Essays on International Finance

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Abstract

I study the exchange rate disconnect puzzle in a two-country DSGE framework that features a financial intermediation sector. An intermediary is subject to two types of financing constraints: 1. a segmented deposit market restricted to local households, and 2. a balance-sheet constraint. These two constraints drive a wedge between marginal decisions of home and foreign intermediaries, which in turn, breaks the link between exchange rates and consumption differences in the Backus-Smith relationship. In contrast to traditional models which find a tight link between exchange rate growth and the consumption growth rate differential, the calibrated model produces a correlation of around -0.16, reconciling the model with the empirical evidence.

We study asset prices, exchange rates, and consumption dynamics in a general equilibrium two-county macro-finance model that features limited stock market participation as well as non-traded goods and distribution cost. The model generates a high price of risk, smooth exchange rates, and makes substantial progress towards explaining the empirically observed low consumption growth correlation between countries. We find that distribution cost plays a central role for reducing international consumption co-movement while also amplifying risk premia.

Chapter 1:
Intermediary-Determined Exchange Rates

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Abstract

I study the exchange rate disconnect puzzle in a two-country DSGE framework that features a financial intermediation sector. An intermediary is subject to two types of financing constraints: 1. a segmented deposit market restricted to local households, and 2. a balance-sheet constraint. These two constraints drive a wedge between marginal decisions of home and foreign intermediaries, which in turn, breaks the link between exchange rates and consumption differences in the Backus-Smith relationship. In contrast to traditional models which find a tight link between exchange rate growth and the consumption growth rate differential, the calibrated model produces a correlation of around -0.16, reconciling the model with the empirical evidence.

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1 Introduction

A prominent and long-standing puzzle in international finance since the introduction of floating exchange rates is that exchange rates seem largely disconnected from macroeconomic fundamentals. Exchange rates move around independently from major macroeconomic variables that are in theory, connected to exchange rates. Exchange rates seem to have a "life of their own," and whether or not exchange rate models can outperform a simple random walk model has been a subject of intense debate.[1] Such observations have led Obsfeld and Rogoff (2001) to label the puzzle as one of the six main puzzles in international macroeconomics, calling for macro models that can better explain exchange rate behavior.

Exchange-rate disconnect manifests itself in a variety of different ways, depending on how one thinks of "disconnect," and which macro fundamentals one is interested in. The well-known Backus-Smith puzzle is a statement about exchange rates being disconnected from cross-country consumption differences, in which the exchange rates are tied to the cross-country consumption differences through first-order conditions of representative households. It stems from a complete markets assumption and time separable utility of consumption. It alludes to the discrepancy first documented in Backus-Smith (1993), namely that the high implied correlation between the exchange rate growth and the ratio of two countries' consumption growth rates is vastly at odds with the data.

It is worth emphasizing that at the heart of the Backus-Smith puzzle lies the assumption of who the marginal investors are. As the puzzle is about the discrepancy that arises when one assumes the marginal investors to be representative households and links the exchange rate growth to the ratio of their stochastic discount factors (SDF) , one can think about how the puzzle can be resolved if the marginal investors were not households so that the exchange rate growth is tied to the SDFs of different type of marginal investors.

In this paper, I ask whether the Backus-Smith puzzle is related to the fact that, in reality, virtually every foreign-currency transaction is *intermediated* by a bank of some type.

[1] See, for example, Mussa (1976), Frankel and Froot (1986), Taylor and Allen (1992), and Frankel and Rose (1995).

If banks are marginal investors, the exchange rate growth would then be linked to the SDFs of banks. Traditionally, however, financial intermediaries were treated as a "veil," in which intermediation is frictionless and intermediary decisions perfectly mirror those of households. In such a case, the intermediaries' marginal decisions on asset choices would coincide with the households', were the households allowed to invest directly. Thus, even with the intermediaries being marginal, it further requires that intermediation is subject to some friction so that their SDFs are different from those of households that delegate their decisions to the intermediaries. I consider such a setting in my model.

Specifically, I consider a model in which banks as as specialists and intermediate investments in risky assets with riskless deposits taken from households. The banks are subject to a financing constraint where their borrowing capacity is limited by their net worths. The borrowing constraint introduces a wedge from the original household pricing kernel, and provides a source for explaining the low correlation between exchange rates and aggregate consumption.

Such a modeling choice adopted for the financial sector in my study is also motivated from recently developed literature on intermediary asset pricing. The literature points out that traditional models of asset pricing is based on the assumption that everyone is alike and equally sufficiently sophisticated to participate in all asset markets and carry out complex trading strategies. In reality, a large share of the investments are intermediated through specialists. As most foreign-currency transactions are intermediated, assets involving exchange rates fit this description perfectly.

This line of research has been spurred by the recent 2007 financial crisis which was characterized by a significant disruption of the financial intermediation sector. Since the crisis, there has been a burgeoning literature where banks' financial health plays a central role in asset pricing. There is a shared view, as well as empirical evidence, in this literature that recent financial crises are characterized by dramatic spikes in the risky premia, and that they were closely related to the sudden deterioration of the banking sector's ability to borrow. As banks' balance sheet weakened, their risk taking capacity dropped and risk premia spiked. As such, models in this literature feature banks as marginal investors where their intermediation capacity is limited by their net worths due to financing constraints. The resulting SDFs of banks are different from those of households, in which the borrowing constraint introduces a wedge that provides a source of variation to price assets that the banks are marginal with. Such models have been applied extensively to a wide range of asset classes and shown to be successful in explaining asset dynamics during banking crises episodes.[2]

Adrian, Etula, and Muir (2014) take this a step further to show that the intermediary pricing kernel explains the cross-section of asset returns such as stock and bond returns, as they are that their single factor model based on intermediary pricing kernel outperforms known multi-factor pricing models. Muir (2014) documents risk premia spike dramatically in banking crisis episodes and not so much during other types of recessions. He also shows that the movements of consumption and consumption volatility cannot account for these risk premia, whereas the net worth of the banking sector has a strong forecasting power for stock and bond returns. All of these findings suggest that the overall health of the banking sector is uniquely important as a state variable for asset pricing, and that it is tied to risk premia unconditionally as well and not just during extreme crisis times.

To summarize, the financial crises and the ensuing literature on intermediary asset pricing provides support for the specification of intermediary pricing kernel that depends on how constrained they are. The constraint creates a wedge from the pricing kernels of households that only depends on aggregate consumption.

By the same token, intermediaries should be the marginal decision makers on asset choices involving exchange rates. If so, it might break the tight link between consumption growth and exchange rate growth in the Backus-Smith puzzle, as the representative household's consumption growth rate no longer prices asset returns involving exchange rate growth.

The Backus-Smith equation is typically derived from equating the home and foreign households' marginal values of an asset return, denominated in common currency units.

[2]See, for example, Brunnermeire and Pedersen (2008), He and Krishnamurthy (2011), Maggiori (2012), Adrian, Etula, and Muir (2013), and Muir (2014) to name a few.

Similarly, to resolve the puzzle in my model, a natural starting point should be the equations for the marginal valuations of a common risky asset by home and foreign *intermediaries*. The existence of the balance-sheet constraint, coupled with an assumption of local deposit financing, leads to a wedge between the two marginal values implied by home and foreign intermediaries.

Dedola, Karadi, and Lombardo (2013) and previous studies using a similar framework have pointed to financial market integration as a source of international spillovers of country-specific shocks. Specifically, as intermediaries frictionlessly take offshore deposits and invest overseas, their balance-sheet conditions are highly synchronized. As a result, when a negative shock hits one country, their domestic intermediaries' balance sheets are constrained, leading the foreign country's intermediaries' balance sheets to tighten. Hence, shocks spill over through intermediaries' balance-sheet conditions.

For the Backus-Smith puzzle, however, I find that segmentation on the deposit side, as opposed to perfect integration, plays a crucial role by creating the wedge mentioned above. Roughly speaking, the presence of balance-sheet constraint makes the intermediaries a "different" marginal investor from the households that delegate their decisions. What I show in this study is that there needs to be some level of difference *between intermediaries* across countries, in order to explain the Backus-Smith puzzle in this framework. In addition to the local deposit financing, a key element comes from the shock to the balance-sheet constraint on intermediaries, which is meant to capture "financial" shocks originating from the intermediary sector. It will be shown that this shock needs to be sufficiently volatile to amplify the wedge, thereby inducing the FX disconnect.

Quantitatively, the baseline model generates a correlation of -0.16 between the growth of real exchange rates and the ratio of consumption growth rates, calibrated to the U.S. and Canada pair, successfully reconciling the model with the data. An extended version of the baseline model is also considered, in which money is introduced via a cash-in-advance (CIA) constraint. The extended model also produces a correlation between *nominal* exchange rates and the consumption difference ratio (in this case adjusted by the cross-country inflation differential) around 0.21, making substantial progress along both the nominal and real dimensions of the Backus-Smith puzzle. The remainder of the paper is organized as follows. Section 2 recasts the Backus-Smith relation in light of FX disconnect. Section 3 develops the model in detail and discusses how intermediaries' marginal decisions and constraints shed light on resolving the puzzle. Section 4 provides analysis of the quantitative results. Section 5 extends the framework by considering money. Section 6 concludes.

2 Intermediation and the Backus-Smith Puzzle

This section reviews the Backus-Smith equation as it is the central focus of this paper. First, we start from the first-order conditions of home and foreign representative agents on the common asset they can frictionlessly trade in,

$$E_t(M_{t+1}\frac{\mathbb{S}_{t+1}}{\mathbb{S}_t}\mathcal{R}^*_{t+1}) = E_t(M^*_{t+1}\mathcal{R}^*_{t+1}), \tag{2.0.1}$$

where M_{t+1} is the home agent's SDF, M^*_{t+1} is the foreign agent's SDF, and $\mathcal{R}^*_{t+1}$ is the return on the asset they can both invest in, denominated in units of foreign numeraire. $\mathbb{S}_t$ is the exchange rate defined as the number of home numeraire units per unit of the foreign numeraire. Note that the exposition of the SDFs, exchange rate, and the asset return is quite general. M_{t+1} and M^*_{t+1} can denote either the households' SDFs or intermediaries' SDFs, depending on who are the marginal investors in (2.0.1). $\mathbb{S}_t$ can take both nominal and real values, as long as the SDFs and the return are defined accordingly.

The Backus-Smith puzzle stems from the implication of (2.0.1) after we impose more structure. Each country is populated by homogenous households. If the households make marginal decisions with standard CRRA utility and complete markets, Equation (2.0.1) becomes,

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{\mathbb{S}_{t+1}}{\mathbb{S}_t} = \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}, \tag{2.0.2}$$

where γ is the relative risk aversion (RRA) coefficient. The SDFs are expressed in terms of consumption growths, which are then equalized up to the exchange rate growth, in all future states of the economy. Taking logs on both sides we have,

$$\Delta \mathbf{s}_{t+1} = \gamma(\Delta c_{t+1} - \Delta c^*_{t+1}), \tag{2.0.3}$$

using lowercase for logs. Clearly, exchange rates move in tandem with the consumption growth differential.

The perfect correlation between exchange rates and consumption differential in (2.0.3) is technically a product of the complete market assumption. However, without the complete market assumption, (2.0.1) implies the SDFs (after adjusting for different units) are equalized up to first order in expectation. This implies,

$$\Lambda_{t+1}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \approx \Lambda^*_{t+1},$$

where Λ_{t+1} denotes the household SDF, assuming the households as marginal investors. Expressing $\Lambda_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ with the CRRA utility again, the resulting correlation between exchange rates and consumption differential is lower than one. Most traditional incomplete models, however, fail to generate substantial deviation from the complete market case, and produce correlations that are still close to one.[3]

In my baseline model, the testable implication is modified to

$$\Omega_{t+1} \times \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \approx \Omega^*_{t+1}, \tag{2.0.4}$$

where Ω_{t+1} and Ω^*_{t+1} are home and foreign *intermediary* SDF, respectively. In order to examine the implication in (2.0.4) more closely, we need to derive the marginal decisions of intermediaries, which will be addressed in the next section. The key insights to resolving the Backus-Smith puzzle will essentially come from analyzing the intermediary SDFs.

[3]With incomplete market assumption, the correlation is less than one. In many specifications of incomplete markets, however, the difference is known to be small. See Chari, Kehoe, and McGrattan (2002) and Corsetti, Dedola, and Leduc (2008) for further discussions.

3 Model

In this section, I introduce the model in detail. The model develops a framework with financial intermediation and endogenously determined exchange rates in a two-country DSGE model. The model is a version of the work of Dedola, Karadi, and Lombardo (2013) (henceforth DKL), modified to incorporate exchange rates. I also follow the models in Gertler and Karadi (2011) (henceforth GK1) and Gertler and Kiyotaki (2011) (henceforth GK2) closely. In order to have endogenously determined exchange rates, I consider two types of country-specific final goods, each produced locally. Households consume both types of goods as they have a constant elasticity of substitution (CES) preference over them.

Each country is populated by households and intermediaries, both of unit measures. There are also two types of non-financial firms, capital producers and final-good producers. Each country produces their country-specific final goods according to a Cobb-Douglas technology.

For simplicity, I refer to the first country as the "home" country and the second as the "foreign" country. For tractability, I further assume the two countries are symmetric. An asterisk ($*$) will be used to denote variables decided on by foreign agents. For example, C^* denotes the amount of basket consumed by foreign households, and M^* denotes the aggregate stock of foreign currencies. c_F denotes the amount of foreign goods consumed by home households, whereas c_H^* denotes the amount of home goods consumed by foreign households.

3.1 Model Primitives

I present first some primitives of the model. In the interest of simplicity, I will focus on the home country. The foreign economy is symmetrically defined. Households maximize their expected lifetime standard utility of,

$$E_t \sum_{\tau=t}^{\infty} B_{\tau-t} \left[\frac{C_\tau^{1-\gamma}}{1-\gamma} - \chi \frac{L_\tau^{1+\varphi}}{1+\varphi} \right], \tag{3.1.1}$$

where the period utility is derived from a consumption basket C_t, and (disutility of) labor, L_t. B_t is an endogenous discount factor à la Schmitt-Grohé and Uribe (2003), which is defined as,

$$B_{t+1} = B_t \times \beta(C_t) = B_t \times b(C_t - \bar{C} + 1)^{-\upsilon}, \tag{3.1.2}$$

where C_t is the per-capita consumption and $\bar{C}$ is its steady-state value. b and υ are chosen in a way so that, $0 < \beta(C_t) < 1$, and $\beta'(C_t) \leq 0$.[4] The multiplicative $\beta(C_t)$ is known at period t, and hence can be treated as constant conditional on the period-t expectation. For notational convenience, I will suppress $\beta(C_t)$ as β for what follows. The consumption basket is defined by the following constant elasticity of substitution (CES) aggregator:

$$C_t = (\lambda_c^{\frac{1}{\theta_c}} c_{H,t}^{\frac{\theta_c - 1}{\theta_c}} + (1 - \lambda_c)^{\frac{1}{\theta_c}} c_{F,t}^{\frac{\theta_c - 1}{\theta_c}})^{\frac{\theta_c}{\theta_c - 1}}.$$

I assume $\lambda_C > 0.5$ to capture home bias in consumption. A standard static cost minimization yields the price of the basket as,

$$P_t = (\lambda_c p_{H,t}^{1-\theta_c} + (1 - \lambda_c) p_{F,t}^{1-\theta_c})^{\frac{1}{1-\theta_c}}, \tag{3.1.3}$$

The prices are in terms of the numeraire which I assume to be the sum of half of home good and half of foreign good so that,

$$\frac{1}{2} p_{H,t} + \frac{1}{2} p_{F,t} = 1. \tag{3.1.4}$$

The exchange rate is defined as the ratio of the price of the foreign consumption basket to the price of the home consumption basket in terms of the common numeraire so that,

$$\mathcal{S}_t = \frac{P_t^*}{P_t}, \tag{3.1.5}$$

where $\mathcal{S}_t$ denotes the exchange rate.

Final output Y_t (Y_t^* for the foreign country) is produced according to a standard Cobb-

[4]In international macro models, incomplete financial markets imply a unit root in the first-order approximate solution. Endogenizing the subjective discount factor this way ensures a stationary solution. See also Corsetti, Dedola, and Leduc (2008) and Devereux and Sutherland (2011).

Douglas technology as,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{3.1.6}$$

by a continuum of perfectly competitive final-good producing firms in the home country. Capital evolves according to the standard law of motion,

$$S_t = (1-\delta)K_t + I_t, \tag{3.1.7}$$

where S_t is the capital in progress at the end of period t, to be used for production in period $t+1$. As in GK1, GK2, and DKL, the effective capital for production is determined upon realization of capital quality shock, ψ_{t+1}, at the beginning of the next period, so that,

$$K_{t+1} = \psi_{t+1} S_t. \tag{3.1.8}$$

Following GK1, GK2, and DKL, the existence of the capital quality shock serves as a channel for a direct shock to the return on risky capital to be defined later.

I_t is a CES composite of the two types of final output defined as

$$I_t = (\lambda_I^{\frac{1}{\theta_I}} i_{H,t}^{\frac{\theta_I - 1}{\theta_I}} + (1-\lambda_I)^{\frac{1}{\theta_I}} i_{F,t}^{\frac{\theta_I - 1}{\theta_I}})^{\frac{\theta_I}{\theta_I - 1}}, \tag{3.1.9}$$

where $i_{F,t}$ denotes the amount of foreign final good used as input to produce the home capital stock.

3.2 Households

There exists a representative household with a continuum of members of unit measure. As in GK1, GK2, and DKL, there are two types of members: "workers" and "bankers." In every period, each banker faces an exogenous i.i.d. survival probability of θ, so that $(1-\theta)$ of existing bankers retire. The same measure of non-banker members become bankers to keep the fraction of banker members constant. This assumption is necessary to prevent intermediaries from accumulating enough net worth to grow out of their balance-sheet constraints.

The household owns intermediaries through their banker members. It is assumed that households make deposits with intermediaries they do not own. There is perfect risk sharing among members of a household, so a banker member returns their profits back to the household they belong to. Intermediaries can raise funds from households in their own country (other than the one they belong to) only in the form of one-period riskless deposit (D_t), subject to a balance-sheet constraint to be defined later. The deposit is assumed to pay off in consumption baskets of home households.[5]

Accordingly, the representative household maximizes the lifetime expected utility in (3.1.1) subject to the following budget constraint. Note that the constraint is expressed in terms of the common numeraire.

$$P_t C_t + D_t = w_t L_t + \Pi_t + R_t D_{t-1}, \tag{3.2.1}$$

Setting up the Lagrangian of the representative household gives,

$$\begin{aligned}\mathcal{L} = {} & E_t \sum_{\tau=t}^{\infty} B_{\tau-t} \left[\frac{C_\tau^{1-\gamma}}{1-\gamma} - \chi \frac{L_\tau^{1+\varphi}}{1+\varphi} \right] \\ & + B_t \lambda_t (w_t L_t + \Pi_t + R_t D_{t-1} - P_t C_t + D_t) \\ & + B_{t+1} \lambda_{t+1} (w_{t+1} L_{t+1} + \Pi_{t+1} + R_{t+1} D_t - P_{t+1} C_{t+1} - D_{t+1}) + ...\end{aligned} \tag{3.2.2}$$

where λ_t is the multiplier on the period-t budget constraint.

The first-order condition with respect to C_t yields,

$$\lambda_t = \frac{C_t^{-\gamma}}{P_t}, \tag{3.2.3}$$

Then, the inter-temporal savings decision by the household yields,

$$E_t \beta \Lambda_{t+1} R_{t+1} = E_t \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} R_{t+1} = 1, \tag{3.2.4}$$

[5] One unit of deposit held in period $t-1$ pays the household one consumption basket in period t in all states of the economy. In other words, the deposit is riskless in terms of the basket, not in terms of the numeraire.

where $\Lambda_{t+1} \equiv \dfrac{\lambda_{t+1}}{\lambda_t}$ is the household SDF in terms of the numeraire.[6]

w_t is the wage rate that enters the labor supply equation of households

$$\chi L_t^{\varphi} = \lambda_t w_t, \tag{3.2.5}$$

which is the first-order condition of the household with respect to labor supply. Π_t in the budget constraint (3.2.1) denotes net profit distributions from owning intermediaries and capital-producing firms, both of which will be specified in the next subsections.

3.3 Non-Financial Firms

Final-good firms in the home country produce Y_t as in (3.1.6) and sell to home and foreign households and capital-good firms. Due to perfect competition, they choose labor and capital inputs so that wage and rent on capital (Z_t) are determined by their respective marginal products as

$$w_t = (1-\alpha) p_{H,t} \frac{Y_t}{L_t},$$

and

$$Z_t = \alpha p_{H,t} \frac{Y_t}{K_t}.$$

The final-good firms must acquire capital from capital-good firms. Since the final-good firms make zero profit owing to perfect competition, they must finance these purchases by taking loans from intermediaries. This is the channel through which intermediaries invest in risky capital, as they get claims on all the future marginal products of capital. Put differently, intermediaries effectively own the capital stock. Note that both home and foreign intermediaries can invest in the home capital stock.

Capital-good firms use both home and foreign final output (Y_t and Y_t^*) to produce I_t, a CES composite of the two types of final output defined in (3.1.9).

The cost to produce a unit of I_t, $P_{I,t}$, is also determined by static cost minimization so

[6] Note, in terms of *consumption baskets,* (3.2.4) can be rewritten as $E_t \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} r_t = 1$, where now r_t is the price-adjusted return in terms of baskets which is known in period t.

that,

$$P_{I,t} = (\lambda_I p_{H,t}^{1-\theta_I} + (1-\lambda_I)(p_{F,t})^{1-\theta_I})^{\frac{1}{1-\theta_I}}. \tag{3.3.1}$$

As in GK1, GK2, and DKL, I assume convex adjustment costs of investment $\Phi_t(\frac{I_t}{I_{t-1}})$, so that $\Phi_t(\cdot)I_t$ of home final output is used for adjustment as capital-good firms change $\frac{I_t}{I_{t-1}}$. A capital-good firm chooses I_t to maximize discounted profits:

$$\max E_t \sum_{\tau=t}^{\infty} B_{\tau-t}\Lambda_{t,\tau} \left[Q_t I_t - P_{I,t} I_t - p_{H,t}\Phi_t(\frac{I_t}{I_{t-1}})I_t \right], \tag{3.3.2}$$

where Q_t is the price of a unit of I_t the capital-good firm sells to a final-good firm. Thus, $\left[Q_t I_t - P_{I,t} I_t - p_{H,t}\Phi_t(\frac{I_t}{I_{t-1}})I_t\right]$ is one component of the Π_t term in (3.2.1).

The gross rate of return $R_{k,t}$ on a unit of home risky capital denominated in the home currency, earned by intermediaries from $t-1$ to t is given by,

$$R_{k,t} = \psi_t \frac{Z_t + Q_t(1-\delta)}{Q_{t-1}}. \tag{3.3.3}$$

The return on a unit of foreign risky capital, denominated in the foreign currency is given by,

$$R^*_{k,t} = \psi^*_t \frac{Z^*_t + Q^*_t(1-\delta)}{Q^*_{t-1}}. \tag{3.3.4}$$

3.4 The Intermediary's Problem

For modeling the intermediary sector, I closely follow the setup in GK1 and DKL.

Financial intermediaries take deposit from households and invest in capital stocks of the home and foreign countries. After configuring the asset holdings and deposit, the intermediary's balance sheet becomes:

$$\mathcal{W}_t = N_t + D_t, \tag{3.4.1}$$

where $\mathcal{W}_t$ is the total asset held by the intermediary, N_t is its internal net worth, and D_t is the deposit from households.

The total asset can be written as

$$\mathcal{W}_t \equiv Q_t s_t^h + Q_t^* s_t^f, \tag{3.4.2}$$

where Q_t (Q_t^*) is the price of a unit of home (foreign) risky capital. s_t^h is the amount of home capital stock and s_t^f is the amount of foreign capital stock, each held by the home intermediary. The asset holdings by foreign intermediaries are denoted by s_t^{h*} and s_t^{f*}, respectively.

Assuming an intermediary can take deposit only from local households, the net worth N_t of the intermediary can be expressed as the difference between earnings on risky assets and repayments of household deposit so that,

$$N_t = \left[R_{k,t-1} Q_{t-1} s_{t-1}^h + R_{k,t-1}^* Q_{t-1}^* s_{t-1}^f - R_t D_{t-1}\right], \tag{3.4.3}$$

where again R_t is the deposit rate from $t-1$ to t. $R_{k,t}$ is the return on home capital and $R_{k,t}^*$ is the return on foreign capital, as previously defined in (3.3.3) and (3.3.4). Using Equation (3.4.1) and (3.4.2), (3.4.3) can be rewritten as the following law of motion for the evolution of net worth,

$$N_t = \left[(R_{k,t} - R_t) + \frac{Q_{t-1}^* s_{t-1}^f}{\mathcal{W}_{t-1}}(R_{k,t}^* - R_{k,t})\right] \mathcal{W}_{t-1} + R_t N_{t-1}. \tag{3.4.4}$$

Recall that bankers face an i.i.d. survival probability of θ each period so that the balance-sheet constraint is always binding. As a result of the binding balance-sheet constraint, an intermediary faces positive economic spreads between return on the risky loan they make and the return on deposit to households. Thus, it is in the best interest of its household that the intermediary reinvests all of its retained earnings until the time of its exit. All of the accumulated net worth of an intermediary is returned to its household only once upon its exit. Accordingly, the objective of a banker at the end of period t is to maximize the intermediary's present value of its terminal net worth for its household so that,

$$V_t = \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i B_{i+1} \Lambda_{t,t+1+i} (N_{t+i+1}), \tag{3.4.5}$$

$$= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i B_{i+1} \Lambda_{t,t+1+i} \Big[(R_{k,t+1+i} - R_{t+1+i}) \mathcal{W}_{t+i} + Q^*_{t+i} s^f_{t+i} (R^*_{k,t+1+i} - R_{k,t+1+i}) + R_{t+1+i} N_{t+i} \Big], \tag{3.4.6}$$

where V_t is the maximized value of the intermediary.

Finally, the balance-sheet constraint is introduced as follows.

$$V_t \geq \kappa_t \mathcal{W}_t. \tag{3.4.7}$$

This constraint is motivated by an agency problem where the banker can steal the assets of the intermediary. The constraint is given as an incentive compatibility constraint on the banker such that the banker can divert a fraction of the bank's assets, after the asset holdings have been configured at the end of the period. The maximized value of the bank should be higher than the divertible fraction in order for a banker to remain operating.[7]

The fraction κ_t is assumed to be stochastic, following a mean-reverting exogenous process as,

$$\log \kappa_t = (1-\rho_\kappa) \log \bar{\kappa} + \rho_\kappa \log \kappa_{t-1} + \varepsilon_{\kappa,t}. \tag{3.4.8}$$

The variable κ_t is meant to capture a "financial shock" which, unlike the productivity shock, originates from the financial sector. The recent financial crises have featured the overall health of the banking sector as the source, where the depositors' perceived risk of getting their deposits repaid played a significant role. The idea behind this assumption is that depositors' view of the health of the intermediary sector varies over time. As illustrated in DKL, an unexpected positive shock to κ_t can be interpreted as depositors' sudden loss of confidence in the ability of the intermediary to protect their deposit, as witnessed during

[7] The timing assumption of the "stealing" is that it happens after the asset and deposit holdings are configured. GK1 and DKL provide a rationale for this assumption by describing the stealing as happening "during the night."

the 2007 financial crisis. The justification for the modeling of the financial shock also comes from recent works in the literature on financial accelerator and intermediary asset pricing. These papers provide evidence for the existence as well as the significance of the separate financial shock process, different from the productivity shock process. In his working paper, Muir (2014) documents that risk premia increase significantly during financial crisis episodes. Jermann and Quadrini (2010) show in their paper that the financial shock is important for understanding the movements of various macro variables and the business cycle. In my paper, the financial shock process is a key element in resolving the Backus-Smith puzzle, as will be made clear in the following sections.

To solve the problem of the intermediary, we first rewrite the objective function in (3.4.5) recursively and apply a standard guess-and-verify method.

Equation (3.4.5) can be written recursively as,

$$V_t = \max E_t \beta \Lambda_{t+1} \left[(1-\theta) N_{t+1} + \theta V_{t+1} \right]. \tag{3.4.9}$$

We can solve the intermediary's problem by a standard guess-and-verify method as in GK2 and DKL. Guess a linear solution V_t in the holdings of assets and deposit as,

$$V_t = V_{sh,t} Q_t s_t^h + V_{sf,t} Q_t^* s_t^f - \eta_t D_t. \tag{3.4.10}$$

We can show that the optimal choice on the holdings of the risky assets gives $V_{sh,t} = V_{sf,t}$ and $V_t = V_{sh,t}\mathcal{W}_t - \eta_t D_t = (V_{sh,t} - \eta_t)\mathcal{W}_t + \eta_t N_t$. Plugging the expression in (3.4.9), and after matching the undermined coefficients assuming (4.1.4) binds near the steady state, we can derive the following optimality conditions for an intermediary.

$$E_t(\beta \Omega_{t+1} R_{t+1}) = \eta_t, \tag{3.4.11}$$

$$E_t \left[\beta \Omega_{t+1} (R_{k,t+1} - R_t) \right] = V_{sh,t} - \eta_t \equiv \nu_t > 0, \tag{3.4.12}$$

$$\Omega_{t+1} = \Lambda_{t+1} \left[1 + \theta(\eta_{t+1} + \nu_{t+1}\phi_{t+1} - 1) \right], \tag{3.4.13}$$

where ϕ_t is the leverage ratio of the intermediary such that,

$$\mathcal{W}_t = \frac{\eta_t}{\kappa_t - \nu_t} N_t = \phi_t N_t. \tag{3.4.14}$$

Note that Ω_{t+1} is the household SDF Λ_{t+1}, scaled by the $[1 + \theta(\eta_{t+1} + \nu_{t+1}\phi_{t+1} - 1)]$ term, and can be interpreted as the effective SDF of the intermediary. Hence, η_t is the marginal value of deposit for the intermediary. ν_t denotes the marginal value of the economic spread between risky assets and the riskless deposit earned by the intermediary. Absent the financing friction, Ω_{t+1} collapses to Λ_{t+1}, as η_t becomes one and ν_t becomes zero.

From (3.4.14) we can observe that marginally, an increase in κ_t lowers the leverage ratio as the balance-sheet constraint tightens. An increase in the marginal values of net worth (η_t) and the economic credit spread (ν_t) increases the leverage.

Due to the existence of survival probability, the evolution of aggregate intermediary net worth differs from the net worth of an individual intermediary. The law of motion for the aggregate net worth can be derived as

$$\mathcal{N}_t = \theta \left[\left[(R_{k,t} - R_t) - \frac{Q^*_{t-1} s^f_{t-1}}{\mathcal{W}_{t-1}} (R_{k,t} - R^*_{k,t}) \right] \phi_{t-1} + R_{t-1} \right] \mathcal{N}_{t-1} + \mathcal{N}_{n,t}. \tag{3.4.15}$$

I use the curly $\mathcal{N}_t$ to distinguish it from the individual intermediary net worth, N_t. $\mathcal{N}_{n,t}$ is a small startup transfer to incoming bankers from the household. As in DKL, $\mathcal{N}_{n,t}$ is given by $\mathcal{N}_{n,t} = \omega \mathcal{W}_{t-1}$, where the small fraction ω is used to pin down the steady-state leverage ratio and economic spread. The profit coming from the intermediary part in Π_t in (3.2.1) is the accumulated net worths of exiting intermediaries less the startup transfer to incoming intermediaries. In line with the expression given in (3.4.15), the distributed profit is given by, $(1 - \theta) \left[[(R_{k,t} - R_t) + \frac{Q^*_{t-1} s^f_{t-1}}{\mathcal{W}_{t-1}} (R^*_{k,t} - R_{k,t})]\phi_{t-1} + R_t \right] \mathcal{N}_{t-1} - \mathcal{N}_{n,t}$.

3.5 A Closer Look at the Mechanism

Throughout my analysis, I rely on a first-order Taylor expansion of the model by log-linearizing it around its deterministic steady state. The moments I study are based on the policy function obtained by Dynare.[8]

The set of intermediary optimality conditions laid out in (3.4.11) and (3.4.13), along with the condition $V_{sh,t} = V_{sf,t}$, provide the necessary grounds for the FX disconnect. Note that $V_{sh,t} = V_{sf,t}$ implies that the managing banker allocates the wealth of the intermediary between the home capital stock and the foreign capital stock in such a way that the intermediary's marginal values of the two returns are equal. In other words, the banker decides on the optimal portfolio choice of $\alpha_t^p \equiv \frac{Q_t^* s_t^f}{\mathcal{W}_t}$ so that the following condition holds:

$$E_t \Omega_{t+1} \left(R^*_{k,t+1} - R_{k,t+1} \right) = 0. \tag{3.5.1}$$

The optimal portfolio choice α_t^p can be solved for as in Devereux and Sutherland (2011).[9] The intermediary cannot balance the marginal values between the risky asset (home or foreign) and the riskless deposit as shown in (3.4.12),

$$E_t \left[\beta \Omega_{t+1} (R_{k,t+1} - R_{t+1}) \right] = \nu_t > 0. \tag{3.5.2}$$

This is due to the balance-sheet constraint that restricts the intermediary from levering up to the point where $\nu_t = 0$.

Note again, as in (3.2.4), we can rewrite the optimal portfolio choice equation in (3.5.1)

[8] As a robustness check, I have also solved the model using a second-order Taylor expansion of the model around its steady state using Dynare. The implied correlation between log exchange rate growth and cross-country log consumption difference is nearly identical to the results from the first-order solution. Admittedly, it has been shown in recent studies on financial amplification such as Brunnermeire and Sannikov (2014) that non-linear effects can be quite sizable in models with financing constraints. These effects, however, typically arise in models that exhibit the economy drifting far away from the steady state and hence spending a substantial amount of time far away from the steady state. Apparently, the second-order solution cannot capture such effects. Corsetti et al. (2008), who study the Backus-Smith puzzle with an approximate solution based on Taylor expansion, also report that their results are very similar across the first-order and second-order solutions.

[9] When solving a model by perturbation methods, an optimal portfolio choice problem suffers from an indeterminacy issue due to the first-order certainty equivalence between risky assets. See Deveruex and Sutherland (2010), Devereux and Sutherland (2011), Tille and Van Wincoop (2010), and Evans and Hnatkova (2012) for discussions and solution methods.

in terms of the home consumption basket as,

$$
\begin{aligned}
E_t\Omega_{t+1}(R^*_{k,t+1} - R_{k,t+1}) &= E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}}\left[1+\theta(\eta_{t+1}+\nu_{t+1}\phi_{t+1}-1)\right]\left(R^*_{k,t+1}-R_{k,t+1}\right)\\
&= E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[1+\theta(\eta_{t+1}+\nu_{t+1}\phi_{t+1}-1)\right]\big(\frac{P_t}{P_{t+1}}R^*_{k,t+1}-\frac{P_t}{P_{t+1}}R_{k,t+1}\big)\\
&= E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[1+\theta(\eta_{t+1}+\nu_{t+1}\phi_{t+1}-1)\right]\left(r^*_{k,t+1}-r_{k,t+1}\right)\\
&= E_t\tilde{\Omega}_{t+1}\big(\frac{S_{t+1}}{S_t}r^*_{k,t+1}-r_{k,t+1}\big)=0, \qquad (3.5.3)
\end{aligned}
$$

where $\tilde{\Omega}_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[1+\theta(\eta_{t+1}+\nu_{t+1}\phi_{t+1}-1)\right]$, $r_{k,t+1}$, and $r^*_{k,t+1}$, are the intermediary SDF, home risky asset return, and foreign risky asset return, each deflated by the respective country's price of consumption baskets.

Now, assuming the balance-sheet constraint in (4.1.4) binds, the maximized value of an intermediary can be written as,

$$
\begin{aligned}
V_t = \kappa_t\mathcal{W}_t &= E_t\beta\Lambda_{t+1}\left[(1-\theta)N_{t+1}+\theta V_{t+1}\right] && (3.5.4)\\
&= E_t\beta\Omega_{t+1}N_{t+1} && (3.5.5)\\
&= E_t\beta\Omega_{t+1}\left[(R_{k,t+1}-R_{t+1})\phi_t+\alpha^p_t(R^*_{k,t+1}-R_{k,t+1})\phi_t+R_{t+1}\right]N_t && (3.5.6)\\
&= E_t\beta\tilde{\Omega}_{t+1}\left[(r_{k,t+1}-r_t)\phi_t+\alpha^p_t(\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}r^*_{k,t+1}-r_{k,t+1})\phi_t+r_t\right]N_t && (3.5.7)\\
&= E_t\beta\tilde{\Omega}_{t+1}\left[\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}r^*_{k,t+1}\phi_t-r_t(\phi_t-1)\right]N_t, && (3.5.8)
\end{aligned}
$$

where (3.5.3) was used to go from (3.5.7) to (3.5.8).

Using $\mathcal{W}_t = \phi_t N_t$ and (3.4.11), (3.5.8) can be expressed as,

$$
\kappa_t + (1-\frac{1}{\phi_t})\eta_t = E_t\beta\tilde{\Omega}_{t+1}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}r^*_{k,t+1}. \qquad (3.5.9)
$$

From the optimality conditions of a foreign intermediary, we can derive the equation that is analogous to (3.5.9),

$$
\kappa^*_t + (1-\frac{1}{\phi^*_t})\eta^*_t = E_t\beta\tilde{\Omega}^*_{t+1}r^*_{k,t+1}. \qquad (3.5.10)
$$

Equations (3.5.9) and (3.5.10) are the equations central to understanding the mechanism by which, 1. restricted local deposit and 2. relatively volatile financial shocks (i.e., shocks to κ_t and κ_t^*) work toward the disconnect between exchange rates and consumption growths.

To see this, first notice that the "intermediary" variables on the LHSs of the two equations drive a wedge between the marginal values across home and foreign intermediaries of the common risky asset (i.e., the RHSs of (3.5.9) and (3.5.10)). Absent the financial frictions, the intermediaries would equate their marginal values so that,

$$E_t\tilde{\Omega}_{t+1}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}r^*_{k,t+1} = E_t\tilde{\Omega}^*_{t+1}r^*_{k,t+1}, \tag{3.5.11}$$

giving rise to,

$$\tilde{\Omega}_{t+1} \times \frac{\mathcal{S}_{t+1}}{\mathcal{S}_t} \approx \tilde{\Omega}^*_{t+1},$$

as in (2.0.4). Then, to the extent $\frac{\tilde{\Omega}^*_{t+1}}{\tilde{\Omega}_{t+1}}$ resembles $\frac{(\frac{C^*_{t+1}}{C^*_t})^{-\gamma}}{(\frac{C_{t+1}}{C_t})^{-\gamma}}$, the ratio of deflated household SDFs, the exchange rate and consumption differences will exhibit a close link.

Keeping this in mind, it is useful to consider one polar case where the deposit market is fully integrated to allow for frictionless deposit taking from overseas, and home and foreign intermediaries face the same deposit rate. This is the case considered in DKL. In their model, the exchange rate does not exist since their model features a single-good economy. In this case, DKL show that the balance-sheet conditions of home and foreign intermediaries are perfectly synchronized so that up to first order,

$$\eta_t \approx \eta_t^*, \tag{3.5.12}$$

$$\nu_t \approx \nu_t^*, \tag{3.5.13}$$

and,

$$\phi_t - \phi_t^* \approx \kappa_t - \kappa_t^*. \tag{3.5.14}$$

So, the marginal values of deposit and credit spread are equalized, and the leverage ratios are equalized up to the first-order difference in the collateral fractions. DKL cite this strong synchronization of home and foreign intermediaries as a channel through which country-specific shocks spill over to other countries. The effect of perfect integration on the Backus-Smith relationship is clear. As the marginal valuations of home and foreign intermediaries co-move closely, the wedge expressed as the difference between the LHSs of (3.5.9) and (3.5.10) becomes negligible, restoring the relationship in (3.5.11).[10] Moreover, the ratio of intermediary SDFs in $\frac{\Omega^*_{t+1}}{\Omega_{t+1}}$ will closely mimic the movements of the ratio of household SDFs.[11] This is clear from the expression for the intermediary SDF in (3.4.13), where the wedge term $[1+\theta(\eta_{t+1}+\nu_{t+1}\phi_{t+1}-1)]$ consists of the variables η_t, ν_t, and ϕ_t. As these variables closely move together across countries, much of the variation in $\frac{\Omega^*_{t+1}}{\Omega_{t+1}}$ will come from the ratio of household SDFs.

The polar case of a fully integrated deposit market serves as a useful benchmark for assessing the importance of the assumption of restricted offshore deposit on the FX disconnect. Suppose that a home intermediary can also take deposit that pays in foreign consumption baskets, from its foreign offices. This will alter the linear guess of the intermediary value in (3.4.10) to,

$$V_t = V_{sh,t}Q_t s^h_t + V_{sf,t}Q^*_t s^f_t - \eta_{h,t}D^h_t - \eta_{f,t}D^f_t, \tag{3.5.15}$$

where $\eta_{h,t}$ and $\eta_{f,t}$ denote the marginal values of local deposit D^h_t and offshore deposit D^f_t, respectively. It can be shown that $\eta_{h,t} = \eta_{f,t}$. Put differently, the intermediary chooses the optimal mix of local and offshore deposit so that,

$$E_t\Omega_{t+1}\left(R^*_{t+1} - R_{t+1}\right) = 0 \tag{3.5.16}$$

$$\Rightarrow E_t\tilde{\Omega}_{t+1}\left(\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}r^*_t - r_t\right) = 0. \tag{3.5.17}$$

The quantitative results in Section 4 reveal that even with the existence of exchange rates and different deposit rates across the two countries, the financial variables η_t, ν_t, and ϕ_t co-

[10]To be precise, $\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}$ is always one in this polar case, as the exchange rate does not exist.

[11]And the ratio of deflated SDFs, $\frac{\tilde{\Omega}^*_{t+1}}{\tilde{\Omega}_{t+1}}$, will resemble the ratio $\frac{(\frac{C^*_{t+1}}{C^*_t})^{-\gamma}}{(\frac{C_{t+1}}{C_t})^{-\gamma}}$.

move closely with their foreign counterparts. This is because the additional layer of optimal portfolio choice between liabilities in (3.5.17) makes the deposit rates essentially equal in expectation,[12] closely resembling the polar case in which the two deposit rates are exactly identical. Consequently, allowing for offshore deposit brings the model close to the polar case, thereby reinforcing the tight link between exchange rates and consumption differences.

If we reflect again on how the original Backus-Smith relationship (and the related puzzle) is derived, it is a product of *international risk sharing* by home and foreign households. With complete markets, households share risks so that growths of the marginal utility are aligned state by state. They do so by trading an array of common assets where exchange rate growth ensures the equalization of the marginal utility growths in their respective numeraires.

Having a bank balance-sheet constraint introduces a wedge between households and the bank *within the country.* With banks being the marginal investors, this wedge alters the usual household SDF to capture how constrained the banks are. The logic is analogous to the usual household SDF. As the household marginal utility growth varies counter-cyclically and assets that pay off in bad times have low expected returns, the wedge term (the shadow value of net worth, which is $\left[1+\theta\left(\eta_{t+1}\nu_{t+1}\phi_{t+1}-1\right)\right]$ shown above) also works in the same way. With banks as marginal investors, bad times are when a bank's net worth shrinks and its balance-sheet constraint tightens, which means the shadow value of net worth to the bank is high. The intermediary asset pricing works in such a way that assets that pay less in the bad times (low bank net worth) have high premia.

Keeping this in mind, the international risk sharing now becomes a problem between banks across the border. The banks seek to enter into a contract so that when one country's bank is hit by a negative shock, so that its financial constraint tightens, net worth becomes low and the shadow value of net worth becomes high, it is shared by the other country's bank so that their shadow values are aligned. It can be argued that the bank shadow values are analogous to marginal utility growths in the case of households being marginal investors.

[12]That is, the deposit rates are equal up to the first-order accuracy, and up to the adjustment by the exchange rate growth. Linearizing (3.5.17), we have $E_t(\widehat{\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}} + \widehat{r^*}_t) \approx E_t\hat{r}_t$, where the hat denotes first-order component of the variable.

The risk-adjusted intermediation spread, $E_t\Omega_{t+1}\left(R_{k,t+1} - R_t\right)$, is closely related to how constrained a bank is, and to its marginal value of net worth. Typically during a banking crisis, net worth goes down and the intermediation spread spikes. The increase in the intermediation spread improves the franchise value of the bank by increasing the profitability of a unit of net worth, and hence loosens the binding financial constraint.

What the segmentation of deposit market is telling us is that, as the international risk sharing problem is shifted from that between households to that between banks, frictions *between banks* matter. In addition to the bank financial constraint which differentiates a bank from households within the country, the market structure between banks needs to be sufficiently incomplete.

The role of volatile financial shock can also be easily understood from (3.5.9) and (3.5.10). Taking the difference of the two equations we have,

$$E_t\beta\tilde{\Omega}_{t+1}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}r^*_{k,t+1} - E_t\beta\tilde{\Omega}^*_{t+1}r^*_{k,t+1} = (\kappa_t - \kappa^*_t) + (1 - \frac{1}{\phi_t})\eta_t - (1 - \frac{1}{\phi^*_t})\eta^*_t. \qquad (3.5.18)$$

Given the assumption of segmented deposit market and the resulting difference between leverages and marginal values, the terms $(\kappa_t - \kappa^*_t)$ in (3.5.18) suggest that a more volatile shock to κ_t will amplify the wedge. The quantitative results reported in Section 4 reveal that both the segmented deposit and the volatile σ_κ are required to drive down the correlation in the Backus-Smith equation.

As stochastic κ_t (and κ^*_t) is the source of variation for a bank's financial condition (how binding the collateral constraint is), a less volatile κ_t should help banks share risks internationally. As we have seen earlier, the stochastic κ_t is motivated as a sudden change in the confidence of depositors of the bank's ability to repay deposit. It is then natural that a more stable level of depositors' perceived risk of a bank's default allows banks to easily align their financial healths internationally.

Another way to look at the two mechanisms is as follows. Suppose a home country's bank is hit by a positive shock to κ_t, which makes the borrowing constraint tighten. Then, the ratio of foreign to home bank marginal values of net worth would increase, as the home bank is more constrained. According to the international risk sharing by home and foreign banks, this would increase the exchange rate (the home currency depreciates). On the other hand, as the home bank's intermediation capacity shrinks, the home household's consumption would decrease over the course of subsequent periods. This is because as their banks' (the local banks that the households own) intermediated assets are lower, the households get less dividend income, and hence less consumption. This movement is even enforced by an initial spike in consumption due to the feature that households can only consume or deposit with local banks, and the banks cannot take as many deposits. Hence the deposit market segmentation induces a negative co-movement between relative consumption and exchange rate growth.

3.6 Equilibrium

The competitive equilibrium of this economy is defined as the set of prices and quantities that satisfy the optimality conditions outlined thus far and make all the markets clear.

The home-goods market clearing condition is,

$$c_{H,t} + c^*_{H,t} + i_{H,t} + i^*_{H,t} + \Phi_t(\frac{I_t}{I_{t-1}})I_t = Y_t, \tag{3.6.1}$$

and the foreign-goods market clearing condition is,

$$c_{F,t} + c^*_{F,t} + i_{F,t} + i^*_{F,t} + \Phi^*_t(\frac{I^*_t}{I^*_{t-1}}) = Y^*_t. \tag{3.6.2}$$

The capital market clearing conditions are,

$$s^h_t + s^{h*}_t = S_t, \tag{3.6.3}$$

$$s^f_t + s^{f*}_t = S^*_t, \tag{3.6.4}$$

where again, S_t is linked to the capital stock K_{t+1} by $K_{t+1} = \xi_{t+1}S_t$.

The deposit market clearing conditions are,[13]

[13]Under the specification of a fully integrated deposit market, this market clearing condition changes to $D_t + D^*_t = (\phi_t - 1)\mathcal{N}_t + (\phi^*_t - 1)\mathcal{N}^*_t$.

$$D_t = (\phi_t - 1)\mathcal{N}_t, \tag{3.6.5}$$

$$D_t^* = (\phi_t^* - 1)\mathcal{N}_t^*. \tag{3.6.6}$$

4 Empirical Analysis

4.1 Data and Calibration

The baseline model outlined in Section 3 is calibrated to the U.S. and Canada country pair. Table 1 presents the sample moments of selected main macroeconomic variables of the two countries from 1976Q1 to 2011Q4. As indicated by their aligned output volatilities and high cross-country correlations of the macro variables, the Canadian business cycle shows a notable degree of synchronization with the U.S. economy. The high degree of synchronization between the two economies is also well-documented by other studies in the international business cycles literature. As reported by their extensive documentations on cross-country correlations of macro aggregates in Backus, Kehoe, and Kydland (1995) and Ambler, Cardia, and Zimmermann (2004), the U.S. and Canada pair stands out as arguably the most synchronized pair. In Ambler, Cardia, and Zimmermann (2004), they show that the Canadian economy shows the highest correlation with the U.S. economy in terms of output, consumption, and Solow residual, among all the country pairs with the U.S. considered in their sample.[14]

Given the relatively high level of business-cycle integration, the two countries yet show substantial "disconnect" in the sense of the Backus-Smith puzzle. Table 2 is from Corsetti, Dedola, and Leduc (2008), which displays a series of correlations in the Backus-Smith puzzle for G-7 countries against the U.S., using Hodrick-Prescott (HP) filtered data. With their business-cycle integration and the highly negative Backus-Smith correlation, the U.S.

[14] The considered countries were Australia, Austria, Canada, France, Germany, Italy, Japan, Switzerland, the U.K., and an aggregate of European countries, from 1960:1-2000:4. They also report the unweighted average of 190 cross-country correlations between macro aggregates, which is again much lower than those from the U.S. and Canada pair.

- Canada pair therefore provides a natural environment to study the Backus-Smith puzzle.

The productivity processes for the two countries are assumed to follow a bivariate VAR(1) process given below.

$$\begin{bmatrix} \log A_t^{us} \\ \log A_t^{can} \end{bmatrix} = \begin{bmatrix} \rho_a & \rho_{a,a^*} \\ \rho_{a,a^*} & \rho_a \end{bmatrix} \begin{bmatrix} \log A_{t-1}^{us} \\ \log A_{t-1}^{can} \end{bmatrix} + \begin{bmatrix} e_t^{us} \\ e_t^{can} \end{bmatrix}$$

The parameters governing the processes are calibrated to match some selected moments of the two countries' output. Specifically, the autoregressive persistence coefficient, the spillover coefficient, the standard deviation of productivity shock, and the correlation between the two productivity shocks are calibrated to match the four moments in Table 3. The correlation between output in levels was difficult to match, and the model-implied value is still low relative to its empirical counterpart. Apart from this correlation, the calibrated model matches the moments closely. Note that the sample moments of the volatilities of output and output growth were close to identical across the two countries.

Table 1: Business cycle moments for the U.S. and Canada

1976:1 - 2011:4	Canadian data		U.S. data		Cross-country
variable:	a	b	a	b	correlations
Consumption	0.79	0.85	0.81	0.88	0.63
Hours	0.77	0.82	1.32	0.89	0.65
Investment	3.39	0.75	3.56	0.82	0.60
Output	1.53*		1.48*		0.79

*: standard deviation; a: standard deviation relative to output; b: contemporaneous correlation with output. All moments are Hodrick-Prescott filtered, and calculated at the quarterly frequency.

The values of the parameters used to solve this model are summarized in Table 4. The parameter values are at a quarterly frequency. The four parameters governing the productivity processes are from the calibration to the output moments as explained above.

The steady-state subjective discount factor b, capital share in the Cobb-Douglas production α, and depreciation rate δ are set to conventional values. The relative risk aversion coefficient γ is set to one, implying a log utility over the consumption basket. The

Table 2: Backus-Smith Correlations against the U.S.

	Correlation with U.S.
Country	HP-filtered
Canada	-0.52
France	-0.20
Germany	-0.51
Italy	-0.28
Japan	0.05
U.K.	-0.51

From Corsetti, Dedola, and Leduc (2008).

Table 3: U.S.-Canada Output Moments

	Model	Data
$\sigma(y)$	0.014	0.015
$\text{corr}(y,\, y^*)$	0.51	0.80
$\sigma(\Delta y)$	0.0085	0.0073
$\text{corr}(\Delta y,\, \Delta y^*)$	0.43	0.48

All moments are Hodrick-Prescott filtered, and calculated at the quarterly frequency.

values for the relative utility weight of labor, χ, and the inverse Frisch-elasticity of labor supply, φ, are taken from DKL. The value of χ is set to match the long-run hours worked of 1/3 in the steady state. The banker survival probability θ is set to match a banker's average tenure of ten years, as in GK2. The banker's steady-state divertible fraction $\bar{\kappa}$ and the startup transfer parameter ω are jointly set to match the steady-state leverage ratio (ϕ) of four and the steady-state annual credit spread of 0.01 as in GK2 and DKL.

The parameters related to the CES aggregators of consumption and investment are particularly known to have wide ranges of values used in the international macroeconomics literature and therefore are difficult to calibrate. I used 0.85 for the domestic weight to account for home bias. 1.5 for the elasticity of substitution is from Backus, Kehoe, and Kydland (1995). These parameter values are certainly in the acceptable range of values previously found in the literature.

Table 4: Parameterization

Preference and Production		
steady-state discount factor	b	0.99
endogenous discount factor, curvature	υ	0.001
risk aversion	γ	1
relative utility weight of labor	χ	3.4
inverse Frisch-elasticity of labor supply	φ	0.276
capital share	α	0.33
depreciation rate	δ	0.025
inverse elasticity of investment to the price of capital	η_i	1.728
CES basket		
weight on domestic consumption good in a CES basket	λ_c	0.85
home vs. foreign consumption CES elasticity parameter	θ_c	1.5
weight on domestic investment good in a CES basket	λ_I	0.85
home vs. foreign investment CES elasticity parameter	θ_I	1.5
Intermediary		
steady-state divertible fraction	$\bar{\kappa}$	0.382
banker continuation probability	θ	0.976
start-up transfer	ω	0.002
persistence financial shock	ρ_κ	0.8
standard deviation financial shock	σ_κ	0.013
Productivity		
spill-over coefficient	ρ_{a,a^*}	0.016
persistence TFP shock	ρ_a	0.973
standard deviation TFP shock	σ_a	0.007
cross-country correlation of TFP shock	σ_{a,a^*}	0.65
Capital quality		
persistence capital-quality shock	ρ_ψ	0.62
standard deviation capital-quality shock	σ_ψ	0.007

Parameters that are crucial for the quantitative results are the ones that govern the stochastic process of κ_t, in particular the volatility of the "financial shock" σ_κ relative to the volatility of the usual productivity (TFP) shock. As shown earlier in Section 3.5, the result for the Backus-Smith puzzle in large part depends on the feature of σ_κ being greater than σ_a. Given the sensitivity of the results to σ_κ, I investigate three different methods to identify this parameter.

Firstly, I use the fact that the value of σ_κ is linked by the model to variables that are more observable than σ_κ. In the model, the intermediation spread $R_{k,t} - R_t$ is primarily affected by the specification of the balance-sheet constraint. In particular, the ex-ante value of the intermediation spread to an intermediary is given by:

$$E_t\Omega_{t+1}\left[R_{k,t+1} - R_t\right] = \frac{\kappa_t\lambda_t}{1+\lambda_t}, \tag{4.1.1}$$

where λ_t denotes the Lagrange multiplier on the intermediary's balance-sheet constraint. While this relationship does not exactly identify κ_t due to the unobservability of Ω_{t+1} and λ_t, it still established a relationship between the observable intermediation spread of $R_{k,t+1} - R_t$ and κ_t. Accordingly, σ_κ is set to 0.013 to match the quarterly standard deviation of the intermediation. For the following analyses based on this identification, the U.S. spread of Baa corporate yield relative to the Federal funds rate was used as a proxy for the credit spread earned by intermediaries. The standard deviation of the spread from 1986.1 to 2014.3 is around 1.7%.

The model also produces the following relationship between κ_t and the leverage of an intermediary, ϕ_t as shown in Equation (3.4.14):

$$\phi_t = \frac{\eta_t}{\kappa_t - \nu_t}. \tag{4.1.2}$$

Hence, the value of σ_κ can be set so that the model-implied volatility of ϕ_t matches the empirically observed volatility of the leverage of the banking sector.

Secondly, I apply the identification method used in Jermann and Quadrini (2009) to extract the series of κ_t from a binding balance-sheet constraint. The idea is as follows. Jermann and Quadrini (2009), in their model, assume their collateral constraint is binding, where the constraint is given as:

$$\xi_t\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right) \geq l_t, \tag{4.1.3}$$

where l_t denotes an intra-period loan, and $\left(k_{t+1} - \frac{b_{t+1}}{1+r_t}\right)$ denotes the collateral. ξ_t denotes the probability the lender can recover the full value of the collateral, which is similar to the κ_t in my model in that it captures the "financial shock." The identification strategy is, as in the case of extracting the Solow residuals from the Cobb-Douglas production function with observable inputs, the unobservable ξ_t can be recovered if the collateral constraint is binding and the rest of the terms in the constraint are observable in the data.

Carrying this over to my model, the identification strategy should be, extracting the time series of $\{\kappa_t\}$ from the binding balance-sheet constraint introduced in Equation (4.1.4),

$$V_t = \kappa_t \mathcal{W}_t. \tag{4.1.4}$$

For this method to work, we need to identify V_t and $\mathcal{W}_t$ first. $\mathcal{V}_t$ is the value of the *configured* net worth of the representative intermediary. In other words, it is the ex-ante risk-adjusted value of the next period's net worth N_{t+1}, *accounting for the balance-sheet constraint.* Note that $\mathcal{V}_t \neq N_t$, as the marginal values of assets and net worth are greater once they are put into operation, compared to their stand-alone values. This reasoning is analogous to the q-theory of capital stocks in which a unit of *installed* capital is worth more than the same unit of *uninstalled* capital where there is capital adjustment cost. Similarly, absent the balance-sheet constraint, $\mathcal{V}_t = N_t$.

$\mathcal{W}_t$ is the total assets held by the intermediary, and is equal to the sum of the intermediary's holdings of the home country's capital stock and foreign country's capital stock.

My strategy is to identify $\mathcal{V}_t$ as the market capitalization of the banking sector, and $\mathcal{W}_t$ as the banks' total assets (debt plus equity on the RHS of balance sheet). I collect the data from six of the ten largest banks (ordered by their market capitalization) for which the relevant data were available from Compustat for the time span of interest, 1970.1-2011.4. The six banks are *Bank of New York Mellon Corp., Citigroup, U.S. Bancorp., Bank of America Corp., Wells Fargo & Co., and PNC Financial Services Group.* For the method of using the banking sector leverage, the same six banks' data were used.

The fitted values of $\rho_\kappa = 0.8$ and $\sigma_\kappa = 0.013$ given in Table 4 are taken from the calibration strategy using the intermediation spread. As will be clear from Table 6, the Backus-Smith correlation gets increasingly negative as σ_κ increases. The calibrated values of σ_κ from using the bank leverage and from using the Jermann-Quadrini (2009) method, respectively, are 0.107 and 0.1281, almost an order of degree higher than 0.013. The two calibrated values imply Backus-Smith correlations of -0.9587 and -0.9681, respectively, which are too extreme. Therefore, I use the values from the method of using the intermediation spread as the baseline calibration.

4.2 Results

First, I present model-implied cross-correlations of variables of interest from, 1. the baseline model with segmented deposit market, and 2. the benchmark model with integrated deposit market, where offshore deposit is allowed. To highlight the role of volatile financial shock, the two models are examined with different values of σ_κ, while fixing the values of the other parameters.

Table 5: Matrix of Correlations, Baseline Model, $\sigma_a = 0.007$, $\sigma_\kappa = 0.013$

Variables	Δs	ΔC	ΔC^*	$\Delta C - \Delta C^*$	$\widetilde{\Omega}^* - \widetilde{\Omega}$	wedge_diff
Δs	1	-0.07	0.07	-0.16	0.32	0.34
ΔC	-0.07	1	0.62	0.44	-0.06	-0.12
ΔC^*	0.07	0.62	1	-0.44	0.06	0.12
$\Delta C - \Delta C^*$	-0.16	0.44	-0.44	1	-0.14	-0.27
$\widetilde{\Omega}^* - \widetilde{\Omega}$	0.32	-0.06	0.06	-0.14	1	0.99
wedge_diff	0.34	-0.12	0.12	-0.27	0.99	1

Correlation values are hp-filtered, calculated at the quarterly frequency.
Δs: log growth of exchange rate
ΔC : log growth of home consumption
ΔC^* : log growth of foreign consumption
$\Delta C_{\text{diff}} \equiv \Delta c - \Delta c^*$
$\widetilde{\Omega}^* - \widetilde{\Omega}$: $\log \tilde{\Omega} - \log \tilde{\Omega}^*$
wedge_diff $\equiv \log \frac{[1+\theta(\eta^*_{t+1} + \nu^*_{t+1}\phi^*_{t+1} - 1)]}{[1+\theta(\eta_{t+1} + \nu_{t+1}\phi_{t+1} - 1)]}$

Table 6: Correlation with Varying σ_κ, Baseline Model

Financial Shock	$\sigma_\kappa = 0.007$	$\sigma_\kappa = 0.013$	$\sigma_\kappa = 0.02$	$\sigma_\kappa = 0.03$
corr(Δs, $\Delta c - \Delta c^*$)	0.36	-0.16	-0.51	-0.73

Table 7: Matrix of Correlations, Benchmark Model (Offshore Deposit), $\sigma_a = 0.007$, $\sigma_\kappa = 0.013$

Variables	Δs	ΔC	ΔC^*	$\Delta C - \Delta C^*$	$\widetilde{\Omega}^* - \widetilde{\Omega}$	wedge_diff
Δs	1	0.3413	-0.3413	0.9998	0.9872	0.0065
ΔC	0.3413	1	0.7669	0.3414	0.3363	-0.0022
ΔC^*	-0.3413	0.7669	1	-0.3414	-0.3363	0.0022
$\Delta C - \Delta C^*$	0.9998	0.3414	-0.3414	1	0.9852	-0.0064
$\widetilde{\Omega}^* - \widetilde{\Omega}$	0.9872	0.3363	-0.3363	0.9852	1	0.1652
wedge_diff	0.0065	-0.0022	0.0022	-0.0064	0.1652	1

Table 5 show the results from the baseline model. First, notice that the correlation between $\Delta \mathbf{s}_t$ and $\gamma(\Delta c_t - \Delta c_t^*)$ is -0.16, which is substantially lower than what is implied by traditional models, and in line with the empirical evidence. Hence in the case of restricted deposit and volatile financial shocks, the first-pass results suggest the model successfully rationalizes the Backus-Smith puzzle.

The correlation between exchange rate growth and the ratio of intermediary SDFs (in terms of consumption baskets) is positive, although modest, suggesting the movement of the exchange rate growth is related to the intermediary SDFs rather than household SDFs. The somewhat modest correlation of 0.32 reflects the incomplete market structure between *intermediaries,* as the home intermediary SDF scaled by the exchange rate growth is aligned with the foreign intermediary SDF only in expectation and not state by state. The correlation between the ratio of the intermediary SDFs and the ratio of intermediary "wedge" terms is virtually one, suggesting that most of the cross-country difference in the intermediary SDFs come from the difference in the "wedge" terms that arise from balance-sheet constraints, and not from the difference in consumption growths.

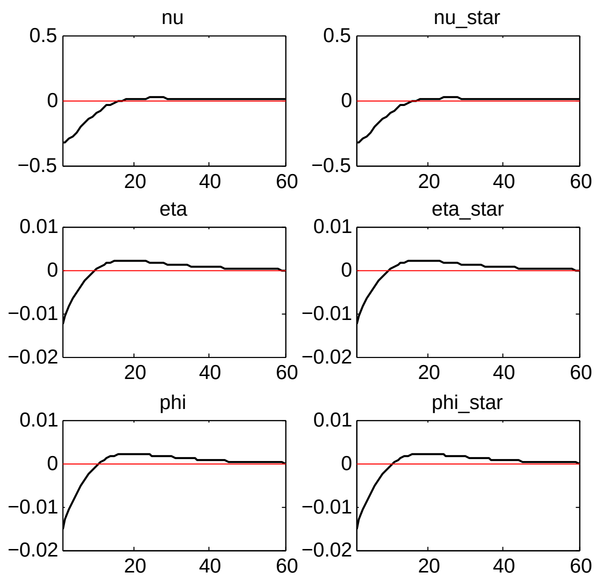
To summarize, exchange rate growth co-moves with the difference between intermediary SDFs, as the intermediaries are marginal with respect to exchange rate growth. Moreover, the exchange rate growth co-moves with the difference in the part of the intermediary SDF that arises from balance-sheet constraints, and not with the difference in the consumption growth. The consumption growth part of the intermediary SDF, which captures the marginal decision of the household, is negatively correlated with the wedge part, and so with the exchange rate growth.

Table 6 shows how the Backus-Smith puzzle is affected by the value of σ_κ. We can observe there is a clear pattern as the correlation becomes significantly more negative as σ_κ increases, confirming the mechanism outlined in Section 3.5.

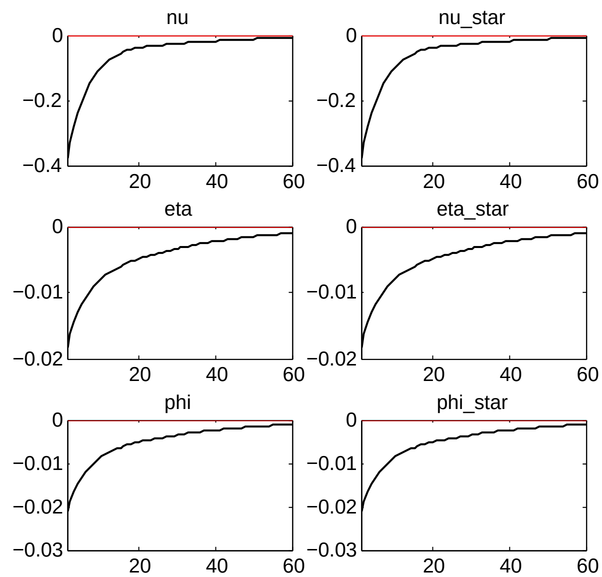
Table 7, under the assumption of a perfectly integrated deposit market, shows the correlation in the Backus-Smith equation is virtually one, implying the model fails to rationalize the Backus-Smith puzzle. We can see from the table that the cross-country difference of intermediary SDF essentially coincides with that of household SDF, as shown in Section 3.5, re-establishing the near perfect correlation between exchange rate growth and consumption differential.

Comparing Table 5 against both the integrated deposit market case (Table 7) and the low σ_κ case in (Table 6), we can see that both forces are in play. Still, the effect of restricting deposit to local households appears to be of much higher significance, as there is virtually no FX disconnect in Table 7 even with a high value of σ_κ. We can infer that there is not much wedge in place to be amplified by increasing σ_κ, as the intermediary variables are highly synchronized across countries.

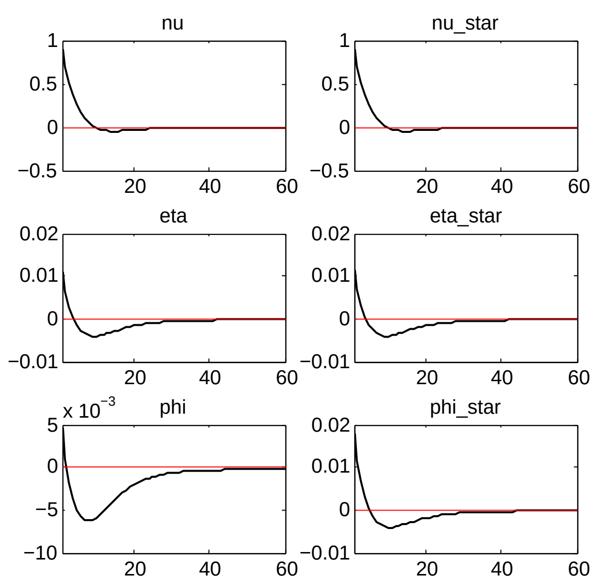
This observation is made clear from the impulse response plots in Figure 1 and Figure 2. Both figures contain a series of impulse responses of η : marginal value of deposit by intermediary, ν : marginal value of credit spread, and ϕ : equilibrium leverage of the intermediary. The plots are impulse responses to, 1. one-standard-deviation shock to the TFP, 2. one-standard-deviation shock to the capital quality, and 3. one-standard-deviation shock to the balance-sheet constraint. Figure 1 corresponds to the benchmark model with offshore deposit. Figure 2 corresponds to the baseline model with a volatile κ. We can see from the impulse response plots in Figure 1 that the intermediary variables are strongly connected. As we move to Figure 2, the variables show clear divergence upon the financial shock (i.e., shock to κ).



(a) Offshore Deposit ($\sigma_\kappa = 0.013$), Shock to e_a

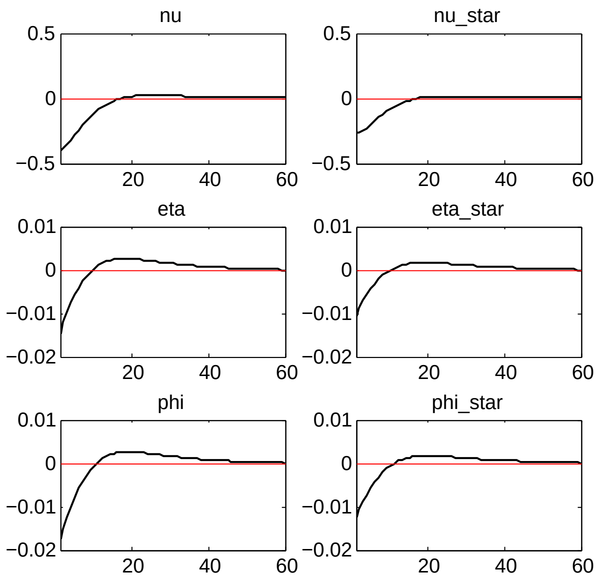


(b) Offshore Deposit ($\sigma_\kappa = 0.013$), Shock to e_ψ

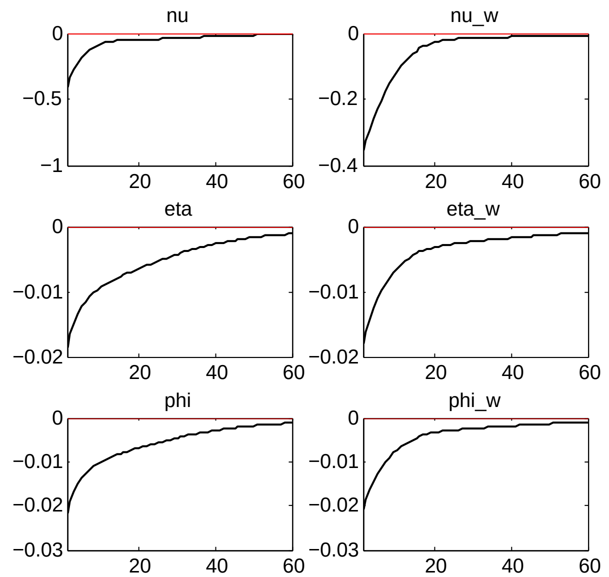


(c) Offshore Deposit ($\sigma_\kappa = 0.013$), Shock to e_κ

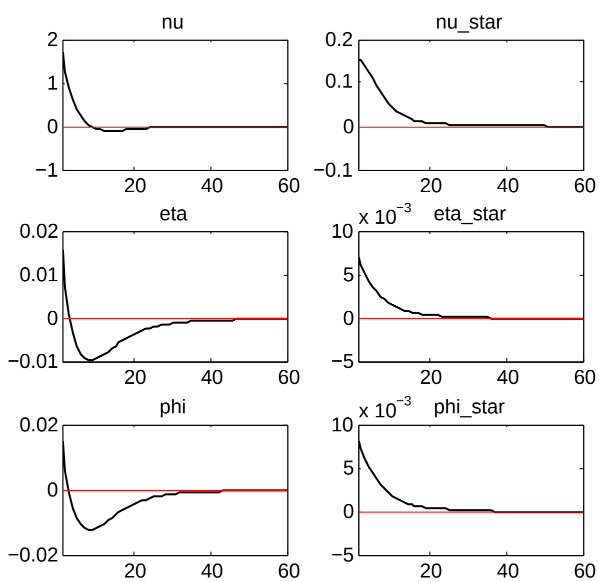
Figure 1: Benchmark Model - Impulse Response Plots



(a) Baseline Model ($\sigma_\kappa = 0.013$), Shock to e_a



(b) Baseline Model ($\sigma_\kappa = 0.013$), Shock to e_ψ



(c) Baseline Model ($\sigma_\kappa = 0.013$), Shock to e_κ

Figure 2: Baseline Model - Impulse Response Plots

5 Extended Model with Money

In this section, I consider an extended version of the model outlined in Section 3 in which national moneys are introduced via cash-in-advance (CIA) constraints. With the same complete market and CRRA utility assumptions, Backus-Smith equation now becomes,

$$\frac{\mathbb{S}_{t+1}}{\mathbb{S}_t} = \frac{(\frac{C^*_{t+1}}{C^*_t})^{-\gamma} \frac{\mathbb{P}^*_t}{\mathbb{P}^*_{t+1}}}{(\frac{C_{t+1}}{C_t})^{-\gamma} \frac{\mathbb{P}_t}{\mathbb{P}_{t+1}}}, \tag{5.0.1}$$

where $\mathbb{S}_t$ is the nominal exchange rate, $\mathbb{P}_t$ is the home-currency price of a unit of home consumption basket, and $\mathbb{P}^*_t$ is the foreign-currency price of a unit of foreign consumption basket. Taking logs on both sides we have,

$$\Delta \mathbf{s}_{t+1} = \gamma(\Delta c_{t+1} - \Delta c^*_{t+1}) + (\Delta \mathbf{p}_{t+1} - \Delta \mathbf{p}^*_{t+1}), \tag{5.0.2}$$

using lowercase for logs. Similarly, we can derive an implication for the log of the real exchange rate growth, denoted by $\mathbf{\Delta rfx}$,

$$\Delta \mathbf{rfx}_{t+1} = \gamma(\Delta c_{t+1} - \Delta c^*_{t+1}). \tag{5.0.3}$$

We can now examine both the nominal and real versions of the Backus-Smith puzzle, and check if the model mechanism in Section 3 can still rationalize the puzzle.

5.1 The Household's Problem

The home representative household maximizes the same objective function in (3.1.1) subject to the following constraints. Note that the constraints are now nominal, denominated in units of the home currency.

$$p_{H,t}c_{H,t} + \mathbb{S}_t p_{F,t} c_{F,t} + M_{H,t} + \mathbb{D}_t = M_{H,t-1} + (\mathbf{M}_t - \mathbf{M}_{t-1}) + \mathbf{w}_t L_t + \mathbf{\Pi}_t + \mathbb{R}_{t-1}\mathbb{D}_{t-1}, \tag{5.1.1}$$

$$p_{H,t}c_{H,t} + \mathbb{S}_t p_{F,t} c_{F,t} \leq M_{H,t-1} + (\mathbf{M}_t - \mathbf{M}_{t-1}) \tag{5.1.2}$$

Equation (5.1.1) is the budget constraint and (5.1.2) is the CIA constraint. $p_{H,t}$ and $p_{F,t}$ are the prices in their respective currencies of home and foreign final goods. $M_{H,t}$ is the amount of home currencies demanded by the (home) household in period t to be used in period $t+1$.

As can be seen from (5.1.2), the CIA constraints are specified in a way that *domestic* currencies must be reserved from the previous period in order to purchase both types of final goods for consumption. In other words, a household must reserve their own country's currencies in order to consume both the domestically produced goods and goods produced overseas.[15] The CIA constraints are only placed on consumption. Namely, consumption goods are cash goods, and investment goods are credit goods.[16] The timing assumption of the CIA constraints used in this extension resembles Svensson (1985) or Cooley and Hansen (1989), in that households decide on cash holdings before observing the shocks.[17][18]

The money supplies vary through time according to the following stochastic processes.

$$\log \mathbf{M}_t - \log \mathbf{M}_{t-1} = (1-\rho_m)\pi + \rho_m(\log \mathbf{M}_{t-1} - \log \mathbf{M}_{t-2}) + \varepsilon_{m,t}, \tag{5.1.3}$$

$$\log \mathbf{M}^*_t - \log \mathbf{M}^*_{t-1} = (1-\rho_m)\pi + \rho_m(\log \mathbf{M}^*_{t-1} - \log \mathbf{M}^*_{t-2}) + \varepsilon^*_{m,t}, \tag{5.1.4}$$

where $\mathbf{M}_t$ and $\mathbf{M}^*_t$ denote home and foreign aggregate money supplies, respectively. π denotes the steady-state inflation. I assume the change in the money supply $M_t - M_{t-1}$ is transferred to the home representative household as a helicopter drop, entering the budget constraint (5.1.1) and also relaxing the CIA constraint (5.1.2).

The nominal deposit $\mathbb{D}_t$ in the budget constraint (5.1.1) is assumed to pay off in the home currency. Therefore, the nominal riskless return on deposit from $t-1$ to t is known

[15] Here, I have in mind a "veil" entity providing frictionless intermediation between the home currency and the foreign currency when households pay foreign firms.

[16] The CIA constraint can be placed on investment or asset purchases. See Helpman and Razin (1985) and Abel (1985) for further discussion.

[17] As an alternative specification, one can consider a setting in which agents decide on cash holdings after observing the shock. In this case, the cash spending in the period is redistributed as income in the next period. See Lucas (1982), Alvarez, Atkeson, and Kehoe (2002), and Alvarez, Atkeson, and Kehoe (2009) for this specification.

[18] Due to the timing assumption, it is possible that in some states of the economy the CIA constraints do not bind, as explained in Svensson (1985). Since the analyses that follow will depend on log-linearizing the model around the steady state, I focus only on the set of equilibria where the CIA constraints always bind. I verify by checking the simulated Lagrange multipliers on the CIA constraint that the constraint always binds near the steady state in my analyses.

in period $t-1$, and denoted by $\mathbb{R}_{t-1}$ in (5.1.1).

Setting up the Lagrangian of the representative household as before, we have,

$$\mathcal{L} = E_t \sum_{\tau=t}^{\infty} B_{\tau-t} \left[\frac{C_\tau^{1-\gamma}}{1-\gamma} - \chi \frac{L_\tau^{1+\varphi}}{1+\varphi} \right]$$
$$+ B_t \lambda_t (M_{H,t-1} + \mathbb{S}_t M_{F,t-1} + \mathbf{w}_t L_t + \mathbf{\Pi}_t + \mathbb{R}_{t-1}\mathbb{D}_{t-1} - p_{H,t}c_{H,t} - \mathbb{S}_t p_{F,t} c_{F,t} - M_{H,t} - \mathbb{S}_t M_{F,t} + \mathbb{D}_t)$$
$$+ B_{t+1}\lambda_{t+1}(M_{H,t} + \mathbb{S}_{t+1}M_{F,t} + \mathbf{w}_{t+1}L_{t+1} + \mathbf{\Pi}_{t+1} + \mathbb{R}_t\mathbb{D}_t$$
$$- p_{H,t+1}c_{H,t+1} - \mathbb{S}_{t+1}p_{F,t+1}c_{F,t+1} - M_{H,t+1} - \mathbb{S}_{t+1}M_{F,t+1} + \mathbb{D}_{t+1}) + ...$$
$$+ \beta\mu_{H,t}(M_{H,t-1} - p_{H,t}c_{H,t} - \mathbb{S}_t p_{F,t}c_{F,t}) + \beta^2\mu_{H,t+1}(M_{H,t} - p_{H,t+1}c_{H,t+1} - \mathbb{S}_{t+1}p_{F,t+1}c_{F,t+1}) + ... \tag{5.1.5}$$

where λ_t is the multiplier on the period-t budget constraint, and $\mu_{H,t}$ is the multiplier on the CIA constraint.

First-order conditions with respect to $c_{H,t}$ and $c_{F,t}$ yield,

$$\frac{\partial U_t}{\partial c_{H,t}} \frac{1}{p_{H,t}} = \lambda_t + \mu_{H,t}, \tag{5.1.6}$$
$$\frac{\partial U_t}{\partial c_{F,t}} \frac{1}{p_{F,t}} = \mathbb{S}_t\lambda_t + \mathbb{S}_t\mu_{H,t}, \tag{5.1.7}$$

The first-order condition with respect to cash holdings yields,

$$\lambda_t = E_t \beta(\lambda_{t+1} + \mu_{H,t+1}), \tag{5.1.8}$$

implying that the marginal utility of wealth is the sum of discounted marginal utility of the next period's *liquidity,* and the discounted marginal utility of the next period's wealth.

The inter-temporal savings decision by the household yields,

$$E_t \beta \mathbf{\Lambda}_{t+1} R_t = 1, \tag{5.1.9}$$

where $\mathbf{\Lambda}_{t+1} \equiv \dfrac{\lambda_{t+1}}{\lambda_t}$ is the nominal household SDF. With the CIA constraint, $\mathbf{\Lambda}_{t+1}$ is not a function of consumption only, but also a function of the multiplier on the CIA constraint.

5.2 Equilibrium

The goods market clearing condition, asset market clearing condition, and deposit market clearing condition are the same as in the moneyless case. The money market clearing condition is given by,

$$M_{H,t} = M_t, \tag{5.2.1}$$

$$M^*_{F,t} = M^*_t. \tag{5.2.2}$$

M_t and M^*_t are supplies of Home and Foreign currencies, respectively, which I assume to be exogenously given. The LHS of each of the equations is the currency demand. Note that only households are required to demand money, as seen by $M_{H,t}$ and $M^*_{F,t}$, and they leave the period with the entire money stock.

5.3 Results from Extended Model

The parameterization for this exercise follows DKL, where the TFP processes are not calibrated to a specific country pair. The parameters related to the money supply processes in Equations (5.1.3) and (5.1.4) are set to match the inflation moments of the U.S. The steady-state inflation π is set to 0.01, implying an annual steady-state inflation of four percent. Table 8 summarizes the parameter values used for this exercise. The calibration and the results are again at the quarterly frequency.

Analogous to the tables 5 and 7, I present at the end of the paper cross-correlations of variables of interest from, 1. the baseline model with segmented deposit market, and 2. the benchmark model with integrated deposit market, where offshore deposit is allowed. To highlight the role of volatile financial shock, the first model has two cases. One is where σ_κ is set to equal the TFP shock σ_a at 0.01, and the other is where σ_κ is set higher at 0.03.

Since the models assume variable money supplies with steady-state inflation, prices have a trend. All the price variables including the nominal and real exchange rates have to be recovered after obtaining the policy functions for the deflated variables. Thus, the correlations for the nominal and real exchange rates had to be simulated. All the moments are calculated by simulating 1,000 paths of 1,000 periods each, and averaging across the paths.

The primary variables of interest are the nominal and real exchange rate growths, and the RHS variables of the Backus-Smith relation in (5.0.2) and (5.0.3).

Table 9 and Table 10 show the results from the segmented deposit models. Table 10 corresponds to the baseline model in Section 4, as it is from the segmented deposit market assumption and a relatively volatile shock to the balance-sheet constraint. Notice that the correlation between $\Delta \mathbf{s}_t$ and $\gamma(\Delta c_t - \Delta c_t^*) + (\Delta p_t - \Delta p_t^*)$ is 0.21 in Table 10. The value is still substantially lower than what is implied by traditional models, despite showing a weak positive correlation as opposed to the negative correlation in the previous results without money. The real exchange rate growth in Table 10, however, shows an even more pronounced negative correlation of -0.66 with the consumption growth differential in $corr(\Delta \mathbf{rfx}, \Delta C_{t,\text{diff}})$ which is at the heart of the traditional Backus-Smith puzzle. The correlation of the exchange rate growth with the households' SDF differential is significantly negative, while it is positive with the intermediary SDF differential, consistent with the households' restricted participation in the asset market. The correlation of 0.9 between the nominal and real exchange rate growth suggests that the two assumptions of restricted deposit and high σ_κ produce more realistic exchange rates as the two exchange rate growths are almost perfectly correlated in the data.

Table 11, under the assumption of a perfectly integrated deposit market, shows correlations that are still implausibly high compared to empirical evidence, without much improvement toward FX disconnect compared to traditional models. Under these model specifications, the correlations between exchange rates and SDF ratios (both household and intermediary SDFs) are close to one, implying a near perfect risk sharing among all agents. The intermediary SDF is also strongly correlated with household SDF as well, which suggests the wedge between the two differentials is very small.

From Tables 9 through 11, we can clearly observe a similar pattern to the results in

Section 4, and that the same model mechanisms are driving the results for the extended model with money.

6 Concluding Remarks

My baseline model focuses on the role of intermediaries as the marginal investor in the international asset market with endogenously determined exchange rates. Although a large proportion of foreign-currency transactions are intermediated, intermediaries have not received much attention as a potential main force behind the dynamics of exchange rates. In this light, my modeling strategy should provide a novel perspective on understanding the dynamics of exchange rates. Specifically, my model shows that a balance-sheet constraint *per se* is not sufficient to generate FX disconnect. The reason is that, although the existence of balance-sheet constraint creates a wedge between households' marginal decisions and intermediaries' marginal decisions *within the country*, the difference between intermediaries *across countries* also needs to be sufficiently large. When the constraints are combined with the additional incomplete market structure in which intermediaries are restricted to local deposits only, the model makes significant progress toward resolving the Backus-Smith puzzle. The model at the parameter values used for the analysis in Section 4 produces a disconnect between exchange rates and consumption and brings the correlation in the Backus-Smith equation close to the observed data.

Future work will include examining my model in other dimensions of international finance. As an example, studying the implications of my proposed model on the uncovered interest rate parity (UIP) will be interesting. It will be also interesting to explore if the model can produce other facets of the exchange rate disconnect puzzle, such as volatile and persistent real exchange rates.

Table 8: Parameterization: Extended Model

Preference and Production		
steady-state discount factor	b	0.99
endogenous discount factor, curvature	υ	0.001
risk aversion	γ	1
relative utility weight of labor	χ	3.4
inverse Frisch-elasticity of labor supply	φ	0.276
capital share	α	0.33
depreciation rate	δ	0.025
inverse elasticity of investment to the price of capital	η_i	1.728
CES basket		
weight on domestic consumption good in a CES basket	λ_c	0.85
home vs. foreign consumption CES elasticity parameter	θ_c	1.5
weight on domestic investment good in a CES basket	λ_I	0.85
home vs. foreign investment CES elasticity parameter	θ_I	1.5
Intermediary		
steady-state divertible fraction	$\bar{\kappa}$	0.382
banker continuation probability	θ	0.976
start-up transfer	ω	0.002
persistence financial shock	ρ_κ	0.8
standard deviation financial shock	σ_κ	0.013
Productivity		
spill-over coefficient	ρ_{a,a^*}	0.016
persistence TFP shock	ρ_a	0.973
standard deviation TFP shock	σ_a	0.007
cross-country correlation of TFP shock	σ_{a,a^*}	0.65
Capital quality		
persistence capital-quality shock	ρ_ψ	0.62
standard deviation capital-quality shock	σ_ψ	0.007
Money supply		
persistence money growth shock	ρ_m	0.75
standard deviation money growth shock	σ_m	0.002
steady-state inflation	π	0.01

Table 9: Baseline Model, $\sigma_a = 0.01$, $\sigma_\kappa = 0.01$

Variables	ΔS	ΔE	Λ_{diff}	Ω_{diff}	ΔC_{diff}	$\Delta C\Delta P_{\text{diff}}$
ΔS	1	0.5996	-0.4955	0.3252	-0.7874	0.4395
ΔE	0.5996	1	0.3645	0.1911	-0.0169	0.1860
Λ_{diff}	-0.4955	0.3645	1	-0.0511	0.9189	-0.0803
Ω_{diff}	0.3252	0.1911	-0.0511	1	-0.1298	0.6312
ΔC_{diff}	-0.7874	-0.0169	0.9189	-0.1298	1	-0.1907
$\Delta C\Delta P_{\text{diff}}$	0.4395	0.1860	-0.0803	0.6312	-0.1907	1

Correlation values are averages over 1,000 simulated paths. The length of each path is 1,000 periods.
All moments are calculated at the quarterly frequency.
ΔS: log growth of nominal exchange rate
ΔE: log growth of real exchange rate
$\Lambda_{\text{diff}} \equiv \log \Lambda^*_{real,t} - \log \Lambda_{real,t}$
$\Omega_{\text{diff}} \equiv \log \Omega^*_{real,t} - \log \Omega_{real,t}$
$\Delta C_{\text{diff}} \equiv \gamma(\Delta c - \Delta c^*)$
$\Delta C\Delta P_{\text{diff}} \equiv \gamma(\Delta c - \Delta c^*) + (\Delta p - \Delta p^*)$

Table 10: Baseline Model, $\sigma_a = 0.01$, $\sigma_\kappa = 0.03$

Variables	ΔS	ΔE	Λ_{diff}	Ω_{diff}	ΔC_{diff}	$\Delta C\Delta P_{\text{diff}}$
ΔS	1	0.8952	-0.6997	0.3406	-0.9186	0.2144
ΔE	0.8952	1	-0.3295	0.2952	-0.6613	0.0993
Λ_{diff}	-0.6997	-0.3295	1	-0.2006	0.9213	-0.0644
Ω_{diff}	0.3406	0.2952	-0.2006	1	-0.2756	0.3482
ΔC_{diff}	-0.9186	-0.6613	0.9213	-0.2756	1	-0.1201
$\Delta C\Delta P_{\text{diff}}$	0.2144	0.0993	-0.0644	0.3482	-0.1201	1

Correlation values are averages over 1,000 simulated paths. The length of each path is 1,000 periods.
All moments are calculated at the quarterly frequency.
ΔS: log growth of nominal exchange rate
ΔE: log growth of real exchange rate
$\Lambda_{\text{diff}} \equiv \log \Lambda^*_{real,t} - \log \Lambda_{real,t}$
$\Omega_{\text{diff}} \equiv \log \Omega^*_{real,t} - \log \Omega_{real,t}$
$\Delta C_{\text{diff}} \equiv \gamma(\Delta c - \Delta c^*)$
$\Delta C\Delta P_{\text{diff}} \equiv \gamma(\Delta c - \Delta c^*) + (\Delta p - \Delta p^*)$

Table 11: Benchmark Model: Offshore Deposit, $\sigma_a = 0.01$, $\sigma_\kappa = 0.05$

Variables	ΔS	ΔE	Λ_{diff}	Ω_{diff}	ΔC_{diff}	$\Delta C\Delta P_{\text{diff}}$
ΔS	1	-0.2311	-0.2278	-0.1940	-0.3602	0.8884
ΔE	-0.2311	1	0.9981	0.9364	0.9774	0.0076
Λ_{diff}	-0.2278	0.9981	1	0.9308	0.9781	0.0176
Ω_{diff}	-0.1940	0.9364	0.9308	1	0.9077	0.0173
ΔC_{diff}	-0.3602	0.9774	0.9781	0.9077	1	-0.0545
$\Delta C\Delta P_{\text{diff}}$	0.8884	0.0076	0.0176	0.0173	-0.0545	1

Correlation values are averages over 1,000 simulated paths. The length of each path is 1,000 periods.

All moments are calculated at the quarterly frequency.

ΔS: log growth of nominal exchange rate

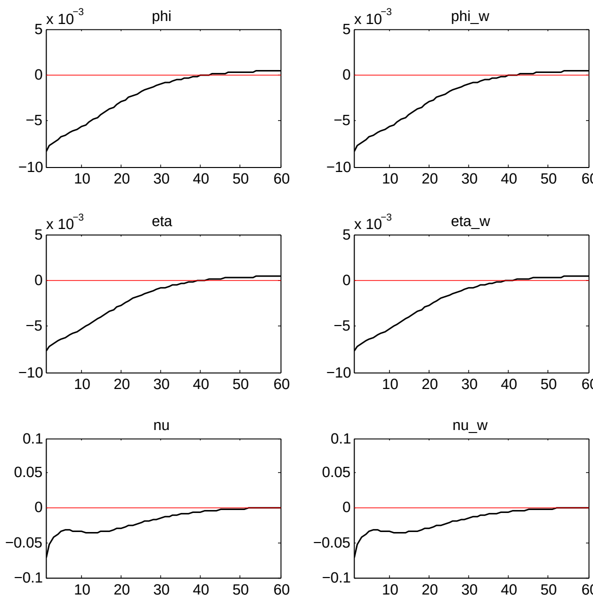
ΔE: log growth of real exchange rate

$\Lambda_{\text{diff}} \equiv \log \Lambda^*_{real,t} - \log \Lambda_{real,t}$

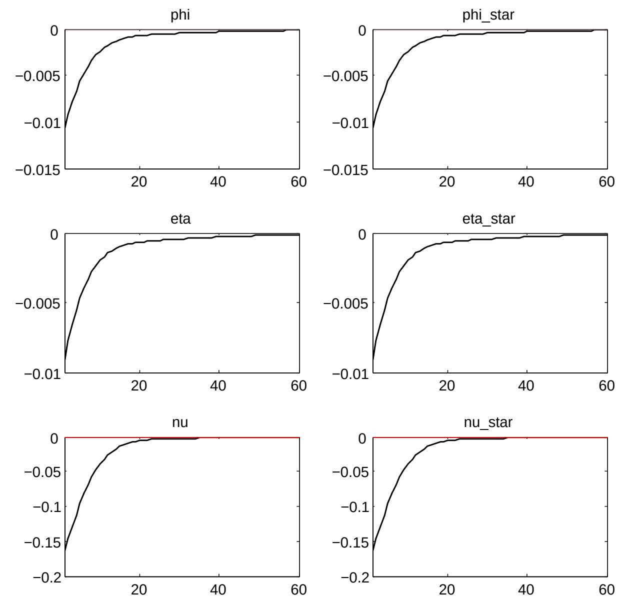
$\Omega_{\text{diff}} \equiv \log \Omega^*_{real,t} - \log \Omega_{real,t}$

$\Delta C_{\text{diff}} \equiv \gamma(\Delta c - \Delta c^*)$

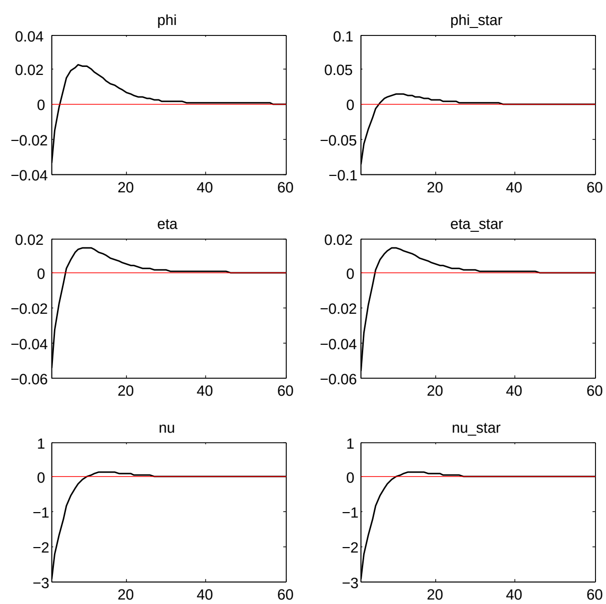
$\Delta C\Delta P_{\text{diff}} \equiv \gamma(\Delta c - \Delta c^*) + (\Delta p - \Delta p^*)$



(a) Offshore Deposit ($\sigma_\kappa = 0.05$), Shock to e_a

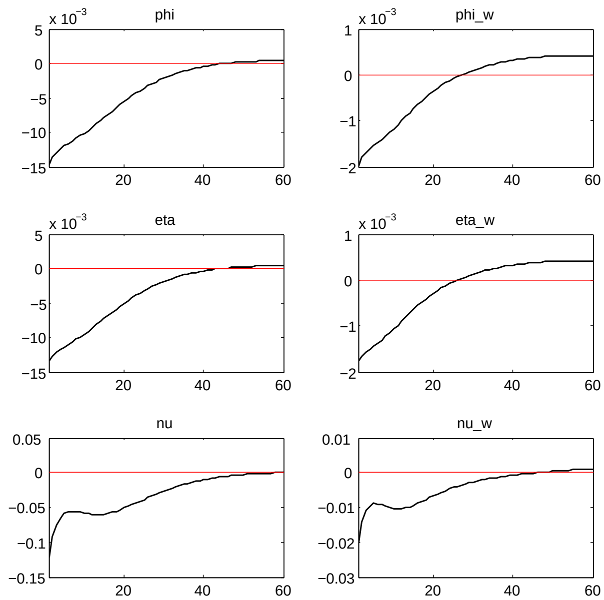


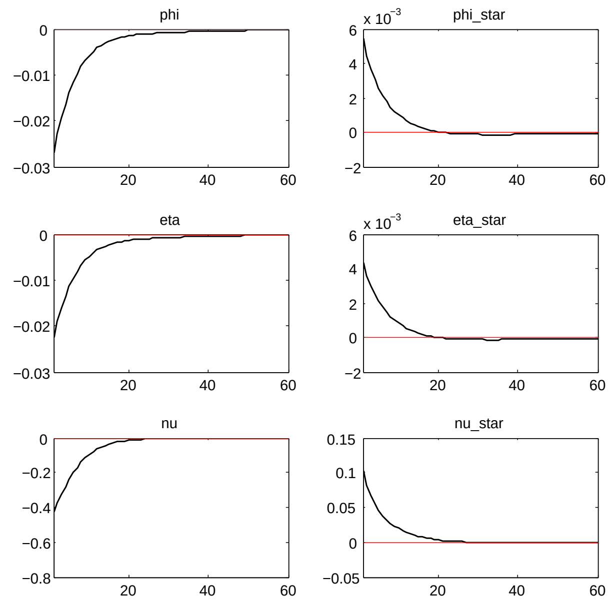
(b) Offshore Deposit ($\sigma_\kappa = 0.05$), Shock to e_m



(c) Offshore Deposit ($\sigma_\kappa = 0.05$), Shock to e_κ

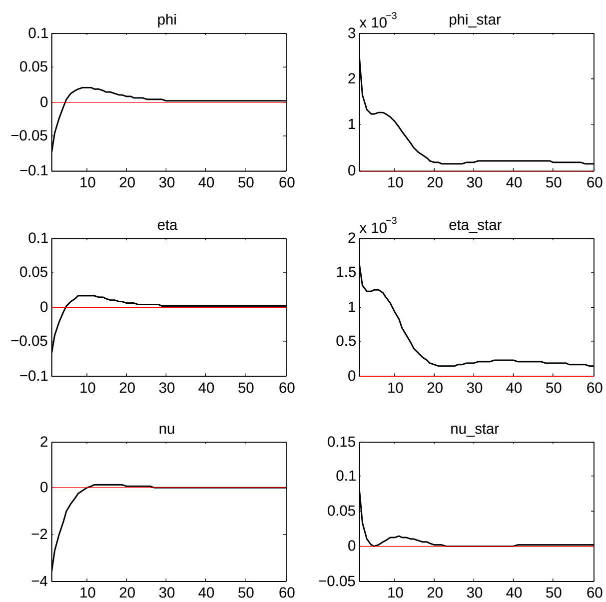
Figure 3: Extended Model, Integrated Deposit Market - Impulse Response Plots





(a) Segmented Deposit ($\sigma_\kappa = 0.03$), Shock to e_a

(b) Segmented Deposit ($\sigma_\kappa = 0.03$), Shock to e_m



(c) Segmented Deposit ($\sigma_\kappa = 0.03$), Shock to e_κ

Figure 4: Extended Model, Segmented Deposit Market - Impulse Response Plots

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Chapter 2:
Limited Stock Market Participation and Goods Market Frictions: A Potential Resolution for Puzzles in International Finance

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Abstract

We study asset prices, exchange rates, and consumption dynamics in a general equilibrium two-county macro-finance model that features limited stock market participation as well as non-traded goods and distribution cost. The model generates a high price of risk, smooth exchange rates, and makes substantial progress towards explaining the empirically observed low consumption growth correlation between countries. We find that distribution cost plays a central role for reducing international consumption co-movement while also amplifying risk premia.

1 Introduction

It has long been a challenge for macro-finance models to jointly explain i) the high equity premium, ii) relatively smooth exchange rates, and iii) the low international correlation of consumption growth.[1] We propose a general equilibrium two-county macroeconomic model that features limited stock market participation as well as non-traded goods and distribution cost to address these salient features of the data.

Our model brings together two strands of literature. Firstly, we build on work showing that the nature of goods markets is an essential determinant of the correlation of consumption growth between countries in general equilibrium models. In particular, Corsetti et al. (2008) show that modeling goods markets to feature non-tradable goods and distribution cost helps decrease the strong international correlation of consumption. These features moderate the international risk sharing mechanism shown in Cole and Obstfeld (1991), in which a country hit by an adverse productivity shock benefits from a natural hedge, as its goods become more scarce and appreciates in price. We model distribution services that are produced with the intensive use of local inputs, allowing the model to generate deviations form the law of one price. The role of such distribution cost in explaining real exchange rate movements has been emphasized by a growing empirical literature, e.g. Crucini et al. (2005).

Secondly, we draw from the literature on limited stock market participation. We adopt the asset market structure of Guvenen (2009), where only a fraction of the population has access to the stock market and the remainder of agents are restricted to trade in a bond.[2] This type of setup has two appealing features for our analysis. First, it is know to generate a realistic price of risk, i.e. Sharpe ratio, which we need in order to jointly study asset prices, international consumption co-movements, and exchange rates. Secondly, it introduces market incompleteness. Brandt et al. (2006) argue that the equilibrium condition linking marginal utility growth to the rate of depreciation in the exchange rate that results in complete market models makes it impossible to generate high risk premia, smooth exchange rates, and moderately correlated consumption growth simultaneously. Allowing for market incompleteness in the form of limited stock market participation breaks this link, making it feasible - at least in principle - for the model to match the three stylized data facts.

There have been previous attempts in the literature to generate the joint dynamics of asset prices, consumption, and exchange rates. Colacito and Croce (2011) study a model that combines cross-country-correlated long-run risk with Epstein-Zin preferences. In their paper,

[1] See, for example, Brandt et al. (2006).

[2] See also Vissing-Jorgensen (2002).

the two countries' exogenous consumption growth processes are calibrated to be moderately correlated but include a persistent predictable component that is highly correlated across counties. This leads to moderate consumption growth correlation and high pricing kernel correlation, allowing the model to successfully match the stylized data facts. Stathopoulos (2012) uses preferences with external habit formation and home-bias in consumption to address the puzzle. The present paper differs from previous work in that we do not resort to non-standard preferences. Rather, we ask whether goods market frictions that have been studied in the international macroeconomics literature can generate a similar result. Our results also do not rely on exogenous driving processes such as long-run risk or external habits that are empirically difficult to observe.

2 The Role of Asset and Goods Markets

The stylized fact in the data that our model attempts to rationalize is the co-existence of moderately correlated consumption growth between countries with smooth exchange rates and high equity risk premia. Our model features two main ingredients that help it address these data facts. On the asset market side, we assume that only a limited fraction of agents in each country can participate in equity markets. On the goods market side, we model a traded and non-traded sector in each country featuring distribution cost for the consumption of tradables.

In this section, we motivate the choice of these two key model components and provide intuition for why they help the model come closer to the data. Since an analytical solution is not available for the full model, we conduct this analysis with respect to two benchmarks. First, we study a complete markets model. This analysis highlights the importance of introducing some form of market incompleteness - in our case limited stock market participation - as a necessary condition for models of a wide class to be able to fit the data. Then we study another benchmark, a model *without* financial markets and with just traded goods. In that setup, we revisit the result from Cole and Obstfeld (1991) that consumption tends to co-move strongly between countries even in the absence of financial markets and provide intuition for how non-traded goods with distribution cost weaken this effect.

2.1 Asset Markets

To motivate our modeling of asset markets, we first consider a representative agent model with complete financial markets in which agents have power utility $U(C)$. In this type of framework, one of the equilibrium conditions requires that exchange rates depreciate by the difference between foreign and marginal utility grow,

$$\ln\frac{Q_{t+1}}{Q_t} = \ln U'\left(\frac{C^*_{t+1}}{C^*_t}\right) - \ln U'\left(\frac{C_{t+1}}{C_t}\right). \tag{1}$$

Here, C_t denotes the home consumption index at time t and the * superscript indicates that a variable refers to the foreign country (as we will adopt throughout the paper). The real exchange rate is $Q = \frac{P_F}{P_H}$, where P_H denotes the consumer price index for the aggregate domestic consumption basket and P_F denotes the same index for the foreign consumption basket.

Brandt et al. (2006) use equation (1) to document a tension that exists between the data and the theory. On the one hand, we know that observed high risk premia in financial markets require marginal utility growth to be highly volatile at home and in the foreign country. On the other hand, observed exchange rates are comparatively smooth. With regards to equation (1), the only way that exchange rates on the left hand side can exhibit low volatility while the two marginal utility terms on the right hand side can exhibit high volatility is if the marginal utilities are highly correlated. This implication of theory is not borne out by the data, however, as the correlation of consumption growth across countries is around 0.6 (depending on the measure and country pair), much lower than required to satisfy equation (1).

In order to break the tight link between exchange rates and consumption growth in equation (1), we assume that only a fraction of the population in each country has access to the stock market. Hence, we introduce a particular type of market incompleteness in which stockholders have access to more complete markets than non-stockholders. In addition, using a limited stock market participation setup has proven successful in explaining high equity risk premia. The model in Guvenen (2009) represents the current state-of-the-art framework for asset pricing with limited stock market participation and we will adopt his specification of the asset market in our full model below.

While this risk sharing condition in equation (1) only arises if financial markets are complete, it turns out that it still holds approximately true even in incomplete markets. The goods market frictions that we turn to next play an important role in that respect.

2.2 Goods Markets

While having incomplete markets is necessary to reconcile asset prices with consumption and exchange rate dynamics, it is not sufficient. In particular, Cole and Obstfeld (1991) famously point out that consumption growth tends to be highly correlated across countries even without financial markets. We will in turn summarize their argument in a model with no financial markets and only traded goods. Note that the exact relationships derived below will not hold in our model. However, analyzing the role of distribution cost in a simple framework that is analytically tractable is useful to build intuition for the results from our full model that we present in the next section.

There are two symmetrical countries, called "home" and "foreign". Aggregate consumption (also referred to as the consumption index) in the home county is given by the constant elasticity of substitution (CES) goods aggregator

$$C = C_T \equiv \left[a_D^{1-\rho}\left(c_H\right)^{\rho} + \left(1-a_D\right)^{1-\rho}\left(c_F\right)^{\rho}\right]^{1/\rho}, \qquad \rho < 1, \tag{2}$$

where c_H denotes domestic consumption of the home tradable good and c_F denotes domestic consumption of the foreign tradable good. The elasticity of substitution between the two varieties of traded goods is given by $\omega = \frac{1}{1-\rho}$ and the weight of home tradables in aggregate consumption is a_D. Further, define the terms of trade $\tau = \frac{p_F}{p_H}$ as the ratio of the price of the foreign tradable p_F to that of the domestic tradable p_H. Corsetti et al. (2008) show that the response of domestic demand for the home good to a fall in its price (increase in τ) can be decomposed into a substitution effect (SE) and income effect (IE), such that

$$\frac{\partial C_H}{\partial \tau} = \underbrace{\omega \frac{a_H\left(1-a_H\right)\tau^{-\omega}}{\left[a_H + \left(1-a_H\right)\tau^{1-\omega}\right]^2} Y^T}_{\text{SE}} \underbrace{- \frac{a_H\left(1-a_H\right)\tau^{-\omega}}{\left[a_H + \left(1-a_H\right)\tau^{1-\omega}\right]^2} Y^T}_{\text{IE}}, \tag{3}$$

where Y^T is the home endowment of the domestic tradable good.

Corsetti et al. (2008) illustrate that Equation (3) allows us to analyze how supply shocks propagate between countries. If the elasticity of substitution between domestic and foreign tradables ω is larger than 1, then the substitution effect dominates the income effect. This is the case studied by Cole and Obstfeld (1991). If the home country is hit with a negative supply shock, the home good needs to become more expensive (τ decreases, the terms of trade improve) for domestic demand to fall and match supply. This partially compensates domestic agents for the adverse supply shock by raising the value of their tradable endowment in international goods markets. Hence, with a large trade elasticity, goods market prices endogenously adjust to provide insurance against negative supply shocks. For this reason, consumption growth will be highly correlated across countries even in the absence of financial markets. This mechanism is amplified by the fact that foreign demand for the home good unambiguously decreases in its price, i.e. $\frac{\partial C_H^*}{\partial \tau}$, as the substitution and income effects are both positive regardless of the trade elasticity.

Now consider the case where the trade elasticity is below one. Then, the income effect dominates. If the home country is hit with a negative shock, the price of the home tradable has to fall in order to induce home agents to reduce their demand to match the restricted supply. Hence, the value of their income drops further, amplifying the effect of the negative endowment shock on their consumption. This is the key mechanism that pushes the consumption growth correlation below unity. Note, however, that the domestic income effect not only has to be stronger than the domestic substitution effect - it also has to outweigh the positive income and substitution effects of the foreign agents.

In the next section, we develop the full model whose goods market side also includes non-tradable goods and distribution cost. It turns out that distribution cost amplify the size of the income effect relative to the substitution effect and play a quantitatively important role for our results. We will discuss the role of the distribution cost in more detail in Section 6.2. Further, recall that equation (3) only holds without financial markets. The full model will have nearly complete asset markets, allowing agents to share consumption risk more effectively between countries. The quantitative results from the full model will hence shed light on the question of how much of the income effect discussed in this section survives after adding financial markets.[3]

3 Model

This section presents the full model featuring limited stock market participation and distribution cost with non-traded goods. Each country is now endowed with *two* Lucas trees, producing a country-specific tradable and non-tradable good, respectively. Furthermore, each country is inhabited by two types of agents according to the set of financial securities they are allowed to hold. A fraction μ of agents in each country has access to the home and foreign stocks as well as the one international bond, a fraction $1-\mu$ can only hold the international bond. In the interest of brevity, we focus our presentation on the domestic economy with the understanding that the foreign counterparts are defined symmetrically.

3.1 Preferences

As above, the consumption index for tradable consumption C_T is given by equation (2). Aggregate consumption now consists of traded and non-traded goods with CES aggregator

$$C \equiv \left[a_T^{1-\phi} \left(C_T\right)^{\phi} + \left(1-a_T\right)^{1-\phi} \left(c_N\right)^{\phi} \right]^{1/\phi}, \qquad \phi < 1,$$

[3]Note that although our model features essentially the same goods market frictions as in Corsetti et al. (2008), the financial market structure is vastly different as the only tradable asset is uncontingent international bond in their study. Although a reasonable level of incompleteness is introduced by limited participation, our financial market structure features both the home and foreign stock markets.

where c_N is domestic consumption of the home non-tradable good. The elasticity of substitution between tradables and non-tradables is $\frac{1}{1-\phi}$ and agents assign a weight of a_T to tradables in aggregate consumption.

Agents have power utility and maximize

$$\mathrm{E}\left[\sum_{t=0}^{\infty}\theta_t\frac{C_t^{1-\gamma}}{1-\gamma}\right],$$

where γ controls relative risk aversion. The discount factor θ_t is endogenous and evolves as $\theta_{t+1} = \theta_t\omega C_t^{-\eta}$, with $0 \leq \eta \leq \gamma$ and $0 < \omega\bar{C}^{-\eta} < 1$ and where $\bar{C}$ denotes the steady-state value of consumption.[4]

3.2 Distribution Cost and Goods Prices

In addition to distinguishing between tradable and non-tradable goods, our economy features distribution cost such that for every unit of either the home or foreign tradable good consumed in the home (foreign) country, ν units of the home (foreign) non-tradable good are needed to distribute the tradable good to consumers. This drives a wedge between prices for tradable goods at the producer and consumer level. Taking consumption of the home-tradable in the home country as an example,

$$p_H = \bar{p}_H + \nu p_N$$

gives the relation between the producer price of the home tradable, $\bar{p}_H$, its consumer price, p_H, and the price of the home non-tradable, p_N.

The utility-based consumer price indices for the home basket of tradable goods is

$$P_T = \left[a_D\left(p_H\right)^{\rho/(\rho-1)} + (1-a_D)\left(p_F\right)^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho}.$$

Similarly, the utility-based consumer price index for the aggregate home consumption basket is

$$P_H = \left[a_T\left(P_T\right)^{\phi/(\phi-1)} + (1-a_T)\left(p_N\right)^{\phi/(\phi-1)}\right]^{(\phi-1)/\phi}.$$

[4]Endogenizing the discount factor in this way pins down a unique steady state for the distribution of wealth in the presence of incomplete financial markets. Otherwise, the model would exhibit a unit-root and not be amenable to standard numerical solution techniques. The use of this discount factor is standard in such settings. See Schmitt-Grohe and Uribe (2003), Corsetti et al. (2008), and Devereux and Sutherland (2011) for further discussions.

We choose the home consumption basket as the numeraire, so that $P_H \equiv 1$. The exchange rate is given by $Q = \frac{P_F}{P_H}$, where P_F denotes the consumer price index for the aggregate foreign consumption basket.

3.3 Endowments and Asset Markets

The home country endowments of the tradable good, Y^T, and the non-tradable good, Y^N, evolve as

$$\begin{aligned} \ln Y_t^T &= \Psi^T \ln Y_{t-1}^T + \epsilon_t^T \\ \ln Y_t^N &= \Psi^N \ln Y_{t-1}^N + \epsilon_t^N, \end{aligned}$$

where ϵ_t^T and ϵ_t^N are iid normally distributed disturbances.

Labor income Y_L and capital income Y_K are given by

$$\begin{aligned} Y_{L,t} &= \theta_L \left(\theta_T p_{H,t}^T Y^T + (1-\theta_T)\, p_{H,t}^N Y^N\right) \\ Y_{K,t} &= (1-\theta_L)\left(\theta_T p_{H,t}^T Y^T + (1-\theta_T)\, p_{H,t}^N Y^N\right), \end{aligned}$$

where the parameter θ_L controls the labor share and θ_T controls the share of traded goods.

Denoting asset prices by Z with the appropriate sub and superscripts, the return to a claim to the home capital income is $r_{A,t+1} = \frac{Z_{A,t+1}+Y_{K,t+1}}{Z_{A,t}}$ and the corresponding return to the foreign capital income is $r^*_{A,t+1} = \frac{Z^*_{A,t+1}+Y^*_{K,t+1}}{Z^*_{A,t}}$. Furthermore, there is an international bond which pays off half a unit of the home tradable and half a unit of the foreign tradable with return $r^*_{B,t+1} = \frac{\frac{1}{2}(p_{H,t+1}+p_{F,t+1})}{Z^*_{B,t}}$.

While both the stock market participants and non-participants receive labor income, the capital income is endowed only to the stock market participants.[5] Recalling that the non-participants can only invest in the international bond, the budget constraints for the two types of home agents can be written as

$$\begin{aligned} W_t^p &= r_{b,t}W_{t-1}^p + \alpha_{1,t-1}^p\left(r_{A,t}-r_{b,t}\right) + \alpha_{2,t-1}^p\left(r^*_{a,t}-r_{b,t}\right) - C_t^p + \frac{1}{\mu}Y_{K,t} + Y_{L,t} \\ W_t^{np} &= r_{b,t}W_{t-1}^{np} - C_t^{np} + Y_{L,t}, \end{aligned}$$

where $W^p = \alpha_1^p + \alpha_2^p + \alpha_3^p$ and $W^{np} = \alpha_3^{np}$ denote net financial wealth of domestic participants and non-participants, respectively. The net amounts invested by an agent are denoted by

[5]As pointed out in Guvenen (2009), the 20 % of US households who participate in the stock market own 90 % of the economy's wealth.

α_1 for the home stock, α_2 for the foreign stock, and α_3^p for the international bond, with superscripts p and np referring to participants and non-participants, respectively.

Note that since we defined wealth and asset positions as net positions[6], asset market clearing is given by

$$\begin{aligned} \mu \left(W_t^p + W_t^{p*} \right) + (1-\mu) \left(W_t^{np} + W_t^{np*} \right) &= 0 \\ \alpha_{1,t}^p + \alpha_{1,t}^{p*} &= 0 \\ \alpha_{2,t}^p + \alpha_{2,t}^{p*} &= 0, \end{aligned}$$

where foreign variables are denoted by *.

4 Model Solution

The model is challenging to solve. It is not amenable to standard global solution techniques such as value function iteration because the incompleteness of financial markets requires that we solve for the decentralized equilibrium directly. Due to the large number of state variables, the computational burden of doing this is prohibitive.

The method proposed by Chien et al. (2011) who solve incomplete markets economies with heterogeneous trading technologies using stochastic Lagrange multipliers and measurability constraints is inapplicable in our setup as well. The reason for this is that their aggregation result for consumption fails due to the differentiated goods in our setup and the home bias in preferences.[7]

For these reasons, we solve the model using second-order linearization techniques. This requires solving for the steady-state and first-order portfolio choice, for which we implement the method suggested in Devereux and Sutherland (2010). We rely on Dynare to implement this approach.

[6]As an example, consider the case where the home agents has a zero net position in both stocks and the bond ($\alpha_{1,}^p = \alpha_2^p = W^p = 0$). His gross position would correspond to holding all of the home equity and an amount $l-1$ times the size of his gross equity position in the bond. He would hold non of the foreign stock.

[7]While extending their method to accommodate differentiated goods and heterogeneous specifications of goods aggregators might be possible, doing this would require a substantial methodological contribution.

Table 1: Data Summary

Panel A shows covariances between U.S. and foreign traded production (row 1), U.S. and foreign non-traded production (row 2), and U.S. traded and foreign non-traded production (row 3). The moments refer to real, per-capital production that has been logged and hp-filtered. Panel B shows the correlation of real per-capita consumption growth between the US and the foreign countries for non-durable consumption (row 1) and total consumption (durables plus non-durables and services, row 2). Panel C shows the volatility of the real exchange rate between the US and the foreign countries. Panel D shows the average share of a country's trade with the US as a percentage of total US trade between our set of countries. The "Average" column uses the trade weights from Panel D.
Data is annual and covers 1970 to 2012 with the exception of non-durable consumption, which starts in the 1990's for some of our countries and is not available for the UK at all. See Appendix A for more details.

	Average	Canada	France	Germany	Italy	Japan	UK
Panel A: Correlation of Output by Sector							
Traded	0.74	0.85	0.68	0.59	0.68	0.61	0.85
Non-Traded	0.61	0.75	0.78	0.24	0.43	0.47	0.71
Traded / Non-Traded	0.46	0.58	0.62	-0.05	0.09	0.38	0.84
Panel B: Correlation of Consumption Growth							
Non-Durable	0.67	0.85	0.75	0.53	0.81	0.39	N/A
Total	0.52	0.61	0.49	0.39	0.29	0.39	0.72
Panel C: Real Exchange Rate							
Volatility (%)	9.5	6.8	11.1	11.4	11.0	11.8	11.8
Panel D: Trade Weights							
		0.43	0.06	0.11	0.05	0.26	0.10

5 Data and Calibration

We follow a conservative calibration strategy in that we do not choose the structural parameters of our model to directly match the stylized facts for asset price, consumption, and exchange rate dynamics. Rather, we set the structural parameters to the values that are typically used in the literature. Similarly, the moments of traded and non-traded output are calibrated to directly match their empirical counterparts.

The next section describes the data and summarizes some empirical regularities regarding international production, consumption, and exchange rates. We then discuss the model calibration.

5.1 Data

Our main data sources are the National Accounts database provided by the OECD and the International Financial Statistics and Direction of Trade Statistics databases offered by the IMF. We draw on these international datasources rather than on national ones in order to have data measures that are comparable across countries. Furthermore, we use annual data which allows us to analyze time series that start early, ranging from 1970 to 2012. The only time series that is not available for this time horizon is that for non-durable consumption, which starts in the 1990's for most of our countries. Table 1 summarizes the data.

We analyze the data from the perspective of the US as the home country and focus on the other G7 economies, including Canada, France, Germany, Italy, Japan, and the UK, as the foreign countries. We then use the trade-weights reported in Panel D to average the moments with respect to the foreign countries.

While international co-movements in traded and non-traded production have been previously analyzed in the literature, (e.g. Stockman and Tesar (1995)) the data used in these studies only extends to 1990. Since then, rapid technological progress has facilitated international trade. We hence find it important to study more recent data and update the dataset accordingly.

We start by analyzing the international co-movement of output in the traded and non-traded sectors among the G7 countries. Following the methodology of Kravis et al. (1982) and Stockman and Tesar (1995), we assign output to be either tradable or non-tradable depending on the sector of production. We consider agriculture, fishing, mining, manufacturing, electricity and utilities, retail, hotels, and transportation to be tradable.[8] The remaining categories, including construction, finance, real estate, and other services are assigned to the non-tradable sector.

Panel A shows the resulting correlations for real per-capita output in the two sectors that has been logged and hp-filtered. The first row shows the correlation between traded output in the US and traded output in the foreign country. The correlations range from 0.59 for Germany to 0.85 for Canada and the UK. The trade-weighted average, which takes into account the relative importance of a country for US trade, is quite high at 0.74. This is due in large part to the high correlation with Canada, which is responsible for nearly half of US trade among the G7 countries.

[8] While electricity and utilities are arguably non-tradable, in particular as they refer to the associated distribution services, they are reported together with manufacturing, which is a large component of tradable goods. Since electricity and utilities only make up a small fraction of output, classifying them as tradable rather than non-tradable does not have a significant bearing on the results.

The second row of the panel shows the correlation of US non-traded output with foreign non-traded output. For all our countries, these correlations are lower than those for traded output, with a trade-weighted average of 0.61. Finally, while the correlations within the same sectors between countries tend to be quite high, the correlation of traded output in the US with non-traded output abroad reported in row 3 is significantly lower for most countries and even slightly negative for Germany. The trade-weighted average for this correlation between sectors is only 0.46. Overall, these results are quite comparable in magnitude to what Stockman and Tesar (1995) for their earlier sample.

One of our main moments of interest is the correlation of consumption growth between countries. Since all consumption in our model is non-durable, the appropriate moment to match is the correlation of real per-capita consumption growth between countries. From row 1 of Panel B, this correlation ranges from 0.85 with Canada to 0.39 with Japan, averaging 0.67. Since data on non-durable consumption is only available since 1990 for most countries and unavailable for the UK, we also compute the correlation from total household final consumption expenditure which is available over the period from 1970 to 2012. Row 2 of the panel shows that this correlation is significantly lower for all countries, averaging 0.52.

We are also interested in the volatility of real exchange rate growth. Panel C shows that this volatility is around 11% for all countries except for Canada, where it is almost half, at 6.8%. Since Canada is the most important trade partner for the US, we find the trade-weighted average volatility of real exchange rate growth to be 9.5%.

5.2 Calibration

We calibrate the endowment processes for tradable and non-tradable goods to their empirical counterparts in the data. For the within-country moments, we calibrate to US data. This leads us to setting the persistence parameter equal to $\Psi^T = 0.27$ for the tradable sector and $\Psi^N = 0.45$ to the non-tradable sector, matching the first order autocorrelation of hp-filtered US production. Similarly, we choose the volatility of the endowment shocks so that the implied standard deviation of traded output, $\text{std}\left(Y^T\right) = 0.023$, and non-traded output, $\text{std}\left(Y^N\right) = 0.010$, match that from US data. We calibrate the size of the traded sector to match the average share of traded goods in US production, leading to a tradables share of $\theta_T = 0.35$. Finally, we chose the covariance of the shocks to traded and non-traded goods within a country to match the correlation of traded and non-traded production in the US, setting $\text{cor}\left(Y^N, Y^T\right) = 0.64$.

We calibrate between-country moments of output to the average correlation between US and foreign production for a given sector. The moments we match are $\text{cor}\left(Y^T, Y^{T*}\right) = 0.74$ for the correlation of traded production between countries, $\text{cor}\left(Y^N, Y^{N*}\right) = 0.61$ for the

Table 2: Calibration

The model is calibrated at annual frequency.

	Parameter	Source
Risk aversion		
Participants	$\gamma_p = 3$	Guvenen (2009)
Non-participants	$\gamma_{np} = 10$	Guvenen (2009)
Weight on traded goods	$a_T = 0.55$	Corsetti et al. (2008)
Home bias in tradables	$a_D = 0.72$	Corsetti et al. (2008)
Elasticity of substitution		
Home and foreign traded goods	$\frac{1}{1-\rho} = 0.85$	Corsetti et al. (2008)
Traded and non-traded goods	$\frac{1}{1-\phi} = .74$	Corsetti et al. (2008)
Endogenous discount factor		
Curvature	$\eta = .1$	
Steady-state discount rate	$\omega \bar{C}^{-\eta} = 0.95$	Guvenen (2009)
Distribution cost	$\nu = 0.85$	
Labor share	$\theta_L = 0.7$	Corsetti et al. (2008)
Tradables share	$\theta_T = 0.35$	
Stock market participation rate	$\mu = 0.3$	Guvenen (2009)
Endowments		
Autocorrelation of tradables	$\Psi^T = 0.27$	
Autocorrelation of non-tradables	$\Psi^N = 0.45$	
Implied moments	$\text{std}\left(Y^T\right) = 0.023$	
	$\text{std}\left(Y^N\right) = 0.010$	
	$\text{cor}\left(Y^N, Y^T\right) = 0.64$	
	$\text{cor}\left(Y^T, Y^{T*}\right) = 0.74$	
	$\text{cor}\left(Y^N, Y^{N*}\right) = 0.61$	
	$\text{cor}\left(Y^N, Y^{T*}\right) = 0.46$	

correlation of non-traded production between countries, and $\text{cor}\left(Y^N, Y^{T*}\right) = 0.46$ for the correlation of domestic non-tradable output with foreign tradable output.

We follow the working paper version of Guvenen (2009) in calibrating relative risk aversion, the time discount rate, and stock market participation.[9] Specifically, we set relative risk aversion to $\gamma_p = 3$ for stockholders and $\gamma_{np} = 10$ for non-stockholders. The steady-state discount rate is $\omega \bar{C}^{-\eta} = 0.95$.[10] We calibrate the stock market participation rate to $\mu = 0.3$.

[9]The working paper version of Guvenen (2009) differs from the published paper in that it uses power utility (as this paper) instead of recursive Epstein-Zin preferences.

[10]We use a value of $\eta = .1$ for the curvature of the endogenous discount factor, which is reasonably

While Guvenen (2009) uses a parameter value of 0.2, he also points to recent evidence that stock market participation has increased. We take this into account by choosing a slightly higher participation rate than him as we calibrate the model to more recent data.

The remaining utility parameters refer to agent's preferences over the different types of goods. Here, we follow Corsetti et al. (2008). Like them, we set the utility weight of tradables to $\theta_T = 0.35$, matching the share of traded goods in the US consumption basket, and the home bias in tradable goods to $a_D = 0.72$. Similarly, we chose the elasticity of substitution between the two traded goods to be $\frac{1}{1-\rho} = 0.85$ and the elasticity between the traded and non-traded good to be $\frac{1}{1-\phi} = .74$, which the authors obtain by performing a method of moments estimation on a model whose goods market structure is similar to ours.

A key parameter in our model is the distribution cost parameter v. As will become apparent in the next section, the distribution cost are quantitatively the most important feature of the model in reducing the correlation of consumption growth between countries. The higher v, the lower the consumption correlation. While we want to restrict the magnitude of the distribution cost to be consistent with results in previous studies, we chose it to be on the high end of that spectrum. This allows us to evaluate how far the present model can go in matching the low consumption correlation in the data. When we study the quantitative importance of distribution cost for our results, we will then conduct extensive sensitivity analysis with regards to this parameter. There are several studies that estimate the distribution margin, which is defined as $\kappa = v\frac{p_N}{p_H}$. Burstein et al. (2003) find that the share of the retail price accounted for by distribution services is between 40% to 50% in the US, depending on the industry. Anderson and Van Wincoop (2004) find that distribution cost average more than 55% among industrialized countries. Considering this evidence, we set $\nu = 0.85$, which implies a stead-state distribution margin of 64% in our model.

6 Results

6.1 Full Model

Table 3 summarizes the moments implied by the fully featured model. The model matches asset prices rather well. The Sharpe ratio of 0.31 is nearly identical to that in the data. While the model produces a realistic *price* of risk, the equity premium is only 3.06%, about half of what it is in the data. This is not surprising, however, given that there is no financial leverage in the model and hence the *quantity* of risk is less than in the data. This is also

small while still producing a stable model solution.

Table 3: Results

All moments are annual and in percent.

	Model	Data	Source
Asset markets			
Equity premium	3.06	6.17	Guvenen (2009)
Volatility of equity premium	9.81	19.40	Guvenen (2009)
Risk-free rate	0.72	1.94	Guvenen (2009)
Volatility of risk-free rate	9.97	5.44	Guvenen (2009)
Sharpe ratio	0.31	0.32	Guvenen (2009)
Exchange rate growth volatility	3.11	9.50	
Consumption growth			
Aggregate volatility	1.26	1.95	
Volatility participants/non-participants	3.30	> 2	Guvenen (2009)
Cross country correlation	0.73	≈ 0.6	

reflected in the fact the the equity premium is about half as volatile in the model as in the data. The risk-free rate is 0.72%, which is just slightly lower than in the data. The standard deviation of the risk-free rate is higher than in the data, with 9.97% in the model compared to 5.44% in the data.

The model-implied correlation of consumption growth is 0.73, which is slightly larger than the value of 0.67 that is implied using non-durable consumption data and considerably larger than the value of 0.52 the we measure using total household final consumption expenditure. The model hence makes substantial progress in generating less than perfect consumption co-movement between countries though it falls short of fully explaining the low consumption correlation in the data.

Finally, we find that the volatility of exchange rate growth is low. It is less than half than what we measure in the data. This finding is consistent with much of the literature, e.g. the international real business cycle model of Backus et al. (1992).

We proceed by analyzing the role of distribution cost for the model mechanism in Section 6.2 and then quantify the importance of all our main model ingredients for our results in Section 6.3.

Table 4: Distribution Cost

The table shows the correlation of aggregate consumption growth between countries, the Sharpe ratio, and the volatility of the exchange rate in the benchmark model for varying degrees of distribution cost ν.

	Distribution cost ν					
	0	0.2	0.4	0.6	0.8	0.85
Consumption correlation	0.98	0.97	0.95	0.89	0.77	0.73
Sharpe ratio	0.24	0.23	0.22	0.22	0.27	0.31
Exchange rate volatility	1.52	1.75	2.11	2.64	3.10	3.11

6.2 The Importance of Distribution Cost

Table 4 shows how the model results change for varying values of the distribution cost parameter ν. The comparative statics illustrate the importance of distribution cost for the model's ability to produce a consumption correlation below unity. In fact, without distribution cost, consumption co-moves nearly perfectly between countries despite the existence of non-tradable goods and a low trade elasticity. The correlation only drops significantly for values of the distribution cost that are on the high end of the empirically observed spectrum, reaching a correlation of 0.73 in our benchmark calibration with $\nu = 0.85$.

To provide intuition for the effect of distribution cost on the correlation of consumption, we return to our analysis from Section 2.2. In particular, we focus on how distribution cost lower the effective elasticity of substitution between the traded goods at the consumer level and hence amplify the magnitude of the income effect studied in that section.

To see how introducing distribution cost helps lower the correlation of consumption growth, consider the equivalent of Equation 3 with distribution cost,

$$\frac{\partial C_H}{\partial \tau} = \underbrace{\omega\left(1-\kappa\right)\left(1-a_H\right)\left(\frac{P_F}{P_H}\right)^{1-\omega}}_{\text{SE}} \underbrace{-\left(1-a_H\right)\left(\frac{P_F}{P_H}\right)^{1-\omega} - \kappa a_H}_{\text{IE}}.$$

Similar to above, this equation shows the response of domestic demand for the home good to a fall in its price (increase in τ) in a model without financial markets. The expression here however takes account of distribution cost, which are linear in the distribution margin $\kappa = v\frac{p_N}{p_H}$. We see that an increase in the distribution margin lowers the magnitude of the substitution effect and increases the (negative) importance of the income effect.

Next, we turn to the importance of distribution cost for the price of risk in the economy. Without the cost, the Sharpe ratio is 0.24. It increases to 0.31 for the high value of the cost in our benchmark calibration. This increase in volatility due to the distribution cost is also reflected in the volatility of the exchange rate, which increases in the cost as well. It is, however, low compared to the data for the entire range of the parameter studied here.

6.3 Relative Importance of Model Ingredients for the Results

In this section, we quantify the relative importance of our main model ingredients, limited stock market participation, non-traded goods, and distribution cost, for our main results. Table 5 shows our main moments of interest, the consumption growth correlation, the Sharpe ratio, and the exchange rate volatility for six different model specifications. We study three special cases with respect to the goods market: only traded goods, traded and non-traded goods but without distribution cost, and traded and non-traded goods with distribution cost. For each of these three cases, we solve a version of the model with and without limited stock market participation.

First, we find that virtually all of the reduction in the consumption growth correlation comes from distribution cost. Irrespective of the financial market setup, we find that the consumption growth correlation is around 0.73 with distribution cost and close to unity without.

With regards to the Sharpe ratio, we find that the model without stock market participation only produces a small price of risk that does not exceed a Sharpe ratio of 0.06. However, once limited stock market participation is introduced, the price of risk does increase significantly. If all goods are tradable, the Sharpe ratio reaches 0.18 and increases significantly both with the addition of non-traded goods and by introducing distribution cost. The goods market setup matters for asset prices as both non-tradablility and distribution cost raise the volatility of the utility based consumption index.

Finally, the model produces very smooth exchange rates in all versions, ranging from a standard deviation of 0.63% in the model with full stock market participation and all traded goods to a standard deviation of 3.11% in the model with limited stock market participation and non-traded goods with distribution cost.

6.4 Consumption Dynamics by Agent Type

Our analysis thus far has focused on the correlation of aggregate consumption growth between countries and we have shown that the model is capable of producing a correlation as low as 0.73. We next analyze the consumption dynamics in more detail by focusing on the different agent types.

Here, we find that the current model has an unappealing implication. In particular, consumption growth between stockholders and non-stockholders is nearly perfectly *negatively*

Table 5: The Contribution of the Model Ingredients

The table shows the correlation of aggregate consumption growth between countries, the sharpe ratio, and the volatility of exchange rate growth for six different model specifications. The two asset market specifications permit either full stock market participation ($\mu = 1$) or limited stock market participation ($\mu = 0.3$). The goods market either includes only traded goods (T), traded and non-traded without distribution cost (T/NT), or the fully featured goods market specification with traded and non-traded goods as well as distribution cost (T/NT/Dist).

	T	T/NT	T/NT/Dist	T	T/NT	T/NT/Dist
	Consumption correlation			Sharpe ratio		
$\mu = 1.0$	0.98	0.97	0.74	0.02	0.04	0.06
$\mu = 0.3$	1.00	0.98	0.73	0.18	0.24	0.31
	Exchange rate volatility (%)					
$\mu = 1.0$	0.63	1.37	2.07			
$\mu = 0.3$	0.68	1.52	3.11			

correlated. The reason for this is the low persistence of output that we measure in the data and to which we calibrate our endowment processes. As agents are hit by a positive supply shock, they expect output to mean-revert quickly. Hence they expect negative future consumption growth. Non-stockholders would like to increase their precautionary savings to smooth the expected reversion of output. Stockholders, on the other-hand, are reluctant to increase their borrowing substantially. As a result, the interest rate falls and reduces the value of non-stockholders' bond holdings (which are positive, on average). This reduction in wealth forces non-stockholders to reduce their consumption despite the positive endowment shock. While calibrating output to be highly persistent with an auto-correlation above 0.95 annually resolves this problem, we find that output is just not nearly this close to a unit root in the data.

As expected, we find that the correlation of consumption growth between foreign and domestic stockholders is unity as they have access to virtually complete financial markets. The above fact that consumption of non-stockholders is almost perfectly negatively correlated with that of stockholders then implies that the correlation of consumption growth between foreign and domestic non-stockholders is nearly unity as well. Consumption for both groups of non-stockholders hence moves together and against that of stockholders. The result that the *aggregate* consumption growth between countries is only 0.73 despite consumption for stockholders and non-stockholders co-moving nearly perfectly then comes from the cross-correlation of stockholders consumption in one country with that of non-stockholders in the other. This correlation is almost perfectly *negative*, driving down the aggregate consumption growth correlation.

It is worth pointing out that the low aggregate consumption growth correlation the model achieves still obtains if we allow all agents to participate in the stock market, as can be seen from Table 5. Hence, this results does not hinge on the unappealing co-movement of consumption between the different groups of agents. That said, we regard improving the model to ameliorate its implications along this dimension as a crucial next step for future research.

7 Conclusion

We propose a general equilibrium two-county macro-finance model that features limited stock market participation, non-traded goods and distribution cost. The model makes significant progress towards rationalizing the coexistence of three stylized data facts that have been a challenge for theory thus far: i) The high equity premium, ii) relatively smooth exchange rates, and iii) the low international correlation of consumption growth.

Consistent with closed-economy models, the limited stock market participation friction produces a high and realistic price of risk. We further find that distribution cost play a central role for reducing international consumption co-movement while also amplifying risk premia. The model naturally produces a low exchange rate volatility that is even lower than in the data, irrespective of the severity of the frictions we study.

Future research will need to focus on resolving the stark implications for the consumption dynamics between agent types that are implied by the model.

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APPENDIX

A Data

Our main data sources are the OECD National Accounts and the IMF International Financial Statistics. To ensure that we have long time series, we use data at the annual frequency. For most measures, we are able to obtain data from 1970 to 2012. Table 1 provides summary statistics for the data.

A.1 Production of Tradables and Non-Tradables

We use annual data covering 1970 to 2012 from the OECD on value-added by sector for the US, Canada, France, Germany, Italy, Japan, and the UK. We then categorize output to be either tradable or non-tradable, depending on its sector, as described in the main text.

The data we retrieve is measured in constant prices and PPPs fixed in the OECD base year (2005). We then divide by each country's population (also from the OECD) to get sectoral output in per-capita terms. Finally, we detrend the data by taking logs and applying an hp-filter with smoothing parameter 6.25. Table 1 shows the resulting correlations of detrended output across sectors between the US and our set of six foreign countries.

A.2 Consumption

We use two different measures of consumption provided by the OECD, one that measures only non-durable consumption and one that measures total final household consumption.

Our measure for non-durable consumption is the sum of household final consumption expenditure for non-durables and services. These time series are in constant prices (OECD base year = 2005) and hence are additive. This measure is available starting in 1981 for Canada, 1959 for France, 1991 for Germany, 1995 for Italy, 1994 for Japan, and 1970 for the US. It is not available for the UK.

To have a longer time series of consumption, we also obtain final household consumption expenditure in constant prices of the OECD base year. This measure is available from 1970 to 2012 for all countries.

For both measures, we then divide by the country's population, take logs, and compute the growth rate. Panel B of table 1 shows the correlation of US consumption growth with our set of foreign countries for the two measures.

A.3 Exchange Rates

We retrieve annual end-of-period nominal exchange rate data from the IMF IFS Database. Exchange rates are in terms of foreign currency to USD. To convert the nominal exchange rates to real terms, we use the deflator for household final consumption expenditure provided by the OECD. Panel C of Table 1 shows the volatility of real exchange rate growth that we obtain. The data cover 1970 to 2012 for all countries.

A.4 Trade Weights

We obtain data on imports and exports between the US and our set of foreign countries from the IMF Direction of Trade Statistics database. The data are in USD terms and span 1970 to 2012 for all countries. Then, for every year, we determine a country's trade weight with the US as the sum of imports and exports between that country and the US divided by the sum of all imports and exports between the US and our complete set of foreign countries. This procedure yields one trade weight for every country in every year. We then compute the time-series averages for every country. The resulting trade weights are reported in Panel D of table 1.

B Goods Market Clearing

The goods market clearing conditions are for domestic tradables, domestic non-tradables, foreign tradables, and foreign non-tradables (in that order) are

$$\begin{aligned}
&\mu\left(c_H^p + c_H^{p*}\right) + (1-\mu)\left(c_H^{np} + c_H^{np*}\right) = \theta_T Y^T \\
&\mu\left(c_N^p + \nu c_H^p + \nu c_F^p\right) + (1-\mu)\left(c_N^{np} + \nu c_H^{np} + \nu c_F^{np}\right) = (1-\theta_T)\, Y^N \\
&\mu\left(c_F^p + c_F^{p*}\right) + (1-\mu)\left(c_F^{np} + c_F^{np*}\right) = \theta_T Y^{T*} \\
&\mu\left(c_N^{p*} + \nu c_H^{p*} + \nu c_F^{p*}\right) + (1-\mu)\left(c_N^{np*} + \nu c_H^{np*} + \nu c_F^{np*}\right) = (1-\theta_T)\, Y^{N*},
\end{aligned}$$

where * denotes foreign variables.

C Portfolios Choice

This section outlines how we adapt the method in Devereux and Sutherland (2010) and Devereux and Sutherland (2011) to the case with multiple assets and a non-zero exchange rate.

C.1 Portfolio Choice Equations

In what follows, we denote the base asset, corresponding to the international bond in the main text, as asset 4. The exchange rate is denoted by E and all returns are converted to units of the home consumption basket.

For every asset m, the home and foreign portfolio choice equations are

$$\begin{aligned} \mathrm{E}\left[C_{t+1}^{-\gamma}\left(R_{m,t+1}-R_{4,t+1}\right)\right] &= 0 \\ \mathrm{E}\left[C_{t+1}^{*-\gamma}\frac{1}{E_{t+1}}\left(R_{m,t+1}-R_{4,t+1}\right)\right] &= 0. \end{aligned}$$

C.2 Steady-state portfolio

Expanding the portfolio choice equations to the second order accuracy and taking the difference yields

$$\mathrm{E}\left[\left(\hat{C}_{t+1}-\hat{C}_{t+1}^{*}-{}^{\hat{E}_{t+1}}/\gamma\right)\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\right]=0+O\left(\epsilon^{3}\right).$$

The steady-state portfolio is the one that satisfies this equation as outlined in Devereux and Sutherland (2011).

C.3 First-order portfolio

Following Devereux and Sutherland (2010), we expand the third-order portfolio choice equations to the third order. For the domestic country, this is

$$\begin{aligned} &\mathrm{E}\Big[\bar{C}^{-\gamma}\left(\bar{R}_m-\bar{R}_4\right)-\gamma\left(\bar{R}_m-\bar{R}_4\right)\bar{C}^{-\gamma}\hat{C}_{t+1}+\bar{C}^{-\gamma}\bar{R}\hat{R}_{m,t+1}-\bar{C}^{-\gamma}\bar{R}\hat{R}_{4,t+1}-\gamma\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}\hat{R}_{m,t+1} \\ &+\gamma\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}\hat{R}_{4,t+1}+\frac{1}{2}\left(\bar{R}_m-\bar{R}_4\right)\gamma^2\bar{C}^{-\gamma}\hat{C}_{t+1}^2+\frac{1}{2}\bar{C}^{-\gamma}\bar{R}\hat{R}_{m,t+1}^2-\frac{1}{2}\bar{C}^{-\gamma}\bar{R}\hat{R}_{4,t+1}^2 \\ &+\frac{3}{6}\gamma^2\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}^2\hat{R}_{m,t+1}-\frac{3}{6}\gamma^2\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}^2\hat{R}_{4,t+1}-\frac{3}{6}\gamma\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}\hat{R}_{m,T+1}^2+\frac{3}{6}\gamma\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}\hat{R}_{4,t+1}^2 \\ &-\frac{1}{6}\gamma^3\bar{C}^{-\gamma}\left(\bar{R}_m-\bar{R}_4\right)\hat{C}_{t+1}^3+\frac{1}{6}\bar{C}^{-\gamma}\bar{R}\hat{R}_{m,t+1}^3-\frac{1}{6}\bar{C}^{-\gamma}\bar{R}\hat{R}_{4,t+1}^3\Big]=0+O\left(\epsilon^4\right) \end{aligned}$$

The third-order expansion of the foreign portfolio choice equation is (where $C^*$is replaced by C for notational convenience)

$$
\begin{aligned}
&\mathrm{E}\Bigg[\bar{C}^{-\gamma}\frac{1}{\bar{E}}\left(\bar{R}_m-\bar{R}_4\right)-\gamma\left(\bar{R}_m-\bar{R}_4\right)\frac{1}{\bar{E}}\bar{C}^{-\gamma}\hat{C}_{t+1}+\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{R}_{m,t+1}-\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{R}_{4,t+1}\\
&-\bar{C}^{-\gamma}\left(\bar{R}_m-\bar{R}_4\right)\frac{1}{\bar{E}}\hat{E}_{t+1}-\gamma\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{C}_{t+1}\hat{R}_{m,t+1}+\gamma\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{C}_{t+1}\hat{R}_{4,t+1}\\
&+\left(\bar{R}_m-\bar{R}_4\right)\gamma\bar{C}^{-\gamma}\frac{1}{\bar{E}}\hat{C}_{t+1}\hat{E}_{t+1}-\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{E}_{t+1}\hat{R}_{m,t+1}+\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{E}_{t+1}\hat{R}_{4,t+1}\\
&+\frac{1}{2}\left(\bar{R}_m-\bar{R}_4\right)\frac{1}{\bar{E}}\gamma^2\bar{C}^{-\gamma}\hat{C}^2_{t+1}+\frac{1}{2}\left(\bar{R}_m-\bar{R}_4\right)\frac{1}{\bar{E}}\bar{C}^{-\gamma}\hat{E}^2_{t+1}+\frac{1}{2}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{R}^2_{m,t+1}\\
&-\frac{1}{2}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{R}^2_{4,t+1}-\frac{3}{6}\left(\bar{R}_m-\bar{R}_4\right)\gamma^2\bar{C}^{-\gamma}\frac{1}{\bar{E}}\hat{C}^2_{t+1}\hat{E}_{t+1}+\frac{3}{6}\frac{1}{\bar{E}}\gamma^2\bar{C}^{-\gamma}\bar{R}\hat{C}^2_{t+1}\hat{R}_{m,t+1}\\
&-\frac{3}{6}\frac{1}{\bar{E}}\gamma^2\bar{C}^{-\gamma}\bar{R}\hat{C}^2_{t+1}\hat{R}_{4,t+1}-\frac{3}{6}\left(\bar{R}_m-\bar{R}_4\right)\gamma\bar{C}^{-\gamma}\frac{1}{\bar{E}}\hat{C}_{t+1}\hat{E}^2_{1,t+1}\\
&-\frac{3}{6}\frac{1}{\bar{E}}\gamma\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}\hat{R}^2_{m,T+1}+\frac{3}{6}\frac{1}{\bar{E}}\gamma\bar{C}^{-\gamma}\bar{R}\hat{C}_{t+1}\hat{R}^2_{4,t+1}\\
&+\frac{3}{6}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{E}^2_{1,t+1}\hat{R}_{m,t+1}-\frac{3}{6}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{E}^2_{t+1}\hat{R}_{4,t+1}-\frac{3}{6}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{E}_{t+1}\hat{R}^2_{m,t+1}\\
&+\frac{3}{6}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{E}_{t+1}\hat{R}^2_{4,t+1}-\frac{1}{6}\left(\bar{R}_m-\bar{R}_4\right)\frac{1}{\bar{E}}\gamma^3\bar{C}^{-\gamma}\hat{C}^3_{t+1}-\frac{1}{6}\left(\bar{R}_m-\bar{R}_4\right)\bar{C}^{-\gamma}\frac{1}{\bar{E}}\hat{E}^3_{t+1}\\
&+\frac{1}{6}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{R}^3_{m,t+1}-\frac{1}{6}\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{R}^3_{4,t+1}+\gamma\hat{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{C}_{t+1}\hat{E}_{t+1}\hat{R}_{m,t+1}\\
&-\gamma\bar{C}^{-\gamma}\frac{1}{\bar{E}}\bar{R}\hat{C}_{t+1}\hat{E}_{t+1}\hat{R}_{4,t+1}\Bigg]=0+O\left(\epsilon^4\right)
\end{aligned}
$$

Take the Difference of the portfolio choice equations to get

$$
\begin{aligned}
&\mathrm{E}\Bigg[\left(\hat{C}_{t+1}-\hat{C}^{*}_{t+1}-{}^{\hat{E}_{t+1}}/\gamma\right)\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\\
&-\frac{1}{2}\gamma\left(\hat{C}^2_{t+1}-\hat{C}^{2*}_{t+1}-{}^{\hat{E}^2_{t+1}}/\gamma^2\right)\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\\
&+\frac{1}{2}\left(\hat{C}_{t+1}-\hat{C}^{*}_{t+1}-{}^{\hat{E}_{t+1}}/\gamma\right)\left(\hat{R}^2_{m,t+1}-\hat{R}^2_{4,t+1}\right)\\
&\hat{C}^{*}_{t+1}\hat{E}_{t+1}\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\Bigg]=0+O\left(\epsilon^4\right)
\end{aligned}
$$

The first-order portfolio choice is the one that satisfies this equation.

Next, sum the Euler Equations to get

$$
\begin{aligned}
\mathrm{E}_t\left[\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right] \quad = \quad & \mathrm{E}_t\Bigg[-\frac{1}{2}\left(\hat{R}^2_{m,t+1}-\hat{R}^2_{4,t+1}\right)-\frac{1}{6}\left(\hat{R}^3_{m,t+1}-\hat{R}^3_{4,t+1}\right)\\
&+\frac{\gamma}{2}\left(\hat{C}_{t+1}+\hat{C}^{*}_{t+1}\right)\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\\
&-\frac{\gamma^2}{4}\left(\hat{C}^2_{t+1}+\hat{C}^{2*}_{t+1}\right)\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)+\frac{\gamma}{4}\left(\hat{C}_{t+1}+\hat{C}^{*}_{t+1}\right)\left(\hat{R}^2_{m,t+1}-\hat{R}^2_{4,t+1}\right)\\
&+\frac{1}{2}\hat{E}_{t+1}\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)-\frac{1}{4}\hat{E}^2_{t+1}\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\\
&+\frac{1}{4}\hat{E}_{t+1}\left(\hat{R}^2_{m,t+1}-\hat{R}^2_{4,t+1}\right)-\frac{1}{2}\gamma\hat{C}^{*}_{t+1}\hat{E}_{t+1}\left(\hat{R}_{m,t+1}-\hat{R}_{4,t+1}\right)\Bigg]=0+O\left(\epsilon^4\right)
\end{aligned}
$$

The state space solution for $\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - {}^{\hat{E}_{t+1}}/\gamma\right)$ and $\hat{r}_{x,t+1}$ can be expressed as:

$$\begin{aligned}\left(\hat{C} - \hat{C}^* - {}^{\hat{E}_{t+1}}/\gamma\right) &= \left[\tilde{D}_0\right] + \left[\tilde{D}_1\right]\xi + \left[\tilde{D}_2\right]_i [\epsilon]^i + \left[\tilde{D}_3\right]_k \left(\left[z^f\right]^k + [z^s]^k\right) + \left[\tilde{D}_4\right]_{ij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left[\tilde{D}_5\right]_{ki} [\epsilon]^i \left[z^f\right]^k + \left[\tilde{D}_6\right]_{ij} \left[z^f\right]^i \left[z^f\right]^j + O\left(\epsilon^3\right)\end{aligned}$$

$$\begin{aligned}[\hat{r}_x]_m &= \left[\tilde{R}_0\right]_m + \left[\tilde{R}_1\right]_m \xi + \left[\tilde{R}_2\right]_{mi} [\epsilon]^i + \left[\tilde{R}_3\right]_{mk} \left(\left[z^f\right]^k + [z^s]^k\right) + \left[\tilde{R}_4\right]_{mij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left[\tilde{R}_5\right]_{mki} [\epsilon]^i \left[z^f\right]^k + \left[\tilde{R}_6\right]_{mij} \left[z^f\right]^i \left[z^f\right]^j + O\left(\epsilon^3\right)\end{aligned}$$

Up to first-order accuracy, the expected excess return is zero and, up to second-order accuracy, it is a constant. This implies that $\left[\tilde{R}_3\right]_{mk} \left[z^f\right]^k = 0$ and the terms $\left[\tilde{R}_3\right]_{mk} [z^s]^k$ and $\left[\tilde{R}_6\right]_{mij} \left[z^f\right]^i \left[z^f\right]^j$ are constants. It also follows that

$$\left[\tilde{R}_0\right]_m = \mathrm{E}\,[\hat{r}_x]_m - \left[\tilde{R}_3\right]_{mk} [z^s]^k - \left[\tilde{R}_4\right]_{mij} [\Sigma]^{ij} - \left[\tilde{R}_6\right]_{mij} \left[z^f\right]^i \left[z^f\right]^j$$

so

$$\begin{aligned}[\hat{r}_x]_m &= \mathrm{E}\,[\hat{r}_x]_m - \left[\tilde{R}_4\right]_{mij} [\Sigma]^{ij} + \left[\tilde{R}_1\right]_m \xi + \left[\tilde{R}_2\right]_{mi} [\epsilon]^i + \left[\tilde{R}_4\right]_{mij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left[\tilde{R}_5\right]_{mki} [\epsilon]^i \left[z^f\right]^k + O\left(\epsilon^3\right)\end{aligned}$$

Now recognize that ξ is endogenous and given by $\xi = [\gamma]_{mk} \left[z^f\right]^k [\hat{r}_x]^m$. This is a second-order term , so $\hat{r}_x$ can be replaced by its first-order parts, i.e. by $\left[\tilde{R}_2\right]_{mi} [\epsilon]^i$. This implies that $\xi = \left[\tilde{R}_2\right]^m_i [\gamma]_{mk} [\epsilon]^i \left[z^f\right]^k$.

Now, we can write (note, here the asset index that's summed over is q, the one that's held fixed is m):

$$\begin{aligned}\left(\hat{C} - \hat{C}^* - {}^{\hat{E}_{t+1}}/\gamma\right) &= \left[\tilde{D}_0\right] + \left[\tilde{D}_2\right]_i [\epsilon]^i + \left[\tilde{D}_3\right]_k \left(\left[z^f\right]^k + [z^s]^k\right) + \left[\tilde{D}_4\right]_{ij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left(\left[\tilde{D}_5\right]_{ki} + \left[\tilde{D}_1\right] \left[\tilde{R}_2\right]^m_i [\gamma]_{mk}\right) [\epsilon]^i \left[z^f\right]^k + \left[\tilde{D}_6\right]_{ij} \left[z^f\right]^i \left[z^f\right]^j + O\left(\epsilon^3\right)\end{aligned}$$

$$\begin{aligned}[\hat{r}_x]_m &= \mathrm{E}\,[\hat{r}_x]_m - \left[\tilde{R}_4\right]_{mij} [\Sigma]^{ij} + \left[\tilde{R}_2\right]_{mi} [\epsilon]^i + \left[\tilde{R}_4\right]_{mij} [\epsilon]^i [\epsilon]^j \\ &\quad + \left(\left[\tilde{R}_5\right]_{mki} + \left[\tilde{R}_1\right]_m \left[\tilde{R}_2\right]^q_i [\gamma]_{qk}\right) [\epsilon]^i \left[z^f\right]^k + O\left(\epsilon^3\right)\end{aligned}$$

Furthermore, we use the following expressions for consumption

$$\hat{C} = \left[\tilde{C}_2^H\right]_i [\epsilon]^i + \left[\tilde{C}_3^H\right]_k \left[z^f\right]^k + O\left(\epsilon^2\right)$$

$$\hat{C}^* = \left[\tilde{C}_2^F\right]_i [\epsilon]^i + \left[\tilde{C}_3^F\right]_k \left[z^f\right]^k + O\left(\epsilon^2\right),$$

return to asset m

$$[\hat{r}]_m = \left[\tilde{R}_2^m\right]_i [\epsilon]^i + \left[\tilde{R}_3^m\right]_k \left[z^f\right]^k + O\left(\epsilon^2\right),$$

and the exchange rate

$$\hat{E} = \left[\tilde{H}_2\right]_i [\epsilon]^i + \left[\tilde{H}_3\right]_k \left[z^f\right]^k + O\left(\epsilon^2\right).$$

Fixing asset m, we get

$$\begin{aligned}
&\left[\tilde{D}_2\right]_i \left[\tilde{R}_2\right]_{mj} [\Sigma]^{ij} + \left[\tilde{D}_2\right]_i \left(\left[\tilde{R}_5\right]_{mkj} + \left[\tilde{R}_1\right]_m \left[\tilde{R}_2\right]_j^q [\gamma]_{qk}\right) [\Sigma]^{ij} \left[z^f\right]^k \\
&+ \left(\mathrm{E}\left[\hat{r}_x\right]_m - \left[\tilde{R}_4\right]_{mij} [\Sigma]^{ij}\right) \left[\tilde{D}_3\right]_k \left[z^f\right]^k + \left[\tilde{R}_2\right]_{mi} \left(\left[\tilde{D}_5\right]_{kj} + \left[\tilde{D}_1\right] \left[\tilde{R}_2\right]_j^q [\gamma]_{qk}\right) [\Sigma]^{ij} \left[z^f\right]^k \\
&+ \left[\tilde{R}_4\right]_{mij} \left[\tilde{D}_3\right]_k [\Sigma]^{ij} \left[z^f\right]^k - \gamma \left[\tilde{R}_2\right]_{mi} \left(\left[\tilde{C}_2^H\right]_j \left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_2^F\right]_j \left[\tilde{C}_3^F\right]_k\right) [\Sigma]^{ij} \left[z^f\right]^k \\
&+ \frac{1}{\gamma} \left[\tilde{R}_2\right]_{mi} \left[\tilde{H}_2\right]_j \left[\tilde{H}_3\right]_k [\Sigma]^{ij} \left[z^f\right]^k + \frac{1}{2} \left(\left[\tilde{R}_2^m\right]_i \left[\tilde{R}_2^m\right]_j - \left[\tilde{R}_2^4\right]_i \left[\tilde{R}_2^4\right]_j\right) \left[\tilde{D}_3\right]_k [\Sigma]^{ij} \left[z^f\right]^k \\
&+ \left[\tilde{D}_2\right]_i \left[\tilde{R}_2\right]_{mj} \left[\tilde{R}_3^m\right]_k [\Sigma]^{ij} \left[z^f\right]^k \\
&+ \left(\left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj} \left[\tilde{H}_3\right]_k + \left[\tilde{C}_3^F\right]_k \left[\tilde{R}_2\right]_{mj} \left[\tilde{H}_2\right]_i\right) [\Sigma]^{ij} \left[z^f\right]^k = 0 + O\left(\epsilon^4\right)
\end{aligned}$$

Since this is at the steady-state portfolio, $\left[\tilde{D}_2\right]_i \left[\tilde{R}_2\right]_{mj} [\Sigma]^{ij} = 0$ for all assets m and the above equation is homogeneous in $\left[z^f\right]^k$ so that the following equation must be satisfied for all k and m:

$$\left[\tilde{D}_2\right]_i \left(\left[\tilde{R}_5\right]_{mkj} + \left[\tilde{R}_1\right]_m \left[\tilde{R}_2\right]_j^q [\gamma]_{qk}\right) [\Sigma]^{ij}$$
$$+ \left(\mathrm{E}\left[\hat{r}_x\right]_m - \left[\tilde{R}_4\right]_{mij} [\Sigma]^{ij}\right) \left[\tilde{D}_3\right]_k + \left[\tilde{R}_2\right]_{mi} \left(\left[\tilde{D}_5\right]_{kj} + \left[\tilde{D}_1\right] \left[\tilde{R}_2\right]_j^q [\gamma]_{qk}\right) [\Sigma]^{ij}$$
$$+ \left[\tilde{R}_4\right]_{mij} \left[\tilde{D}_3\right]_k [\Sigma]^{ij} - \gamma \left[\tilde{R}_2\right]_{mi} \left(\left[\tilde{C}_2^H\right]_j \left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_2^F\right]_j \left[\tilde{C}_3^F\right]_k\right) [\Sigma]^{ij}$$
$$+\frac{1}{\gamma} \left[\tilde{R}_2\right]_{mi} \left[\tilde{H}_2\right]_j \left[\tilde{H}_3\right]_k [\Sigma]^{ij} + \frac{1}{2} \left(\left[\tilde{R}_2^m\right]_i \left[\tilde{R}_2^m\right]_j - \left[\tilde{R}_2^4\right]_i \left[\tilde{R}_2^4\right]_j\right) \left[\tilde{D}_3\right]_k [\Sigma]^{ij}$$
$$+ \left[\tilde{D}_2\right]_i \left[\tilde{R}_2\right]_{mj} \left[\tilde{R}_3^m\right]_k [\Sigma]^{ij}$$
$$+ \left(\left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj} \left[\tilde{H}_3\right]_k + \left[\tilde{C}_3^F\right]_k \left[\tilde{R}_2\right]_{mj} \left[\tilde{H}_2\right]_i\right) [\Sigma]^{ij} = 0 + O\left(\epsilon^4\right)$$

Furthermore, we can express the second order of the expected excess return of asset m as

$$\begin{aligned}\mathrm{E}_t\left[\hat{R}_{m,t+1} - \hat{R}_{4,t+1}\right] &= \mathrm{E}_t\left[-\frac{1}{2}\left(\hat{R}_{m,t+1}^2 - \hat{R}_{4,t+1}^2\right) + \frac{\gamma}{2}\left(\hat{C}_{t+1} + \hat{C}_{t+1}^*\right)\left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1}\right)\right.\\ &\quad \left. +\frac{1}{2}\hat{E}_{t+1}\left(\hat{R}_{m,t+1} - \hat{R}_{4,t+1}\right)\right] + O\left(\epsilon^3\right)\end{aligned}$$

Evaluating this using the first-order state-space solution for consumption, returns, and the exchange rate yields

$$\begin{aligned}\mathrm{E}_t\left[\hat{R}_{m,t+1} - \hat{R}_{4,t+1}\right] &= -\frac{1}{2}\left(\left[\tilde{R}_2^m\right]_i \left[\tilde{R}_2^m\right]_j [\Sigma]^{ij} - \left[\tilde{R}_2^4\right]_i \left[\tilde{R}_2^4\right]_j [\Sigma]^{ij}\right)\\ &\quad +\frac{\gamma}{2}\left(\left[\tilde{C}_2^H\right]_i + \left[\tilde{C}_2^F\right]_i\right)\left(\left[\tilde{R}_2^m\right]_j - \left[\tilde{R}_2^4\right]_j\right) [\Sigma]^{ij}\\ &\quad +\frac{1}{2}\left[\tilde{H}_2\right]_i \left(\left[\tilde{R}_2^m\right]_j - \left[\tilde{R}_2^4\right]_j\right) [\Sigma]^{ij} + O\left(\epsilon^3\right)\\ &= \frac{1}{2}\left(\left[\tilde{R}_2^4\right]_i \left[\tilde{R}_2^4\right]_j - \left[\tilde{R}_2^m\right]_i \left[\tilde{R}_2^m\right]_j\right.\\ &\quad +\gamma \left[\tilde{C}_2^H\right]_i \left[\tilde{R}_2\right]_{mj} + \gamma \left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj}\\ &\quad \left. + \left[\tilde{H}_2\right]_i \left[\tilde{R}_2\right]_{mj}\right) [\Sigma]^{ij} + O\left(\epsilon^3\right)\end{aligned}$$

Using this, we get

$$\begin{aligned}
\mathrm{E}\left[\hat{r}_x\right]_m \left[\tilde{D}_3\right]_k &= \mathrm{E}\left[\hat{r}_x\right]_m \left(\left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_3^F\right]_k - \frac{1}{\gamma}\left[\tilde{H}_3\right]_k\right) \\
&= \frac{1}{2}\left(\left[\tilde{R}_2^4\right]_i \left[\tilde{R}_2^4\right]_j - \left[\tilde{R}_2^m\right]_i \left[\tilde{R}_2^m\right]_j\right)\left(\left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_3^F\right]_k\right)[\Sigma]^{ij} \\
&\quad -\frac{1}{2}\frac{1}{\gamma}\left(\left[\tilde{R}_2^4\right]_i \left[\tilde{R}_2^4\right]_j - \left[\tilde{R}_2^m\right]_i \left[\tilde{R}_2^m\right]_j\right)\left[\tilde{H}_3\right]_k [\Sigma]^{ij} \\
&\quad +\frac{\gamma}{2}\left(\left[\tilde{C}_2^H\right]_i \left[\tilde{R}_2\right]_{mj} + \left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj}\right)\left(\left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_3^F\right]_k\right)[\Sigma]^{ij} \\
&\quad -\frac{1}{2}\left(\left[\tilde{C}_2^H\right]_i \left[\tilde{R}_2\right]_{mj} + \left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj}\right)\left[\tilde{H}_3\right]_k [\Sigma]^{ij} \\
&\quad +\frac{1}{2}\left[\tilde{H}_2\right]_i \left[\tilde{R}_2\right]_{mj}\left(\left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_3^F\right]_k - \frac{1}{\gamma}\left[\tilde{H}_3\right]_k\right)[\Sigma]^{ij}
\end{aligned}$$

It follows that

$$\begin{aligned}
& \left[\tilde{D}_2\right]_i \left(\left[\tilde{R}_5\right]_{mkj} + \left[\tilde{R}_1\right]_m \left[\tilde{R}_2\right]_j^q [\gamma]_{qk}\right)[\Sigma]^{ij} \\
+ & \left[\tilde{R}_2\right]_{mi}\left(\left[\tilde{D}_5\right]_{kj} + \left[\tilde{D}_1\right]\left[\tilde{R}_2\right]_j^m [\gamma]_{mk}\right)[\Sigma]^{ij} \\
+ & \left(\left[\tilde{C}_2^H\right]_i - \left[\tilde{C}_2^F\right]_i\right)\left[\tilde{R}_2\right]_{mj}\left[\tilde{R}_3^m\right]_k [\Sigma]^{ij} \\
- & \frac{\gamma}{2}\left[\tilde{R}_2\right]_{mj}\left(\left[\tilde{C}_2^H\right]_i - \left[\tilde{C}_2^F\right]_i\right)\left(\left[\tilde{C}_3^H\right]_k + \left[\tilde{C}_3^F\right]_k\right)[\Sigma]^{ij} \\
+ & \frac{1}{\gamma}\left[\tilde{R}_2\right]_{mi}\left[\tilde{H}_2\right]_j \left[\tilde{H}_3\right]_k [\Sigma]^{ij} - \frac{1}{2}\left(\left[\tilde{C}_2^H\right]_i \left[\tilde{R}_2\right]_{mj} + \left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj}\right)\left[\tilde{H}_3\right]_k [\Sigma]^{ij} \\
- & \frac{1}{\gamma}\left(\left[\tilde{H}_2\right]_i \left[\tilde{R}_2\right]_{mj}\left[\tilde{R}_3^m\right]_k - \left[\tilde{H}_2\right]_i \left[\tilde{R}_2\right]_{4j}\left[\tilde{R}_3^4\right]_k\right)[\Sigma]^{ij} \\
+ & \left(\left[\tilde{C}_2^F\right]_i \left[\tilde{R}_2\right]_{mj}\left[\tilde{H}_3\right]_k + \left[\tilde{C}_3^F\right]_k \left[\tilde{R}_2\right]_{mj}\left[\tilde{H}_2\right]_i\right)[\Sigma]^{ij} \\
+ & \frac{1}{2}\left[\tilde{H}_2\right]_i \left[\tilde{R}_2\right]_{mj}\left(\left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_3^F\right]_k - \frac{1}{\gamma}\left[\tilde{H}_3\right]_k\right)[\Sigma]^{ij} = 0
\end{aligned}$$

Next, using the fact that $\left[\tilde{D}_2\right]_i \left[\tilde{R}_2\right]_{mj} [\Sigma]^{ij} = 0$ and that $\left[\tilde{R}_2\right]_{4j} = 0$, this simplifies to

$$\begin{aligned}
& \left[\tilde{D}_2\right]_i \left[\tilde{R}_5\right]_{mkj} [\Sigma]^{ij} \\
+ & \left[\tilde{R}_2\right]_{mi}\left(\left[\tilde{D}_5\right]_{kj} + \left[\tilde{D}_1\right]\left[\tilde{R}_2\right]_j^m [\gamma]_{mk}\right)[\Sigma]^{ij} \\
- & \frac{\gamma}{2}\left[\tilde{R}_2\right]_{mj}\left(\left[\tilde{C}_2^H\right]_i - \left[\tilde{C}_2^F\right]_i\right)\left(\left[\tilde{C}_3^H\right]_k + \left[\tilde{C}_3^F\right]_k\right)[\Sigma]^{ij} \\
+ & \left(\left[\tilde{C}_3^F\right]_k \left[\tilde{R}_2\right]_{mj}\left[\tilde{H}_2\right]_i\right)[\Sigma]^{ij} \\
+ & \frac{1}{2}\left[\tilde{H}_2\right]_i \left[\tilde{R}_2\right]_{mj}\left(\left[\tilde{C}_3^H\right]_k - \left[\tilde{C}_3^F\right]_k\right)[\Sigma]^{ij} = 0,
\end{aligned}$$

which can be solved f or $[\gamma]_{mk}$.