either create or destroy jobs during a period, it can simultaneously have positive hires and separations. Davis, Faberman, and Haltiwanger (2006) report that the monthly job creation and job destruction rates are 2.6% and 2.5%, respectively, which our model replicates closely.

The model also performs well in replicating the dynamics of aggregate labor market tightness. Shimer (2005) estimates average labor market tightness of 0.63, while the model implied one is 0.65. The cyclical behavior of model-generated time series for labor market tightness, aggregate vacancies and unemployment rate match well the correlations in monthly data (for the data see Table 1). Changes in labor market tightness correlate positively with changes in vacancies (0.78), and negatively with changes in the unemployment rate (-0.83). The negative relationship between changes in vacancies and unemployment rate (-0.36) is consistent with the well-known shape of the Beveridge curve.

Given our calibration strategy, the model matches well the high labor share of income (0.72), the low relative volatility of wages to output (0.55), and the small profit margin (0.11). The data for the labor share is from Gomme and Rupert (2007) and the volatility of aggregate wages to aggregate output is from Gertler and Trigari (2009). We compute the average share of corporate profits to national income using the National Income and Product Accounts as in Gourio (2007).

At the firm-level, we compute moments of annual employment growth rates as in Davis, Haltiwanger, Jarmin, and Miranda (2006) for the merged CRSP-Compustat sample for the period 1980 to 2012. The model generates the observed high volatility in annual employment growth, 23.9% in the model relative to 23.6% in the data. The proportional cost structure implies the existence of firms that are neither posting vacancies nor laying off workers. As emphasized by Cooper, Haltiwanger, and Willis (2007), we measure inaction as the fraction of firms with no change in employment, which is 9.7% for the merged CRPS-Compustat sample. In the model, this fraction is 9.9%, lending support for our modeling assumption of proportional costs.

To gauge the aggregate pricing implications, we obtain the monthly series of the value-weighted market return and one-month T-Bill rate from CRSP, and inflation from the Bureau (2011).

of Labor Statistics to compute the annualized first and second moments of the one-month real risk-free rate and real market return for the period 1926 to 2012. The pricing kernel and its calibration give rise to a realistic annual average market return (8.2%) and volatility (17.2%). In addition, the average risk-free rate is low (1%) and smooth (2.1%) as in the data.

4.3 Equilibrium Forecasting Rules

The goal of the model is to endogenously generate a negative relation between loadings on labor market tightness and expected returns, implying a negative factor risk premium for labor market tightness, λ_t^θ . Given that aggregate productivity shocks carry a positive and efficiency shocks a negative price of risk, $\gamma_x > 0$ and $\gamma_{p,0} < 0$, Proposition 1 (Equation (3.32)) states that for the model to generate a negative factor risk premium for labor market tightness, it is necessary that labor market tightness reacts positively to efficiency shocks, i.e., $\tau_p > 0$.

The dynamics of labor market tightness (3.23) are the equilibrium outcome of firm policies and the solution to the labor market equilibrium condition (3.21). In particular, the endogenous response of labor market tightness to efficiency shocks, τ_p , depends on two economic forces, namely, a cash flow and a discount rate effect, which work in opposite directions. To illustrate this trade-off, we compute the Euler equation for job creation, which is given by¹³

$$\frac{\kappa_h}{q(\theta_t, p_t)} = \mathbb{E}_t M_{t+1} \left[e^{x_{t+1} + z_{i,t+1}} \alpha N_{i,t+1}^{\alpha-1} - w_{i,t+1} - N_{i,t+1} \frac{\partial w_{i,t+1}}{\partial N_{i,t+1}} + (1-s) \frac{\kappa_h}{q(\theta_{t+1}, p_{t+1})} \right]. \quad (3.36)$$

The left-hand side is the marginal cost and the right-hand side the marginal benefit of job creation.

In Figure 3, we illustrate this trade-off by plotting labor market tightness as a function of matching efficiency. Consider a positive matching efficiency shock, which shifts p_0 to p_1 . A positive efficiency shock increases the rate at which vacancies are filled and thus reduces the marginal costs of hiring workers, i.e., the left-hand side of the Euler equation (3.36). This cash flow effect implies that firms are willing to post more vacancies after a positive efficiency shock. Consequently, the equilibrium moves along the solid black line and shifts from point A to B, resulting in a higher labor market tightness θ_1 . This effect causes a positive relation between labor market tightness and matching efficiency, i.e., $\tau_p > 0$.

The cash flow effect would be the only equilibrium effect in a setting in which agents are risk-neutral. Since we are interested in the pricing of labor market risks, we assume that efficiency shocks carry a negative price of risk. As a result, a positive efficiency shock leads to an increase in discount rates. This discount rate effect implies that firms reduce vacancy postings, as an increase in discount rates reduces the value of job creation, i.e., the right-hand side of the Euler equation (3.36). In Figure 3, the discount rate effect shifts the equilibrium labor market tightness schedule downward. If the discount rate channel dominates the cash flow channel (blue dotted

¹³For simplicity, we ignore the Lagrange multipliers on vacancy postings $V_{i,t}$ and firing $F_{i,t}$.

line), then the new equilibrium is point D, which is associated with a drop in labor market tightness to θ_3 and thus $\tau_p < 0$.

Our benchmark calibration implies that the cash flow effect dominates the discount rate effect (dashed red line) so that labor market tightness is positively related with matching efficiency (point C in Figure 3). Quantitatively, the equilibrium labor market tightness dynamics are¹⁴

$$\log \theta_{t+1} = -0.0076 + 0.9827 \log \theta_t + 0.0392 \varepsilon_{t+1}^x + 0.0079 \varepsilon_{t+1}^p. \quad (3.37)$$

Labor market tightness is highly persistent and firms increase their vacancy postings after positive aggregate productivity shocks, $\tau_x > 0$, and after positive efficiency shocks, $\tau_p > 0$. Similarly, the equilibrium dynamics of (realized) market excess returns are

$$R_{M,t+1}^e = 0.0060 + 0.0096 \varepsilon_{t+1}^x - 0.0470 \varepsilon_{t+1}^p. \quad (3.38)$$

The average market excess return is 60 basis points per month and market prices increase with aggregate productivity shocks, $\nu_x > 0$, and decrease with efficiency shocks, $\nu_p < 0$, which is consistent with a positive price of risk for productivity shocks and a negative one for efficiency shocks.

These two dynamics allow us to compute stock return loadings on labor market tightness, which we use in the following section to form portfolios. Proposition 1 (Equation (3.31)) states the functional form for labor market tightness loadings, $\beta_{i,t}^\theta$. As the above discussion highlights, efficiency shocks and not productivity shocks are the driver of the labor market tightness premium. To illustrate the intuition behind Equation (3.31), we assume here that loadings on the market are constant. Labor market tightness loadings are negatively correlated with expected returns when $\nu_x/(\tau_p \nu_x - \tau_x \nu_p) > 0$. Because productivity has a positive effect on job creation, $\tau_x > 0$, and on market returns, $\nu_x > 0$, this condition reduces to $\tau_p > \nu_p$, which again emphasizes that the cash flow effect of efficiency shocks has to dominate the discount rate effect.

4.4 Cross-Section of Returns

In the previous section, we have shown that labor market tightness obtains a negative factor risk premium in equilibrium. To assess the extent to which the model can quantitatively explain the empirically observed negative relation between loadings on labor market tightness and future stock returns, we follow the empirical procedure of Section 2 on simulated data. To this end, we sort simulated firms into decile portfolios by their labor market tightness loadings, $\beta_{i,t}^\theta$, as defined in Proposition 1. Table 10 compares the simulated returns with data on industry-neutral portfolios from Panel A of Table 7.¹⁵ As in the data, we form monthly value-weighted portfolios

¹⁴Note that the coefficients on the x and p shocks are normalized by their respective standard deviations as compared to Equation (3.23). The same normalization applies to Equation (3.38).

¹⁵We base our analysis on industry-neutral portfolios because the model does not capture heterogeneities across industries.

with annual rebalancing. The table reports average labor market tightness loadings, returns, and CAPM alphas across portfolios.

The model generates a realistic dispersion in labor market tightness loadings and returns across portfolios. The average monthly return difference between the low- and high-loading portfolios is 0.38% relative to 0.37% in the data. Moreover, the CAPM cannot explain the return differences across portfolios because in the model it does not span all systematic risks. In particular, Proposition 2 states that the CAPM alphas are inversely related to loadings on labor market tightness, as long as the market price of labor market tightness is negative.

The cash flow channel of hiring costs impacts the cross-section of returns in the following way. Due to proportional hiring and firing costs, the optimal firm policy exhibits regions of inactivity, where firms neither hire nor fire workers. Figure 4 illustrates the optimal firm policy. The horizontal black line is the optimal policy when adjusting the workforce is costless. In the frictionless case, firms always adjust to the target employment size independent of the current size. The red curve is the optimal policy in the benchmark model. It displays two kinks. In the middle region, where the optimal policy coincides the dashed line, firms are inactive. In the inactivity region below the frictionless employment target, firms have too few workers but hiring is too costly (*Hiring constrained*). In the inactivity region above the frictionless employment target, firms have too many workers but firing is too costly (*Excess labor*).

Due to the time variation in matching efficiency, ideally firms would like to hire when marginal hiring costs, $\kappa_h/q(\theta, p)$, are low. This holds for the majority of firms, as aggregate vacancy postings increase with efficiency shocks. However, some firms are hit by low idiosyncratic productivity shocks such that hiring is not optimal when matching efficiency is high. For these firms, the discount rate channel dominates the cash flow channel, thereby depressing valuations. Their dividends are reduced not only by low idiosyncratic productivity shocks but also by higher wages, arising from tighter labor markets, and by firing costs. Consequently, these firms have countercyclical dividends and valuations with respect to matching efficiency shocks, which renders them more risky. Since labor market tightness loadings and loadings on matching efficiency are positively related, our model can replicate the negative relation between labor market tightness loadings and expected returns.

To illustrate that the differences in average return across portfolios are driven by the cash flow effect of matching efficiency shocks, we compute the correlation between profitability and the labor market tightness factor for each portfolio. The results are reported in the column denoted by Corr both for the data and model of Table 10.¹⁶ Consistent with the model, firms with low β^θ loadings are risky because their cash flows are counter-cyclical with respect to labor market tightness.

¹⁶Portfolio assignment is done within 48 industries as in Panel A of Table 7 to obtain industry-neutral decile portfolios.

4.5 Robustness

To gain more insights about the driving forces of the model, we consider alternative calibrations in Table 11. Specifically, we are interested in the sensitivity of the return differences across β^θ -sorted portfolios to parameter values.

In specifications (1) to (3), we consider the effects of changing prices of risks, holding the first and second moments of the risk-free rate constant. Specification (1) illustrates the impact of pricing aggregate productivity shocks by setting its price to zero, $\gamma_x = 0$. The portfolio spread is of the correct sign but of smaller magnitude compared to the data. This finding indicates the importance of modeling productivity shocks to generate cross-sectional heterogeneity among firms.

In specification (2), we assume that matching efficiency shocks are not priced, $\gamma_{p,0} = 0$. We also raise the price of risk of productivity shocks to $\gamma_x = 20$, so that the Sharpe ratio of the pricing kernel matches the benchmark calibration. With only productivity shocks being priced, the cross-sectional spread is small and negative -0.18 . This experiment shows that the priced variation in aggregate matching efficiency is crucial for the labor market tightness factor to affect valuations. In the benchmark calibration, we assume that the price of matching efficiency risk increases with adverse shocks. In specification (3), we assume that matching efficiency shocks have a constant price of risk, $\gamma_{p,1} = 0$. As a result, the simulated cross-sectional spread reduces from 0.38% to 0.18%, indicating the importance of the time variation in the price of risk of matching efficiency shocks.

Specifications (4) to (7) analyze the importance of labor search frictions by varying labor market parameters. For these exercises, we hold the dynamics of labor market tightness, Equation (3.37), constant, and study local perturbations of the parameter space. In specification (4), we increase the bargaining power of workers η by 10% to 0.1375. As a result, wages become more cyclical, implying a weaker operating leverage effect. The results suggest that this wage operating leverage channel is weak as the return spread does not change relative to the benchmark.

The next two specifications show that the costs of hiring rather than firing drive the cross-sectional return spread. In particular, reducing the costs of laying off workers κ_f by 10% to 0.315 has little effect on the return spread (specification (5)). In contrast, reducing the costs of hiring works κ_h by 10% to 0.675, decreases the return spread to 0.34% (specification (6)), compared to 0.38% for the benchmark calibration.

In the baseline calibration, we set the fixed operating costs f to match the corporate profit margin in the data. In specification (7), we reduce the fixed operating costs by 10% to 0.2034. Since the ratio of hiring costs to output is very small, $\kappa_h V / N^\alpha = 0.036$, reducing operating costs makes time-varying hiring costs less relevant for firm cash flow dynamics. As a result, the return spread drops to 0.32%.

Optimal firm employment policies depend on the equilibrium dynamics of labor market

tightness (3.37). The log-linear structure shows that, controlling for aggregate productivity, labor market tightness proxies for unobserved matching efficiency shocks. As shown in Table 10, firms' cash flow exposures to variations in labor market tightness are the source for the pricing of labor market tightness in the cross-section of returns. Consequently, the labor market tightness factor should also be a valid aggregate state variable, predicting future aggregate economic conditions.

Table 12 confirms the predictability of future economic activity by labor market tightness both in the data and model. In the data, we obtain quarterly time series for the Gross Domestic Product, Wages and Salary Accruals, and Personal Dividend Income from the National Income and Product Accounts and total factor productivity from Fernald (2012). In the table, we report coefficients on labor market tightness growth, their t -statistics, and adjusted R^2 values from bivariate regressions of output growth (Panel A), wage growth (Panel B), and dividend growth (Panel C) on labor market tightness growth and total factor productivity. We run quarterly forecasting regressions for horizons up to a year.

In the data, labor market tightness predicts positively and significantly output growth, wage growth, and dividend growth for horizons up to a year. This finding is consistent with our model, where changes in labor market tightness measure shocks to the matching efficiency of the labor market. Positive matching efficiency shocks predict an increase in economic activity, wages and dividends. Although being a highly procyclical aggregate variable, labor market tightness effectively captures a dimension of systematic risk absent in total factor productivity.

5 Conclusion

This paper studies the cross-sectional asset pricing implications of labor search frictions. The dynamic nature of the labor market implies that firms face costly employment decisions while searching for and training new employees. The ratio of vacant positions to unemployed workers, termed labor market tightness, determines the likelihood and costs of filling a vacant position.

We show that firms with low loadings on labor market tightness generate higher future returns than firms with high loadings. The return differential, at 6% per year, is economically and statistically important, cannot be explained by commonly considered factor models, and is distinct from previously studied determinants of the cross-section of equity returns.

To provide an interpretation for this result, we develop a Labor Capital Asset Pricing Model with heterogeneous firms making optimal employment decisions under labor search frictions. In the model, equilibrium labor market tightness is determined endogenously and depends on the time-varying firm-level distribution and aggregate shocks. Loadings on labor market tightness proxy for the sensitivity to aggregate shocks to the efficiency of matching workers and firms. Firms with lower labor market tightness loadings are more exposed to adverse matching efficiency shocks and hence require higher expected stock returns.

The model successfully replicates the observed return differential and other empirical firm-

level and aggregate labor market moments. Our results suggest that labor search frictions have important implications for equity returns. Further research into the nature of interactions between labor and financial markets should provide an even more complete picture on the determinants of asset prices.

Appendix

A Data

We use the following definitions of CRSP-Compustat variables: ME is the natural logarithm of market equity of the firm, calculated as the product of its share price and number of shares outstanding. BM is the natural logarithm of the ratio of book equity to market equity. Book equity is defined following [Davis, Fama, and French \(2000\)](#) as stockholders' book equity (SEQ) plus balance sheet deferred taxes (TXDB) plus investment tax credit (ITCB) less the redemption value of preferred stock (PSTKRV). If the redemption value of preferred stock is not available, we use its liquidation value (PSTKL). If the stockholders' equity value is not available in Compustat, we compute it as the sum of the book value of common equity (CEQ) and the value of preferred stock. Finally, if these items are not available, stockholders' equity is measured as the difference between total assets (AT) and total liabilities (LT). RU is the 12-month stock return run-up. HN is the hiring rate, calculated following [Belo, Lin, and Bazdresch \(2014\)](#) as $(N_t - N_{t-1})/((N_t + N_{t-1})/2)$, where N_t is then number of employees (EMP). AG is the asset growth rate, calculated following [Cooper, Gulen, and Schill \(2008\)](#) as $A_t/A_{t-1} - 1$, where A_t is the value of total assets (AT). IK is the investment rate, calculated following [Belo, Lin, and Bazdresch \(2014\)](#) as the ratio of capital expenditure (CAPX) divided by the lagged capital stock (PPENT). Profitability is defined following [Cooper, Gulen, and Schill \(2008\)](#) as $[\text{operating income before depreciation (OIBDP)} - \text{interest expenses (XINT)} - \text{taxes (TXT)} - \text{preferred dividends (DVP)} - \text{common dividends (DVC)}] / \text{total assets (AT)}$.

B Proofs

Proof of Proposition 1: A log-linear approximation of the pricing kernel M_{t+1} is given by

$$\frac{M_{t+1}}{\mathbb{E}_t[M_{t+1}]} = e^{m_{t+1} - \ln(\mathbb{E}_t[M_{t+1}])} \approx 1 + m_{t+1} - \ln(\mathbb{E}_t[M_{t+1}]).$$

Given this approximation, the Euler equation, $\mathbb{E}_t[M_{t+1}R_{i,t+1}^e] = 0$, implies

$$\mathbb{E}_t[R_{i,t+1}^e] \approx -\text{Cov}_t(m_{t+1}, R_{i,t+1}^e). \quad (3.39)$$

For the pricing kernel [\(3.17\)](#), the previous equation specializes to

$$\mathbb{E}_t[R_{i,t+1}^e] \approx \gamma_x \text{Cov}_t(\sigma_x \varepsilon_{t+1}^x, R_{i,t+1}^e) + \gamma_{p,t} \text{Cov}_t(\sigma_p \varepsilon_{t+1}^p, R_{i,t+1}^e). \quad (3.40)$$

Because $\sigma_x \varepsilon_{t+1}^x = x_{t+1} - \rho_x x_t$ and $\sigma_p \varepsilon_{t+1}^p = p_{t+1} - \rho_p p_t$, a two-factor model in x and p holds:

$$\mathbb{E}_t[R_{i,t+1}^e] \approx \beta_{i,t}^x \lambda^x + \beta_{i,t}^p \lambda^p, \quad (3.41)$$

where risk loadings are given by

$$\beta_{i,t}^x = \frac{\text{Cov}_t(x_{t+1}, R_{i,t+1}^e)}{\sigma_x^2} \quad \beta_{i,t}^p = \frac{\text{Cov}_t(p_{t+1}, R_{i,t+1}^e)}{\sigma_p^2}, \quad (3.42)$$

and factor risk premia are

$$\lambda^x = \gamma_x \sigma_x^2 \quad \lambda_t^p = \gamma_{p,t} \sigma_p^2. \quad (3.43)$$

Given the pricing kernel (3.27) and laws of motion (3.24) and (3.26), it follows from (3.39) that

$$\mathbb{E}_t[R_{i,t+1}^e] = (\gamma_{M,t} \nu_x + \gamma_{\theta,t} \tau_x) \text{Cov}_t(\sigma_x \varepsilon_{t+1}^x, R_{i,t+1}^e) + (\gamma_{M,t} \nu_p + \gamma_{\theta,t} \tau_p) \text{Cov}_t(\sigma_p \varepsilon_{t+1}^p, R_{i,t+1}^e). \quad (3.44)$$

Thus, by matching coefficients in terms of covariances between equations (3.40) and (3.44), it follows that

$$\gamma_x = \gamma_{M,t} \nu_x + \gamma_{\theta,t} \tau_x \quad \gamma_{p,t} = \gamma_{M,t} \nu_p + \gamma_{\theta,t} \tau_p,$$

implying (3.28) holds.

Since x_t and p_t are uncorrelated, the factor loadings β^x and β^p satisfy the regression

$$R_{i,t+1}^e - \mathbb{E}_t[R_{i,t+1}^e] = \beta_{i,t}^x \sigma_x \varepsilon_{t+1}^x + \beta_{i,t}^p \sigma_p \varepsilon_{t+1}^p + \epsilon_{i,t+1}, \quad (3.45)$$

with loadings defined in equation (3.42). Similarly, the loadings on the market return and labor market tightness satisfy the regression

$$R_{i,t+1}^e - \mathbb{E}_t[R_{i,t+1}^e] = \beta_{i,t}^M (R_{M,t+1}^e - \mathbb{E}_t[R_{M,t+1}^e]) + \beta_{i,t}^\theta (\vartheta_{t+1} - \mathbb{E}_t[\vartheta_{t+1}]) + \epsilon_{i,t+1}. \quad (3.46)$$

Notice that since $R_{M,t+1}^e$ and ϑ_{t+1} are not independent, it follows that

$$\beta_{i,t}^M \neq \frac{\text{Cov}_t(R_{i,t+1}^e, R_{M,t+1}^e)}{\text{Var}_t(R_{M,t+1}^e)} \quad \beta_{i,t}^\theta \neq \frac{\text{Cov}_t(R_{i,t+1}^e, \vartheta_{t+1})}{\text{Var}_t(\vartheta_{t+1})}.$$

To compute the loadings on the market return and labor market tightness, equate Equations (3.45) and (3.46) and substitute in laws of motion (3.24) and (3.26), obtaining

$$\beta_{i,t}^x \sigma_x \varepsilon_{t+1}^x + \beta_{i,t}^p \sigma_p \varepsilon_{t+1}^p + \epsilon_{i,t+1} = \beta_{i,t}^M (\nu_x \sigma_x \varepsilon_{t+1}^x + \nu_p \sigma_p \varepsilon_{t+1}^p) + \beta_{i,t}^\theta (\tau_x \sigma_x \varepsilon_{t+1}^x + \tau_p \sigma_p \varepsilon_{t+1}^p) + \epsilon_{i,t+1}.$$

By matching the coefficients in terms of $\sigma_x \varepsilon_{t+1}^x$ and $\sigma_p \varepsilon_{t+1}^p$, we get

$$\beta_{i,t}^x = \beta_{i,t}^M \nu_x + \beta_{i,t}^\theta \tau_x \quad \beta_{i,t}^p = \beta_{i,t}^M \nu_p + \beta_{i,t}^\theta \tau_p,$$

implying that (3.30) and (3.31) hold.

Next, substitute (3.30) and (3.31) into (3.29), yielding

$$\mathbb{E}_t[R_{i,t+1}^e] = \frac{\tau_x \beta_{i,t}^p - \tau_p \beta_{i,t}^x}{\nu_p \tau_x - \nu_x \tau_p} \lambda_t^M + \frac{\nu_p \beta_{i,t}^x - \nu_x \beta_{i,t}^p}{\nu_p \tau_x - \nu_x \tau_p} \lambda_t^\theta \quad (3.47)$$

and match coefficients in terms of $\beta_{i,t}^x$ and $\beta_{i,t}^p$ with (3.41), implying

$$\begin{aligned}\lambda^x(\nu_p\tau_x - \nu_x\tau_p) &= \nu_p\lambda_t^\theta - \tau_p\lambda_t^M \\ \lambda_t^p(\nu_p\tau_x - \nu_x\tau_p) &= \tau_x\lambda_t^M - \nu_x\lambda_t^\theta.\end{aligned}$$

Solving for λ_t^θ and λ_t^M confirms (3.32).

Proof of Proposition 2: Given the dynamics for the market excess return (3.26), univariate loadings on the market return can be computed via

$$\begin{aligned}\beta_{i,t}^{CAPM} &= \frac{\text{Cov}_t(R_{i,t+1}^e, R_{M,t+1}^e)}{\text{Var}_t(R_{M,t+1}^e)} \\ &= \frac{\nu_x\text{Cov}_t(R_{i,t+1}^e, \sigma_x\varepsilon_{t+1}^x) + \nu_p\text{Cov}_t(R_{i,t+1}^e, \sigma_p\varepsilon_{t+1}^p)}{\text{Var}_t(R_{M,t+1}^e)} = \frac{\nu_x\sigma_x^2\beta_{i,t}^x + \nu_p\sigma_p^2\beta_{i,t}^p}{\nu_x^2\sigma_x^2 + \nu_p^2\sigma_p^2}.\end{aligned}$$

Notice that the CAPM factor risk premium remains the same in the one-factor or two-factor models, that is, $\lambda_t^{CAPM} = \lambda_t^M = \nu_x\lambda^x + \nu_p\lambda_t^p$. Given the pricing of expected excess returns in terms of independent aggregate risks (3.41), we can calculate the CAPM mispricing as

$$\alpha_{i,t}^{CAPM} = \beta_{i,t}^x\lambda_t^x + \beta_{i,t}^p\lambda_t^p - \beta_{i,t}^{CAPM}\lambda_t^{CAPM} = \frac{(\beta_{i,t}^x\nu_p - \beta_{i,t}^p\nu_x)(\nu_p\gamma_x - \nu_x\gamma_p)\sigma_x^2\sigma_p^2}{\nu_x^2\sigma_x^2 + \nu_p^2\sigma_p^2}.$$

Using the definition of $\beta_{i,t}^\theta$ in (3.31) and $\gamma_{\theta,t}$ in (3.28), it follows that (3.34) holds.

C Computational Algorithm

To solve the model numerically, we discretize the state space. All shocks (x, p, z) follow finite states Markov chains according to Rouwenhorst (1995) with 5 states for x , 9 for p , and 11 for z . We create a log-linear grid of 500 points for current employment N in the interval $[0.01, 20]$. The lower and upper bounds of N are set such that the optimal policies are not binding in the simulation. The choice variable N' is a vector containing 5,000 elements, also log-linearly spaced on the same interval as N . The space of labor market tightness θ is discretized into a linear grid in the interval $[0.1, 1.5]$ with 50 points. The upper and lower bounds for θ are chosen such that the simulated path of equilibrium labor market tightness never steps outside its bounds.

The computational algorithm amounts to the following iterative procedure. To save on notation, we drop the firm index i and time index t .

1. Initial guess: Make an initial guess for the coefficient vector $\tau = (\tau_0, \tau_\theta, \tau_x, \tau_p)$ of the law of motion (3.23). We start from $\tau = (-0.0091, 0.98, 0, 0)$ because labor market tightness tends to be highly persistent and in steady state $\tau_0 = (1 - \tau_\theta)\log(\theta^{ss}) = (1 - 0.98)\log(0.634)$.
2. Optimization: Solve the firm's optimization problem (3.19) given the forecasting rule

coefficients τ . We use value function iteration and linear interpolation to obtain the value function off grid points. Given the discretized state space $\Omega = (N, z, x, p, \theta)$ and proportional hiring and firing costs, the firm value function solves

$$S(\Omega) = \max\{S^h(\Omega), S^f(\Omega), S^i(\Omega)\},$$

where S^h is the value of a firm that expands its workforce

$$S^h(\Omega) = \max_{N' > (1-s)N} \left\{ e^{x+z} N^\alpha - WN - \frac{\kappa_h}{q(\theta, p)} [N' - (1-s)N] + \mathbb{E}[M' S(\Omega') | \Omega] \right\},$$

S^f is the value of a firm that fires workers

$$S^f(\Omega) = \max_{N' < (1-s)N} \left\{ e^{x+z} N^\alpha - WN - \kappa_f [(1-s)N - N'] + \mathbb{E}[M' S(\Omega') | \Omega] \right\},$$

and S^i is the value of an inactive firm

$$S^i(\Omega) = e^{x+z} N^\alpha - WN + \mathbb{E}[M' S((1-s)N, z', x', p', \theta') | \Omega].$$

3. Simulation: Use the firm's optimal employment policies $V(\Omega)$ and $F(\Omega)$ to simulate a panel of 5,000 firms for 5,300 periods. Importantly, we impose labor market equilibrium at each date of the simulation by solving θ as the fixed point in Equation (3.21). In this way, we obtain a time series of realized equilibrium θ .
4. Update coefficients: Delete the initial 300 periods as burn-in and use the stationary region of the simulated data to estimate the vector τ by OLS; update the forecasting coefficients, and restart from the optimization step 2; continue the outer loop iteration until the τ coefficients have converged.

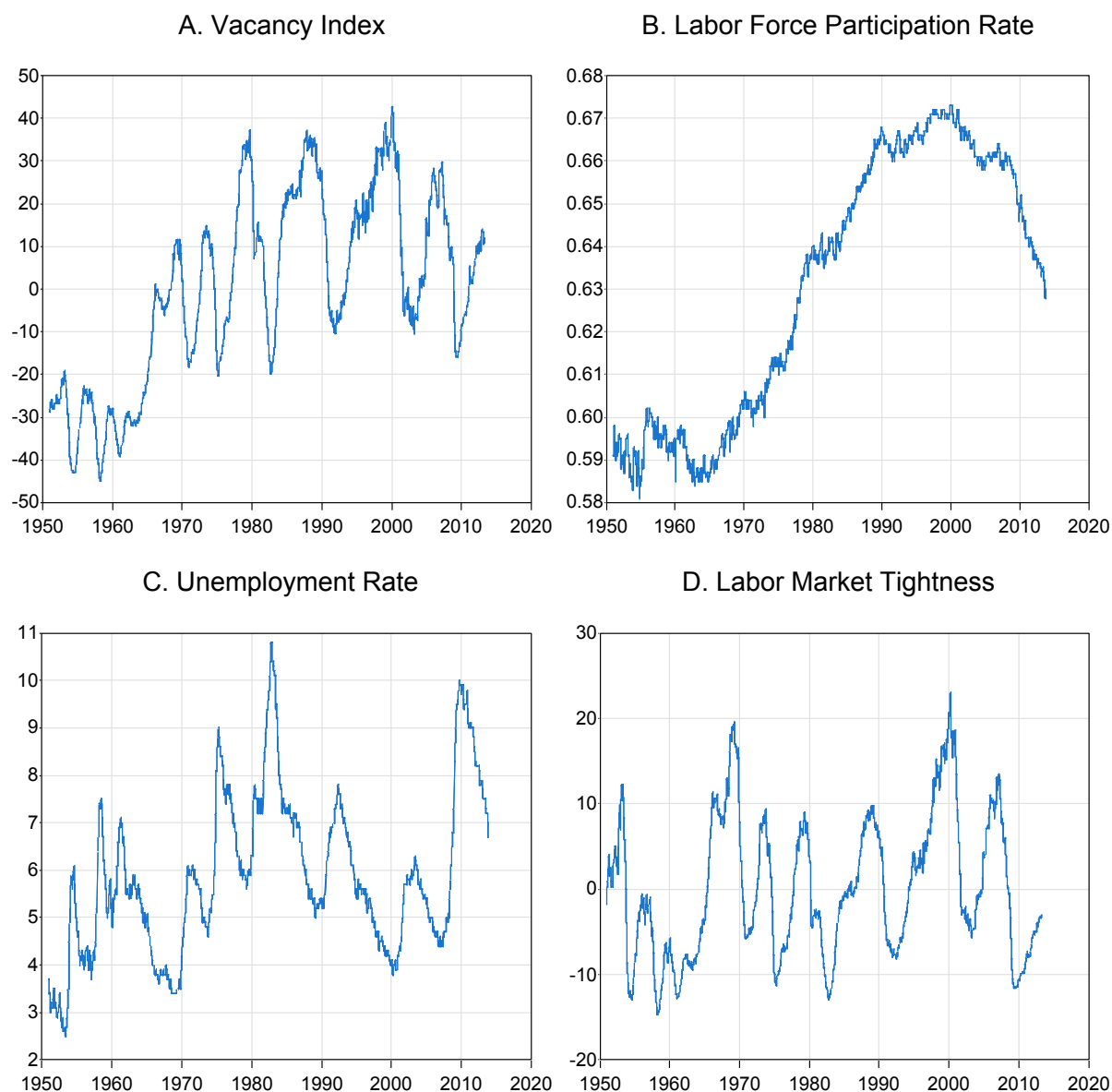


Figure 1. Labor Market Tightness and Its Components

This figure plots the monthly time series of the vacancy index, the labor force participation rate, the unemployment rate, and labor market tightness for the years 1951 to 2012.

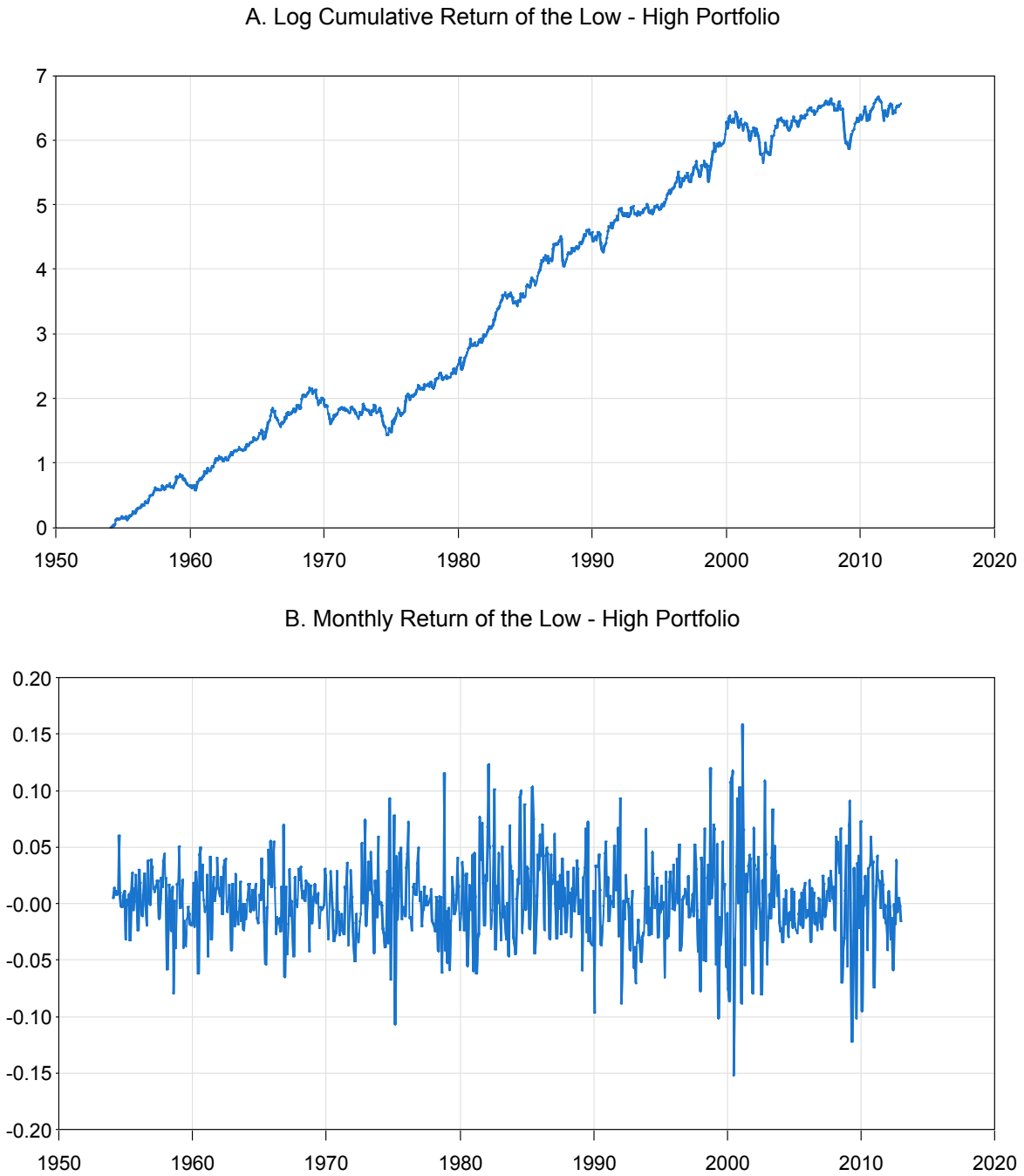


Figure 2. Returns on Long-Short Labor Market Tightness Portfolios
This figure plots the log cumulative (Panel A) and monthly (Panel B) returns on a portfolio that is long the decile of stocks with the lowest exposure to the labor market tightness factor and short the decile of stocks with the highest loadings. The sample spans 1954 to 2012.

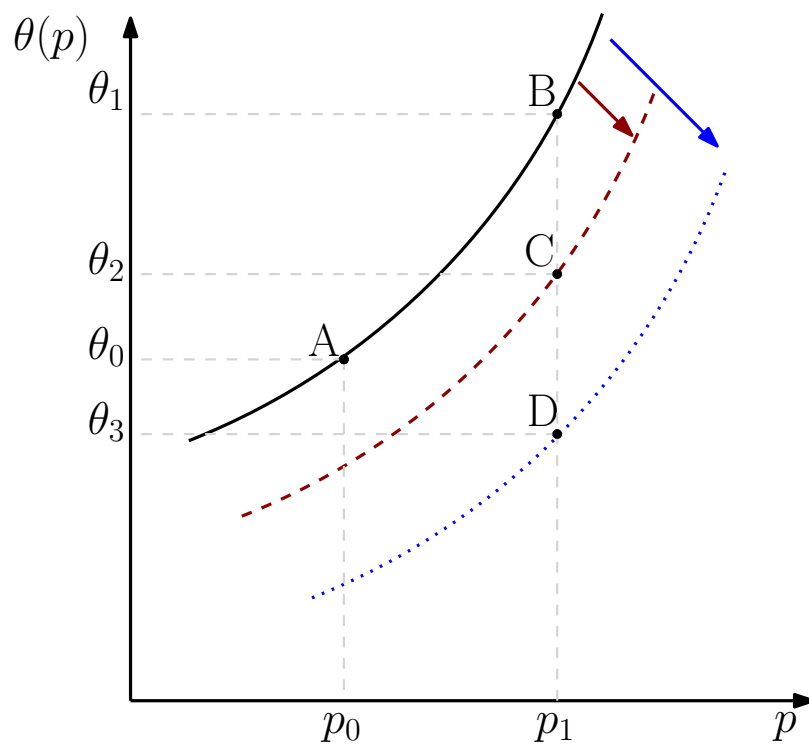


Figure 3. Labor Market Tightness and Matching Efficiency

This figure illustrates the endogenous response of equilibrium labor market tightness $\theta(p)$ to a positive matching efficiency shock p .

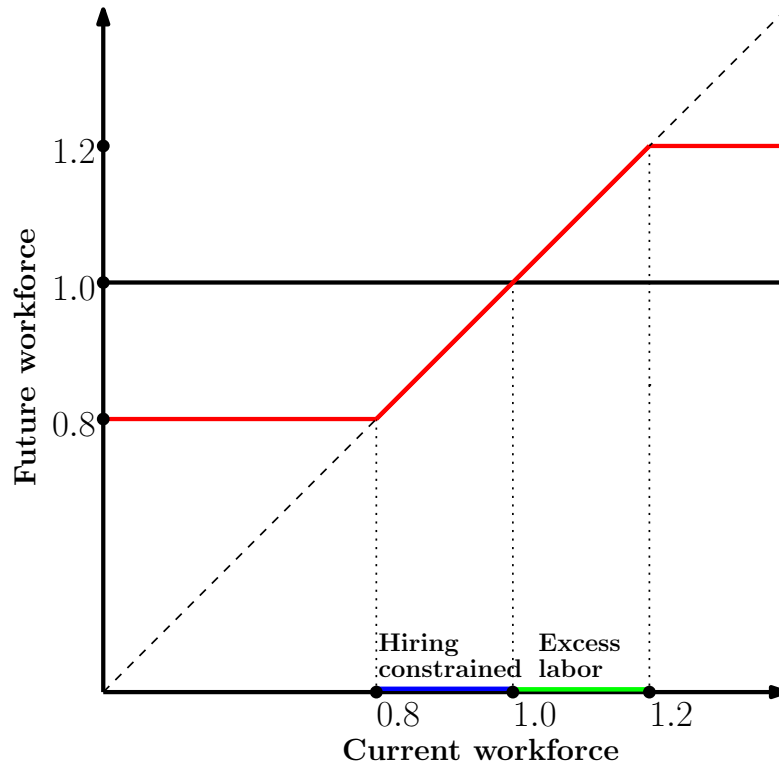


Figure 4. Optimal Employment Policy

This figure illustrates the optimal employment policy. The horizontal black line is the optimal policy when adjusting the workforce is costless. The red kinked curve is the optimal policy in the benchmark model under search frictions. In the middle region, where the optimal policy coincides with the dashed line, firms are inactive. In the inactivity region below the frictionless employment target, firms have too few workers but hiring is too costly (*Hiring constrained*). In the inactivity region above the frictionless employment target, firms have too many workers but firing is too costly (*Excess labor*).

Table 1. Summary Statistics

This table reports summary statistics for the monthly labor market tightness factor (ϑ), changes in the vacancy index (VAC), changes in the unemployment rate (UNEMP), changes in the labor force participation rate (LFPR), changes in industrial production (IP), changes in the consumer price index (CPI), dividend yield (DY), T-bill rate (TB), term spread (TS), and default spread (DS) calculated for the 1954 to 2012 period. Means and standard deviations are in percent.

	Mean	StDev	Correlations								
			ϑ	VAC	UNEMP	LFPR	IP	CPI	DY	TB	TS
ϑ	0.02	5.48									
VAC	-0.10	3.46	0.78								
UNEMP	0.16	3.38	-0.83	-0.36							
LFPR	0.01	0.30	-0.13	0.04	0.15						
IP	0.24	0.89	0.56	0.44	-0.48	0.04					
CPI	0.31	0.32	-0.08	-0.04	0.06	0.05	-0.08				
DY	3.20	1.13	-0.14	0.00	0.12	0.07	-0.10	0.34			
TB	0.39	0.24	-0.12	-0.08	0.04	0.05	-0.09	0.52	0.51		
TS	1.45	1.22	0.11	0.10	-0.05	-0.03	0.04	-0.29	-0.12	-0.39	
DS	0.99	0.45	-0.26	-0.20	0.22	-0.03	-0.28	0.11	0.33	0.33	0.29

Table 2. Characteristics of Labor Market Tightness Portfolios

This table reports average characteristics for the ten portfolios of stocks sorted by their loadings on labor market tightness β^θ . β^M denotes the market beta, BM the book-to-market ratio, ME the market equity decile, RU the 12-month run-up return in percent; AG, IK, and HN are asset growth, investment, and new hiring rates, respectively, all shown in percent. Mean characteristics are calculated annually for each decile and then averaged over time. The sample period is 1954 to 2012 except for variables that use Compustat data (BM, AG, IK, and HN) where it is 1960 to 2012.

Decile	β^θ	β^M	BM	ME	RU	AG	IK	HN
Low	-0.80	1.35	0.89	4.84	15.44	12.92	32.59	6.36
2	-0.38	1.16	0.92	5.73	13.68	13.02	29.39	7.16
3	-0.23	1.07	0.91	6.09	12.67	11.01	27.34	5.70
4	-0.12	1.01	0.92	6.27	12.92	11.36	27.05	6.72
5	-0.03	1.00	0.92	6.22	13.37	11.17	26.08	5.00
6	0.06	1.01	0.94	5.99	13.08	11.51	26.44	5.12
7	0.16	1.04	0.94	5.84	13.35	11.30	27.35	5.94
8	0.27	1.08	0.95	5.52	13.55	11.41	28.17	5.50
9	0.45	1.17	0.94	4.98	13.71	12.23	29.54	6.95
High	0.91	1.33	0.92	3.99	16.13	12.63	32.87	6.86

Table 3. Performance of Labor Market Tightness Portfolios

This table reports average raw returns and alphas, in percent per month, and loadings from the four-factor model regressions for the ten portfolios of stocks sorted on the basis of their loadings on the labor market tightness factor, as well as for the portfolio that is long the low decile and short the high one. The bottom row gives t -statistics for the low-high portfolio. Firms are assigned into deciles at the end of every month and the value-weighted portfolios are held without rebalancing for 12 months. Conditional alphas are based on either Ferson and Schadt (FS) or Boguth, Carlson, Fisher, and Simutin (BCFS). The sample period is 1954 to 2012.

Decile	Raw	Unconditional Alphas			Cond. Alphas		4-Factor Loadings			
	Return	CAPM	3-Factor	4-Factor	FS	BCFS	MKT	HML	SMB	UMD
Low	1.12	0.04	0.06	0.04	0.08	0.08	1.16	-0.11	0.38	0.02
2	1.09	0.13	0.13	0.13	0.11	0.11	1.05	0.01	-0.02	0.00
3	1.05	0.13	0.11	0.13	0.11	0.10	0.99	0.06	-0.08	-0.02
4	1.01	0.11	0.09	0.09	0.10	0.10	0.95	0.07	-0.10	-0.01
5	0.98	0.09	0.05	0.03	0.06	0.05	0.96	0.13	-0.11	0.01
6	0.96	0.06	0.03	0.01	0.04	0.04	0.97	0.09	-0.11	0.03
7	0.96	0.05	0.03	0.04	0.03	0.03	0.98	0.04	-0.07	-0.01
8	0.94	-0.01	-0.02	0.03	-0.01	0.00	1.01	0.00	0.02	-0.05
9	0.83	-0.20	-0.19	-0.13	-0.16	-0.14	1.11	-0.08	0.18	-0.07
High	0.65	-0.47	-0.46	-0.37	-0.40	-0.38	1.18	-0.19	0.62	-0.09
Low-High	0.47	0.51	0.52	0.41	0.48	0.47	-0.02	0.07	-0.24	0.11
t -statistic	[3.41]	[3.78]	[3.83]	[2.99]	[3.56]	[3.46]	[-0.62]	[1.41]	[-5.18]	[3.30]

Table 4. Summary Statistics of Risk Factors

This table reports summary statistics for the difference in returns on stocks with low and high loadings β^θ on the labor market tightness factor as well as for the market excess return, and value, size and momentum factors. All data are monthly. Means and standard deviations are in percent. The sample period is 1954 to 2012.

	Mean	Standard deviation	Sharpe ratio	Correlations			
				Low-high β^θ return	Mkt excess return	Value factor	Size factor
Low-high β^θ return	0.47	3.60	0.13				
Market excess return	0.55	4.40	0.13	-0.11			
Value factor	0.38	2.75	0.14	0.08	-0.27		
Size factor	0.20	2.95	0.07	-0.22	0.28	-0.21	
Momentum factor	0.73	4.06	0.12	0.15	-0.13	-0.17	-0.03

Table 5. Robustness of Performance of Labor Market Tightness Portfolios

This table reports average raw returns and alphas, in percent per month, four-factor loadings, and corresponding t -statistics for the portfolio that is long the decile of stocks with low loadings on the labor market tightness factor and short the decile with high loadings. In Panel A, firms are assigned into deciles at the end of May and are held for one year starting in July. In Panel B, firms are assigned into deciles at the end of every month τ and are held during month $\tau + 2$. In Panel C, firms are assigned into deciles at the end of every month τ and are held without rebalancing for 12 month beginning in month $\tau + 3$. In Panel D, firms are assigned into quintiles rather than deciles. In Panel E, firms below 20th percentile of NYSE market capitalization are excluded from the sample. In Panel F, the labor market tightness factor is defined as the residual from a time-series regression of log-changes in the labor market tightness on changes in industrial production and the consumer price index, dividend yield, T-Bill rate, term spread, and default spread. In Panel G, labor market tightness factor is defined as the residual from an ARMA(1,1) specification. In Panel H, regression (3.3) is amended to also include size, value, and momentum factors. Conditional alphas are based on either Ferson and Schadt (FS) or Boguth, Carlson, Fisher, and Simutin (BCFS). In all panels, portfolios are value-weighted. The sample period is 1954 to 2012.

Decile	Raw	Unconditional Alphas			Cond. Alphas		4-Factor Loadings			
	Return	CAPM	3-Factor	4-Factor	FS	BCFS	MKT	HML	SMB	UMD
A. Non-overlapping portfolios										
Low-High	0.55	0.59	0.51	0.47	0.50	0.50	0.01	0.24	-0.25	0.04
t -statistic	[3.42]	[3.68]	[3.19]	[2.89]	[3.16]	[3.11]	[0.29]	[3.92]	[-4.50]	[0.94]
B. One-month holding period										
Low-High	0.51	0.58	0.61	0.46	0.54	0.52	-0.06	0.04	-0.28	0.16
t -statistic	[2.99]	[3.44]	[3.63]	[2.67]	[3.35]	[3.23]	[-1.42]	[0.67]	[-4.84]	[3.85]
C. Two-month waiting period										
Low-High	0.47	0.52	0.52	0.42	0.48	0.47	-0.02	0.07	-0.24	0.11
t -statistic	[3.49]	[3.84]	[3.90]	[3.08]	[3.62]	[3.52]	[-0.58]	[1.35]	[-5.17]	[3.23]
D. Quintile portfolios										
Low-High	0.32	0.39	0.39	0.30	0.35	0.32	-0.06	0.07	-0.19	0.09
t -statistic	[2.82]	[3.44]	[3.44]	[2.60]	[3.13]	[2.90]	[-2.29]	[1.61]	[-4.91]	[3.33]
E. Excluding micro caps										
Low-High	0.45	0.49	0.51	0.35	0.49	0.46	-0.05	0.01	-0.01	0.17
t -statistic	[3.72]	[4.05]	[4.15]	[2.84]	[4.08]	[3.88]	[-1.79]	[0.31]	[-0.22]	[5.54]
F. Alternative definition 1 of ϑ										
Low-High	0.44	0.48	0.50	0.46	0.47	0.46	-0.04	0.02	-0.19	0.04
t -statistic	[3.22]	[3.55]	[3.65]	[3.32]	[3.53]	[3.46]	[-1.30]	[0.44]	[-4.08]	[1.06]
G. Alternative definition 2 of ϑ										
Low-High	0.45	0.51	0.49	0.41	0.45	0.44	-0.03	0.11	-0.24	0.09
t -statistic	[3.28]	[3.68]	[3.58]	[2.87]	[3.29]	[3.20]	[-0.80]	[2.13]	[-4.92]	[2.70]
H. Alternative computation of β^θ										
Low-High	0.29	0.36	0.38	0.30	0.32	0.31	-0.08	0.03	-0.08	0.20
t -statistic	[2.36]	[2.90]	[3.04]	[2.55]	[2.62]	[2.54]	[-2.55]	[0.70]	[-1.82]	[6.46]

Table 6. Fama-MacBeth Regressions of Annual Stock Returns

This table reports the results of annual Fama-MacBeth regressions. Stock returns from July to June are regressed on lagged labor market tightness loadings β^θ , market betas β^M , log market equity ME, log of the ratio of book equity to market equity BM, 12-month stock return RU, hiring rates HN, investment rates IK, and asset growth rates AG. Reported are average coefficients and the corresponding [Newey and West \(1987\)](#) t -statistics. Details of variable definitions are in Appendix A. The sample period is 1960 to 2012.

Reg	β^θ	β^M	ME	BM	RU	HN	IK	AG
(1)	-0.028 [-2.46]	0.000 [-0.01]	-0.015 [-2.70]					
(2)	-0.030 [-2.30]	0.009 [0.67]	-0.011 [-1.95]	0.035 [4.20]				
(3)	-0.040 [-2.85]	0.010 [0.72]	-0.012 [-2.22]	0.037 [4.65]	0.066 [3.46]			
(4)	-0.043 [-3.01]	0.010 [0.72]	-0.012 [-2.29]	0.032 [4.09]	0.067 [3.38]	-0.050 [-3.48]		
(5)	-0.048 [-2.94]	0.011 [0.78]	-0.013 [-2.30]	0.033 [4.37]	0.066 [3.32]		-0.017 [-1.54]	
(6)	-0.040 [-2.75]	0.012 [0.83]	-0.011 [-2.13]	0.032 [4.10]	0.066 [3.36]			-0.075 [-5.02]
(7)	-0.046 [-3.01]	0.011 [0.77]	-0.012 [-2.25]	0.028 [3.67]	0.068 [3.36]	0.004 [0.26]	0.014 [1.16]	-0.091 [-4.66]

Table 7. Performance of Labor Market Tightness Portfolios: Industry-Level Analysis

This table reports in Panel A average raw returns and alphas, in percent per month, and loadings from the four-factor model regressions for the ten portfolios of stocks sorted within each of the 48 Ken French-defined industries on the basis of their loadings on the labor market tightness factor. Panel B repeats the analysis for the ten portfolios obtained by sorting 48 value-weighted industry portfolios from Ken French's data library on the basis of their loadings on the labor market tightness factor. The table also shows returns, alphas, and loadings for the portfolio that is long the low decile and short the high one. The bottom row of each panel gives t -statistics for the low-high portfolio. Firms (in Panel A) or industries (in Panel B) are assigned into deciles at the end of every month and are held without rebalancing for twelve months. Conditional alphas are based on either Ferson and Schadt (FS) or Boguth, Carlson, Fisher, and Simutin (BCFS). The sample period is 1954 to 2012.

Decile	Raw	Unconditional Alphas			Cond. Alphas		4-Factor Loadings			
	Return	CAPM	3-Factor	4-Factor	FS	BCFS	MKT	HML	SMB	UMD
A. Portfolios of Stocks Sorted by Labor Market Tightness Loadings Within Industries										
Low	1.10	0.09	0.04	0.02	0.10	0.09	1.10	0.06	0.20	0.03
2	1.06	0.10	0.06	0.07	0.09	0.09	1.03	0.06	0.06	-0.01
3	1.01	0.08	0.07	0.11	0.07	0.07	0.99	0.01	-0.05	-0.04
4	0.99	0.08	0.07	0.08	0.06	0.07	0.97	0.03	-0.07	-0.01
5	0.97	0.06	0.06	0.07	0.04	0.04	0.98	0.03	-0.12	-0.02
6	0.99	0.08	0.08	0.08	0.06	0.05	0.98	0.02	-0.12	0.00
7	0.93	0.01	0.00	0.00	-0.01	-0.01	0.99	0.03	-0.08	0.00
8	0.92	-0.01	-0.03	-0.03	-0.01	-0.01	1.00	0.05	-0.05	0.00
9	0.91	-0.06	-0.09	-0.06	-0.04	-0.03	1.05	0.05	0.04	-0.04
High	0.73	-0.27	-0.34	-0.31	-0.25	-0.25	1.08	0.09	0.27	-0.03
Low-High	0.37	0.36	0.39	0.33	0.35	0.34	0.02	-0.03	-0.07	0.06
t -statistic	[3.84]	[3.76]	[4.01]	[3.36]	[3.79]	[3.63]	[0.96]	[-0.78]	[-1.98]	[2.40]
B. Portfolios of Industries Sorted by Labor Market Tightness Loadings										
Low	1.30	0.34	0.23	0.13	0.29	0.27	1.03	0.22	0.25	0.11
2	1.14	0.19	0.10	0.10	0.15	0.14	1.00	0.16	0.16	0.01
3	1.13	0.18	0.08	0.07	0.15	0.14	1.00	0.16	0.21	0.02
4	1.11	0.17	0.08	0.05	0.14	0.13	0.99	0.17	0.23	0.03
5	1.08	0.13	0.04	0.05	0.11	0.10	1.00	0.14	0.20	-0.01
6	1.05	0.09	0.00	0.04	0.06	0.06	1.03	0.14	0.17	-0.04
7	1.04	0.07	-0.03	0.01	0.03	0.03	1.03	0.16	0.18	-0.04
8	1.09	0.12	0.00	0.05	0.06	0.06	1.05	0.19	0.17	-0.06
9	0.90	-0.08	-0.21	-0.11	-0.15	-0.14	1.05	0.19	0.21	-0.11
High	0.96	-0.03	-0.16	-0.13	-0.09	-0.10	1.05	0.19	0.32	-0.03
Low-High	0.34	0.37	0.39	0.26	0.38	0.37	-0.02	0.03	-0.07	0.13
t -statistic	[2.37]	[2.55]	[2.60]	[1.71]	[2.54]	[2.45]	[-0.48]	[0.56]	[-1.41]	[3.67]

Table 8. Benchmark Parameter Calibration

This table lists the parameter values of the benchmark calibration, which is at monthly frequency.

Parameter	Symbol	Value
Labor Market		
Size of the labor force	L	1.63
Matching function elasticity	ξ	1.27
Bargaining power of workers	η	0.125
Benefit of being unemployed	b	0.71
Returns to scale of labor	α	0.735
Workers quit rate	s	0.022
Flow cost of vacancy posting	κ_h	0.75
Flow cost of firing	κ_f	0.35
Fixed operating costs	f	0.226
Shocks		
Persistence of aggregate productivity shock	ρ_x	0.9830
Volatility of aggregate productivity shock	σ_x	0.005
Persistence of matching efficiency shock	ρ_p	0.9583
Volatility of matching efficiency shock	σ_p	0.025
Persistence of idiosyncratic productivity shock	ρ_z	0.965
Volatility of idiosyncratic productivity shock	σ_z	0.095
Pricing Kernel		
Time discount rate	β	0.994
Price of risk of aggregate productivity shock	γ_x	1
Constant price of risk of matching efficiency shock	$\gamma_{p,0}$	-4.7
Time-varying price of risk of matching efficiency shock	$\gamma_{p,1}$	3.6
Interest rate sensitivity	ϕ	-0.0214

Table 9. Aggregate and Firm-Specific Moments

This table summarizes empirical and model-implied aggregate and firm-specific moments. The data on the unemployment rate are from the BLS; the hiring and firing rates are from the JOLTS dataset collected by the BLS; job creation and destruction rates are from [Davis, Faberman, and Haltiwanger \(2006\)](#); labor market tightness is the ratio of vacancies to unemployment, with vacancy data from the Conference Board and [Barnichon \(2010\)](#); the labor share of income is from [Gomme and Rupert \(2007\)](#); the relative volatility of wages to output is from [Gertler and Trigari \(2009\)](#); profits and output data are from the National Income and Product Accounts. At the firm level, we compute moments of annual employment growth rates as in [Davis, Haltiwanger, Jarmin, and Miranda \(2006\)](#) for the merged CRSP-Compustat sample. The first and second moments of real stock returns and real risk-free rate are based on the value-weighted CRSP market return and the one-month T-Bill rate, and inflation from the BLS.

Moment	Data	Model
Aggregate Labor Market		
Unemployment rate	0.058	0.058
Hiring rate	0.036	0.036
Layoff rate	0.014	0.014
Job creation rate	0.026	0.028
Job destruction rate	0.025	0.027
Labor market tightness (LMT)	0.634	0.650
Correlation of LMT and vacancy	0.780	0.747
Correlation of LMT and unemployment rate	-0.830	-0.851
Correlation of unemployment rate and vacancy	-0.360	-0.328
Labor share of income	0.717	0.717
Volatility of aggregate wages to aggregate output	0.520	0.547
Aggregate profits to aggregate output	0.110	0.106
Firm-Level Employment		
Volatility of annual employment growth rates	0.236	0.239
Fraction of firms with zero annual employment growth rates	0.097	0.099
Asset Prices		
Average risk-free rate	0.010	0.010
Volatility of risk-free rate	0.021	0.021
Average market return	0.081	0.082
Stock market volatility	0.176	0.172

Table 10. Labor Market Tightness Portfolios from the Benchmark Model

This table compares the performance of the benchmark model with the data. Reported are loadings on labor market tightness factor, β^θ , average returns of portfolios sorted by loadings on labor market tightness, alphas from the one-factor CAPM, α^{CAPM} , and cash flow correlations of profits and labor market tightness, Corr. Returns and alphas are expressed in percent per month.

Decile	Data				Model			
	β^θ	Return	α^{CAPM}	Corr	β^θ	Return	α^{CAPM}	Corr
Low	-0.74	1.10	0.09	-0.13	-0.98	1.10	0.23	-0.12
2	-0.39	1.06	0.10	-0.03	-0.76	1.02	0.19	-0.10
3	-0.23	1.01	0.08	-0.01	-0.64	0.99	0.16	-0.09
4	-0.12	0.99	0.08	-0.09	-0.52	0.95	0.15	-0.09
5	-0.03	0.97	0.06	-0.01	-0.37	0.91	0.13	-0.05
6	0.06	0.99	0.08	-0.00	-0.30	0.90	0.12	-0.04
7	0.16	0.93	0.01	0.10	-0.10	0.85	0.00	-0.01
8	0.28	0.92	-0.01	0.05	0.02	0.83	-0.01	0.09
9	0.45	0.91	-0.06	0.05	0.26	0.79	-0.04	0.11
High	0.85	0.73	-0.27	0.19	0.66	0.72	-0.08	0.15
Low-High	-1.59	0.37	0.36	-0.32	-1.64	0.38	0.31	0.27

Table 11. Labor Market Tightness Portfolios from Alternative Calibrations

This table summarizes average returns of portfolios sorted by loadings on labor market tightness from alternative calibrations. In specification (1), the aggregate productivity shock is not priced, $\gamma_x = 0$. In specification (2), the matching efficiency shock is not priced, $\gamma_{p,0} = 0$, and in specification (3), $\gamma_{p,1} = 0$ so that the aggregate matching efficiency shock has a constant price of risk. In specification (4), the bargaining power of workers, η , is raised by 10% relative to the benchmark calibration. In specifications (5, 6, 7), the costs of laying off workers, κ_f , the vacancy posting cost κ_h , and the fixed operating costs, f , are lowered by 10%, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Decile	γ_x	$\gamma_{p,0}$	$\gamma_{p,1}$	η	κ_f	κ_h	f
Low	1.09	0.70	0.80	1.10	1.09	1.07	1.05
2	1.03	0.76	0.76	1.02	1.02	1.01	0.99
3	1.00	0.75	0.76	1.00	0.99	0.99	0.98
4	0.96	0.83	0.74	0.96	0.96	0.95	0.95
5	0.91	0.80	0.73	0.92	0.92	0.92	0.92
6	0.91	0.79	0.73	0.92	0.92	0.92	0.92
7	0.86	0.89	0.69	0.85	0.85	0.85	0.86
8	0.85	0.87	0.69	0.84	0.84	0.85	0.85
9	0.79	0.84	0.66	0.78	0.79	0.79	0.79
High	0.73	0.88	0.62	0.72	0.72	0.73	0.73
Low-High	0.36	-0.18	0.18	0.38	0.37	0.34	0.32

Table 12. Forecasting Economic Activity with Labor Market Tightness

This table summarizes the ability of labor market tightness to forecast future economic activity. The quarterly time series for the Gross Domestic Product, Wages and Salary Accruals, and Personal Dividend Income are from the National Income and Product Accounts and total factor productivity from [Fernald \(2012\)](#). The table reports coefficients on labor market tightness growth, their t -statistics, and adjusted R^2 values from bivariate regressions of output growth (Panel A), wage growth (Panel B), and dividend growth (Panel C) on labor market tightness growth and total factor productivity. Forecasting horizons range from one quarter to one year and the data cover the years 1951 to 2012.

	Data				Model			
Horizon (quarters)	1	2	3	4	1	2	3	4
A. Predicting aggregate output growth								
ϑ	0.032	0.040	0.037	0.022	0.041	0.052	0.052	0.047
t -statistic	[7.03]	[5.15]	[3.46]	[1.67]				
R^2	25.13	20.47	14.01	8.59	25.12	11.14	5.94	3.46
B. Predicting aggregate wage growth								
ϑ	0.043	0.063	0.077	0.076	0.046	0.056	0.056	0.052
t -statistic	[9.71]	[8.26]	[7.24]	[5.50]				
R^2	37.80	34.07	30.74	23.97	32.94	16.03	9.45	5.91
C. Predicting aggregate dividend growth								
ϑ	0.078	0.151	0.193	0.199	0.282	0.346	0.324	0.297
t -statistic	[3.81]	[4.99]	[4.69]	[3.88]				
R^2	8.08	14.29	13.89	10.86	23.09	12.59	7.19	4.39

Internet Appendix to A Labor Capital Asset Pricing Model

In this Internet Appendix, we evaluate robustness of the inverse relation between stock return loadings on changes in labor market tightness and future equity returns. We also provide additional empirical results.

A. Controlling for Liquidity and Profitability Factors

Pastor and Stambaugh (2003) show that stocks with higher liquidity risk earn higher returns, and Novy-Marx (2013) documents that more profitable firms generate superior future stock returns. To ensure that our results are not driven by liquidity or profitability risks, we repeat the portfolio analysis of Table 3, while controlling for these two sources of risk. As before, we assign stocks into deciles conditional on their loadings on the labor market tightness factor and obtain a monthly time series of future returns for each of the resulting ten portfolios. We use the same models as before to calculate unconditional and conditional alphas, but include the liquidity (Panel A) or profitability factor (Panel B) as an additional regressor in Table IA.I.¹⁷

The table shows that our results are robust to controlling for the liquidity and profitability factors. The negative relation between labor market tightness loadings and future stock returns is economically important and statistically significant in all regressions. The differences in future returns of portfolios with low and high loadings range from 0.33% to 0.47% monthly.

B. Post-Ranking Loadings on Labor Market Tightness

Table IA.II summarizes post-ranking β^θ loadings of the labor market tightness portfolios. For each portfolio, we obtain monthly time series of returns from January 1954 until December 2012. We then regress excess returns of each group annually on the market and the labor market tightness factors, including two Dimson (1979) lags to account for any effects due to non-synchronous trading. We average betas across years to obtain average β^θ loadings for each portfolio. We show results for decile sorts in Panel A and quintile ones in Panel B. The differences in post-ranking betas of the bottom and top groups are sizable, although muted relative to the spread in betas shown in Table 2. Importantly, in both panels a positive relation emerges between pre-ranking and post-ranking betas.

C. Controlling for Market Beta

In Table IA.III, we evaluate the relation between β^θ loadings and future equity returns, conditional on market betas β^M . We sort firms into quintiles based on their β^θ and β^M loadings computed at the end of month τ and hold the resulting 25 value-weighted portfolios without

¹⁷Liquidity and profitability factors are from <http://faculty.chicagobooth.edu/lubos.pastor/research/> and http://rnm.simon.rochester.edu/data_lib/index.html, respectively. The data on the two factors are available starting only in 1960s, which shortens our sample by as much as 14 years.

rebalancing for 12 months beginning in month $\tau + 2$. Table [IA.III](#) shows that irrespective of whether we consider independent sorts or dependent sorts (e.g., first on β^M and then by β^θ within each market beta quintile), stocks with low loadings on the labor market tightness factor significantly outperform stocks with high loadings.

D. Controlling for Components of Labor Market Tightness and for Industrial Production

Labor market tightness is composed of three components: vacancy index, unemployment rate, and labor force participation rate. The negative relation between labor market tightness loadings and future stock returns can plausibly be driven by just one of these components, rather than the combination of them, that is the labor market tightness. It could also be driven by changes in industrial production, with which labor market tightness is highly correlated (see Table 1). To explore whether this is the case, we first estimate loadings from a two-factor regression of stock excess returns on market excess returns and log changes in either the vacancy index (β^{Vac}), the unemployment rate (β^{Unemp}), the labor force participation rate (β^{LFPR}), or the industrial production (β^{IP}). Following the methodology used in the main body of the paper, we next study future performance of portfolios formed on the basis of these loadings and also run Fama-MacBeth regressions of annual stock returns on the lagged loadings and control variables. Tables [IA.IV](#) and [IA.V](#) show that none of the considered loadings relate robustly to future equity returns. Loadings on the vacancy factor relate negatively but weakly to future stock returns, and loadings on the unemployment rate factor relate positively but also weakly. There is no convincing evidence that loadings on either the labor force participation factor or the industrial production factor relate to future returns. Overall, the results suggest that the inverse relation between labor market tightness loadings and future stock returns is not driven by vacancies, unemployment rates, or labor force participation rates alone, but rather by their interaction: the labor market tightness.

E. Loadings on 48 Industry Portfolios

In Table [IA.VI](#), we summarize labor market tightness statistics for the 48 value-weighted industry portfolios from Ken French’s data library. We report average conditional betas from rolling three-year regressions, their corresponding standard deviations, and the fractions of months an industry falls into the high or the low β^θ quintiles. Differences in loadings on labor market tightness across industries are small, with average conditional betas falling in a tight range from -0.097 (Precious Metals) to 0.071 (Real Estate). All industries exhibit significant time variation in β^θ , suggesting that industry return sensitivities to changes in labor market tightness vary strongly over time, conceivably in response to changes in the underlying economics of the industry. For example, the Precious Metals industry has the lowest average conditional loading but it still falls in the top β^θ quintile 21% of the time. Overall, the results suggest considerable

heterogeneity and time variation in loadings on labor market tightness across industries.

Table IA.I. Performance of Labor Market Tightness Portfolios: Controlling for Liquidity and Profitability Factors

This table reports average raw returns and alphas, in percent per month, and five-factor betas for the ten portfolios of stocks sorted on the basis of their loadings on the labor market tightness factor, as well as for the portfolio that is long the low decile and short the high one. In Panel A, all alphas are computed by including the Pastor-Stambaugh liquidity factor (LIQ). In Panel B, all alphas are computed by including the Novy-Marx profitability factor (PMU). The bottom row of each Panel gives t -statistics for the low-high portfolio. Firms are assigned into deciles at the end of every month and the value-weighted portfolios are held without rebalancing for 12 months. Conditional alphas are based on either Ferson and Schadt (FS) or Boguth, Carlson, Fisher, and Simutin (BCFS). The sample period is January 1968 to December 2012 in Panel A, and July 1963 to December 2012 in Panel B.

A. Controlling for Pastor-Stambaugh liquidity factor											
Decile	Raw	Uncond. Alphas: Liquidity +			Cond. Alphas		5-Factor Loadings				
	Return	Market	3-Factor	4-Factor	FS	BCFS	MKT	HML	SMB	UMD	LIQ
Low	1.00	0.01	0.05	0.03	0.05	0.05	1.17	-0.14	0.38	0.02	0.02
2	1.03	0.13	0.12	0.13	0.11	0.11	1.05	0.01	-0.02	-0.01	0.04
3	0.97	0.10	0.08	0.10	0.08	0.08	0.99	0.06	-0.09	-0.02	0.03
4	0.96	0.11	0.09	0.09	0.09	0.09	0.97	0.08	-0.12	-0.01	0.03
5	0.92	0.10	0.04	0.03	0.07	0.06	0.96	0.15	-0.11	0.01	0.01
6	0.93	0.11	0.08	0.05	0.09	0.08	0.97	0.10	-0.11	0.03	-0.02
7	0.89	0.08	0.07	0.08	0.06	0.06	0.97	0.04	-0.07	-0.01	-0.06
8	0.87	0.03	0.02	0.07	0.02	0.03	1.01	0.00	0.03	-0.06	-0.08
9	0.73	-0.16	-0.15	-0.09	-0.14	-0.12	1.12	-0.08	0.19	-0.07	-0.10
High	0.53	-0.43	-0.40	-0.32	-0.35	-0.34	1.16	-0.22	0.64	-0.09	-0.12
Low-High	0.47	0.44	0.45	0.34	0.40	0.39	0.01	0.08	-0.25	0.11	0.14
t -statistic	[2.82]	[2.62]	[2.67]	[2.03]	[2.41]	[2.35]	[0.24]	[1.29]	[-4.70]	[2.99]	[3.15]

B. Controlling for Novy-Marx profitability factor											
Decile	Raw	Uncond. Alphas: Profitability +			Cond. Alphas		5-Factor Loadings				
	Return	Market	3-Factor	4-Factor	FS	BCFS	MKT	HML	SMB	UMD	PMU
Low	1.05	0.04	0.09	0.07	0.06	0.06	1.17	-0.15	0.39	0.02	-0.10
2	1.03	0.12	0.09	0.09	0.10	0.09	1.05	0.04	-0.01	0.00	0.08
3	0.99	0.12	0.07	0.08	0.10	0.09	1.00	0.09	-0.08	-0.02	0.09
4	0.97	0.11	0.06	0.07	0.10	0.09	0.97	0.10	-0.11	0.00	0.08
5	0.93	0.09	0.00	-0.01	0.07	0.06	0.96	0.17	-0.11	0.01	0.09
6	0.93	0.07	0.01	-0.02	0.06	0.05	0.98	0.13	-0.11	0.03	0.11
7	0.89	0.00	-0.03	-0.03	-0.01	-0.01	0.98	0.07	-0.07	-0.01	0.12
8	0.89	-0.03	-0.06	-0.02	-0.04	-0.02	1.02	0.03	0.02	-0.05	0.12
9	0.77	-0.20	-0.17	-0.10	-0.18	-0.15	1.12	-0.10	0.17	-0.07	-0.04
High	0.59	-0.42	-0.35	-0.26	-0.37	-0.36	1.16	-0.29	0.64	-0.09	-0.22
Low-High	0.45	0.47	0.44	0.33	0.43	0.42	0.02	0.13	-0.25	0.12	0.13
t -statistic	[2.91]	[2.96]	[2.78]	[2.05]	[2.77]	[2.67]	[0.46]	[2.25]	[-4.86]	[3.26]	[1.80]

Table IA.II. Post-Ranking Betas of Labor Market Tightness Portfolios

This table reports post-ranking β^θ loadings of the labor market tightness portfolios. Firms are assigned into deciles by β^θ at the end of every month and the value-weighted portfolios are held without rebalancing for 12 months. Excess returns of each portfolio are then regressed annually on the market and the labor market tightness factors, including two [Dimson \(1979\)](#) lags to account for effects of non-synchronous trading. Betas are averaged across the years to obtain average β^θ loadings for each portfolio. Panel A reports results for decile portfolios, and Panel B for quintile ones. The last column of each Panel shows the differences in post-ranking betas of the bottom and top groups. The sample period is 1954 to 2012.

A. Post-ranking betas of decile portfolios											
	Low	2	3	4	5	6	7	8	9	High	Low - High
β^θ	-0.07	-0.13	-0.13	-0.01	0.00	-0.06	0.13	0.04	0.15	0.18	-0.25

B. Post-ranking betas of quintile portfolios						
	Low	2	3	4	High	Low - High
β^θ	-0.10	-0.06	-0.01	0.06	0.10	-0.20

Table IA.III. Performance of Portfolios Sorted by Loadings on Market and Labor Market Tightness Factors

This table reports average excess returns, in percent per month, for the quintile portfolios of stocks sorted on the basis of their loadings on labor market tightness and market factors, as well as for the portfolio that is long the low quintile and short the high quintile. Firms are assigned into groups at the end of every month and the value-weighted portfolios are held without rebalancing for 12 months. The bottom row and the last column of each Panel give t -statistics for the low-high portfolios. The sample period is 1954 to 2012.

	Low β^M	2	3	4	High β^M	Low-High β^M	
A. Independent sorts							
Low β^θ	0.62	0.64	0.51	0.47	0.30	0.32	[1.79]
2	0.64	0.54	0.54	0.39	0.23	0.41	[2.53]
3	0.58	0.56	0.47	0.34	0.13	0.45	[2.69]
4	0.61	0.51	0.42	0.25	0.20	0.41	[2.43]
High β^θ	0.28	0.34	0.17	0.11	-0.03	0.31	[1.69]
Low-High β^θ	0.34	0.29	0.34	0.36	0.33		
t -statistic	[2.27]	[2.31]	[2.61]	[2.73]	[2.60]		
B. Conditional sorts: first on β^θ, then on β^M							
Low β^θ	0.63	0.60	0.44	0.39	0.30	0.33	[1.78]
2	0.64	0.56	0.56	0.44	0.31	0.33	[2.23]
3	0.59	0.59	0.47	0.38	0.21	0.39	[2.66]
4	0.62	0.53	0.38	0.29	0.17	0.45	[2.86]
High β^θ	0.31	0.31	0.04	0.04	-0.04	0.35	[1.80]
Low-High β^θ	0.32	0.29	0.40	0.35	0.34		
t -statistic	[2.21]	[2.32]	[2.95]	[2.64]	[2.49]		
C. Conditional sorts: first on β^M, then on β^θ							
Low β^θ	0.70	0.60	0.51	0.44	0.27	0.43	[2.34]
2	0.63	0.53	0.55	0.41	0.19	0.44	[2.61]
3	0.53	0.54	0.45	0.32	0.18	0.35	[2.03]
4	0.62	0.53	0.44	0.22	0.14	0.48	[2.79]
High β^θ	0.32	0.39	0.24	0.10	-0.16	0.49	[2.39]
Low-High β^θ	0.37	0.21	0.26	0.34	0.43		
t -statistic	[2.70]	[1.91]	[2.24]	[2.53]	[2.95]		

Table IA.IV. Performance of Portfolios Sorted by Loadings on Components of Labor Market Tightness and Industrial Production

This table reports four-factor alphas, in percent per month, for the ten portfolios of stocks sorted on the basis of β^{Vac} , β^{Unemp} , β^{LFPR} , and β^{IP} , which are loadings from two-factor regressions of stock excess returns on market excess returns and log changes in either the vacancy index, the unemployment rate, the labor force participation rate, or industrial production, respectively. The bottom two rows show the alphas and the corresponding t -statistics for the portfolio that is long the low decile and short the high one. Firms are assigned into groups at the end of every month and the value-weighted portfolios are held without rebalancing for 12 months. The sample period is 1954 to 2012.

Decile	Four-factor alphas of portfolios sorted by			
	β^{Vac}	β^{Unemp}	β^{LFPR}	β^{IP}
Low	0.02	-0.22	-0.05	-0.07
2	0.12	-0.08	-0.02	0.07
3	0.11	0.01	0.02	0.04
4	0.02	0.02	0.03	0.10
5	0.02	0.11	0.03	0.05
6	0.01	0.11	0.04	0.04
7	0.10	0.04	0.04	0.02
8	0.02	0.09	0.08	-0.07
9	-0.05	0.13	0.10	-0.07
High	-0.22	-0.01	0.08	-0.07
Low-High	0.24	-0.20	-0.13	-0.01
t -statistic	[1.58]	[-1.33]	[-0.89]	[-0.06]

Table IA.V. Fama-MacBeth Regressions of Annual Stock Returns

This table reports the results of annual Fama-MacBeth regressions. Stock returns from July to June are regressed on lagged market betas (β^M) and loadings from two-factor regressions of stock excess returns on market excess returns and log changes in either labor force participation rate, unemployment rate, vacancy index, or industrial production (β^{LFPR} , β^{Unemp} , β^{Vac} , or β^{IP} , respectively). Regressions (7) to (12) also control for log market equity, log of the ratio of book equity to market equity, 12-month stock return, hiring rates, investment rates, and asset growth rates. Reported are average coefficients and the corresponding [Newey and West \(1987\)](#) t -statistics. Details of variable definitions are in Appendix A. The sample period is 1960 to 2012.

Reg	β^M	β^{LFPR}	β^{Unemp}	β^{Vac}	β^{IP}	Controls
(1)	0.001 [0.04]	0.001 [1.19]				No
(2)	0.000 [-0.01]		0.011 [1.51]			No
(3)	0.001 [0.05]			-0.007 [-1.13]		No
(4)	0.001 [0.09]				-0.001 [-0.43]	No
(5)	0.000 [-0.00]	0.001 [1.08]	0.015 [1.51]	0.002 [0.35]		No
(6)	0.001 [0.05]	0.001 [1.35]	0.015 [1.15]	0.004 [0.48]	0.000 [-0.10]	No
(7)	0.012 [0.86]	0.001 [1.54]				Yes
(8)	0.011 [0.81]		0.021 [2.21]			Yes
(9)	0.011 [0.77]			-0.024 [-3.36]		Yes
(10)	0.012 [0.86]				-0.004 [-1.04]	Yes
(11)	0.011 [0.79]	0.001 [1.21]	0.012 [1.14]	-0.017 [-1.25]		Yes
(12)	0.012 [0.86]	0.001 [1.34]	0.012 [0.96]	-0.015 [-1.06]	0.000 [-0.11]	Yes

Table IA.VI. Loadings of 48 Industry Portfolios on Labor Market Tightness

This table reports average and standard deviation of conditional loadings on the labor market tightness factor for industry portfolios. Loadings are computed as in regression (3.3), based on rolling three-year windows. The last two columns show the fraction of months each industry was assigned to the low and high β^θ quintiles. Definitions of the 48 industries are from Ken French's data library. The sample period is 1954 to 2012 for all industries except Candy & Soda (1963 to 2012), Defense (1963 to 2012), Fabricated Products (1963 to 2012), Healthcare (1969 to 2012), and Precious Metals (1963 to 2012).

Industry	Average cond β^θ	Standard dev of β^θ	Fraction of months in	
			low β^θ quintile	high β^θ quintile
Precious Metals	-0.097	0.612	0.595	0.211
Tobacco Products	-0.086	0.211	0.416	0.152
Beer & Liquor	-0.063	0.162	0.337	0.060
Utilities	-0.047	0.106	0.271	0.047
Communication	-0.031	0.103	0.123	0.118
Banking	-0.030	0.151	0.287	0.094
Candy & Soda	-0.025	0.187	0.267	0.239
Business Services	-0.023	0.113	0.075	0.079
Food Products	-0.022	0.114	0.213	0.094
Coal	-0.019	0.272	0.355	0.313
Electronic Equipment	-0.019	0.133	0.166	0.152
Shipping Containers	-0.018	0.112	0.137	0.123
Medical Equipment	-0.018	0.155	0.278	0.136
Computers	-0.016	0.175	0.220	0.235
Chemicals	-0.008	0.083	0.073	0.144
Almost Nothing	-0.006	0.220	0.208	0.186
Insurance	0.000	0.133	0.204	0.114
Petroleum and Natural Gas	0.001	0.136	0.220	0.144
Agriculture	0.002	0.206	0.319	0.201
Pharmaceutical Products	0.004	0.128	0.152	0.169
Retail	0.005	0.106	0.069	0.157
Steel Works Etc	0.005	0.149	0.209	0.166
Consumer Goods	0.006	0.093	0.032	0.079
Transportation	0.006	0.120	0.129	0.112
Printing and Publishing	0.007	0.152	0.161	0.220
Construction	0.008	0.173	0.274	0.213
Entertainment	0.010	0.165	0.260	0.209
Fabricated Products	0.011	0.240	0.274	0.277
Personal Services	0.012	0.178	0.202	0.202
Restaraunts, Hotels, Motels	0.013	0.149	0.159	0.233
Trading	0.017	0.113	0.066	0.158
Defense	0.019	0.212	0.284	0.274
Electrical Equipment	0.020	0.093	0.042	0.194
Construction Materials	0.021	0.119	0.065	0.105
Shipbuilding, Railroad Equipment	0.021	0.204	0.262	0.244
Aircraft	0.024	0.128	0.224	0.195
Machinery	0.027	0.103	0.043	0.132
Recreation	0.028	0.247	0.188	0.295
Rubber and Plastic Products	0.031	0.125	0.121	0.216
Business Supplies	0.033	0.119	0.116	0.230
Measuring and Control Equipment	0.036	0.121	0.141	0.230
Apparel	0.038	0.142	0.078	0.190
Healthcare	0.040	0.273	0.297	0.291
Automobiles and Trucks	0.042	0.117	0.069	0.425
Non-Metallic and Industrial Metal Mining	0.042	0.202	0.267	0.258
Wholesale	0.043	0.114	0.026	0.134
Textiles	0.056	0.140	0.073	0.284
Real Estate	0.071	0.195	0.127	0.355

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